Nonlinear Optimal Controller of Three-Phase Induction Motor Using Quasilinearization

Jasem M. Kh. Tamimi and Hussein M. Jaddu

***Abstract*— This paper presents the solution of the nonlinear optimal control problem of three-phase induction motor (IM). A third order nonlinear model described in arbitrary rotating frame of induction motor is used in this paper along with a quadratic performance index. The problem is solved using the quasilineraization approach which converts the nonlinear optimal control problem into sequence of linear quadratic optimal control problems. The optimal trajectories of fluxes, speed, currents, and torque that represent the model states and controls of IM are presented in this paper.**

I. INTRODUCTION

The Induction motors (IM) is widely used in industrial application because of its robustness, efficiency, size, reliability and cost [1], [2], [3], [4].

Induction motors driven by static converters are widely used in industrial applications [1], [2], due to the progress in power electronics.

On the other hand, the induction motor is highly nonlinear system, and it requires complex control algorithm to control. One of these algorithms is called Field Oriented Control (FOC) or Vector Control algorithm.

Vector control algorithm uses a dynamic equivalent circuit of the induction motor and convert the three-phase stator currents into two DC currents, then the motor speed and torque are calculated similar to DC motor. Vector control algorithm uses two primary famous transformations, first is called *Clark* transformation that transform the three stator currents into two DC current in a stationary frame (*DQ*), then converted into two DC currents in arbitrary rotating frame (*dq*) using *Park* transformation [3], [4].

Based on the vector control algorithm many researchers proposed methods to control the induction motor. For example, Kim *et al* [5] presented a theoretical and experimental comparison between two recent nonlinear controllers for speed regulation of current-fed induction motors: the passivity-based controller (PBC) and the observer based-adaptive controller (OBAC).While Hovingh *et al [6]* presented an algorithm to estimate the rotor's speed and torque from the terminal voltage and input current to the motor. They showed that measurement of the stator voltage and currents are sufficient to determine the rotor position, speed and torque of an induction motor during any conditions, whether transient or steady state. Their work is being performed to analyze the response of a Field Orientated Control system when the estimated waveforms are used as an input into the control loop.

Moreover, Georges *et al* [7], [9] presented a nonlinear control design for both the H_2 and H_∞ optimal control for current-fed induction motor drives, they derived the controller using the analytical stationary solutions obtained in [8] that minimizes a generalized convex energy cost function including the stored magnetic energy and the coil losses, they present experimental results of a nonlinear torque-flux optimal control for induction motor drive, and their controller is a cascade-based scheme with tree main loops: an inner high-gain current control loop that permits reduce the motor model into the reduced-order current-fed induction motor model; a middle loop for torque and flux tracking and an external loop for speed tracking control;

In this paper the quasilineraization approach is used to solve the nonlinear optimal control problem of the induction motor. the method is based on converting the nonlinear optimal control problem into sequence of linear quadratic optimal control problems which can be solved by solving sequence of Riccati equations. The optimal trajectories of the induction motor's fluxes , currents , torque, and speed are presented in this paper.

This paper consists of six parts, parts two and three describe the induction motor nonlinear model and the optimal control problem, respectively. While part four presents the problem formulation and the quasilinearization of the induction motor problem. The simulation and some conclusion remards are presented in parts five and six respectively .

II. INDUCTION MOTOR MODEL:

The differential equations that describe the dynamics of induction motor can be obtained by reducing the motor into two-axis coil (*dq*) model on both stator and rotor as described by Krause and Thoms [10].

Figure 1 shows the d-q equivalent circuit for a three phase symmetrical squirrel cage induction motor in arbitrary rotating frame with zero sequence component neglected [3], [11], [12].

The nonlinear differential equations that describe the dynamics of an ideal symetrical inductin motor in a rotating frame is as follows [3], [11]:

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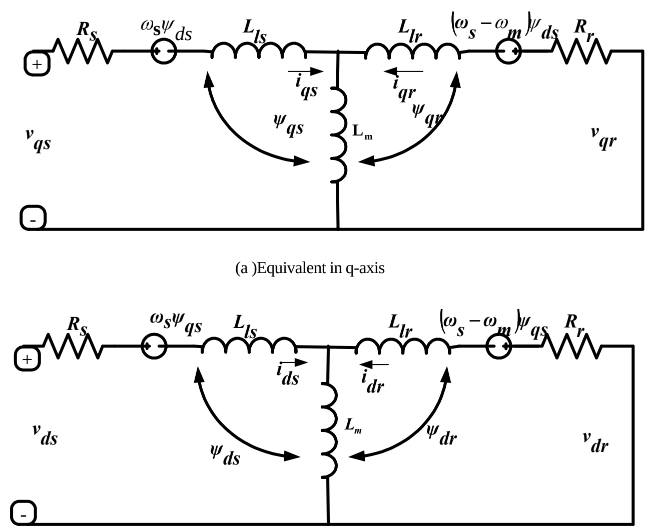
J. M. Kh. Tamimi is with Faculty of Engineering, Alquds University , P.O. Box 20002, Abu Dies, Jerusalem, Palestine. jmt_tam@yahoo.com

H. M. Jaddu is with Faculty Engineering, Alquds University, P.O. Box 20002, Abu Dies, Jerusalem, Palestine. jaddu@eng.alquds.edu



(a)Equivalent in q-axis

(b)Equivalent in d-axis

Fig. 1. Dynamic equivalent circuit for induction motor.

$$\begin{bmatrix} v_{qs} \\ v_{ds} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} R_s + pL_s & \omega_s L_s & pL_m & \omega L_m \\ -\omega_s L_s & R_s + pL_s & -\omega_s L_s & -\omega_s L_s \\ -\omega_s L_s & (\omega_{s-}\omega_m) L_m & R_r + pL_r & (\omega_{s-}\omega_m) L_m \\ -(\omega_{s-}\omega_m) & pL_m & -(\omega_{s-}\omega_m) & R_r + pL_r \end{bmatrix} \times \begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{qr} \\ i_{dr} \end{bmatrix} \quad (1)$$

Where R_s, R_r are stator, rotor resistance per phase respectively, L_s, L_r are stator, rotor inductance per phase respectively, $p = \frac{d}{dt}$ operator, ω_s, ω_m are synchronous and rotor speeds respectively.

Moreover, the rotor flux linkages are given by equations (2)&(3):

$$\begin{bmatrix} \psi_{qr} \\ \psi_{dr} \end{bmatrix} = \begin{bmatrix} i_{qr} & i_{qs} \\ i_{dr} & i_{ds} \end{bmatrix} \begin{bmatrix} L_r \\ L_m \end{bmatrix} \quad (2)$$

Solving for i_{qr}, i_{dr} we obtain:

$$\begin{bmatrix} i_{qr} \\ i_{dr} \end{bmatrix} = \begin{bmatrix} \psi_{qr} & i_{qs} \\ \psi_{dr} & i_{ds} \end{bmatrix} \begin{bmatrix} \frac{1}{L_r} \\ \frac{-L_m}{L_r} \end{bmatrix} \quad (3)$$

Substituting equation (2) in equation (1) and using equation (3), the following state equation model can be obtained:[3]

$$\begin{bmatrix} \dot{\psi}_{qr} \\ \dot{\psi}_{dr} \end{bmatrix} = \begin{bmatrix} -\frac{L_r}{R_r} & -(\omega_{s-}\omega_m) \\ (\omega_{s-}\omega_m) & -\frac{L_r}{R_r} \end{bmatrix} \begin{bmatrix} \psi_{qr} \\ \psi_{dr} \end{bmatrix} + \frac{L_m}{L_r} R_r \begin{bmatrix} i_{qs} \\ i_{ds} \end{bmatrix} \quad (4)$$

And from the result presented in [13] the following velocity differential equation is obtained

$$\dot{\omega} = \frac{z^2 L_m}{I L_r} \left(i_{sq}\psi_{rd} - i_{sd}\psi_{rq} \right) - \frac{F}{I}\omega_m - \frac{z}{I} T_l \quad (5)$$

where *z* is the number of the poles of the induction motor, *I* is the moment of inertia, *F* is the viscous friction coefficient, T_l is the torque load.

Rewriting equations (4) and (5) in matrix form, we get the following differential equation:

$$\begin{bmatrix} \dot{\psi}_{qr} \\ \dot{\psi}_{dr} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} -\frac{L_r}{R_r} & -\omega_s & \psi_{dr} \\ \omega_s & -\frac{L_r}{R_r} & -\psi_{qr} \\ -\frac{z^2 L_m}{I L_r} i_{sd} & \frac{z^2 L_m}{I L_r} i_{sq} & -\frac{F}{I} \end{bmatrix} \begin{bmatrix} \psi_{qr} \\ \psi_{dr} \\ \omega_m \end{bmatrix} + \begin{bmatrix} \frac{L_m}{L_r} R_r i_{qs} \\ \frac{L_m}{L_r} R_r i_{ds} \\ -\frac{z}{I} T_l \end{bmatrix} \quad (6)$$

Equation (6) represents the nonlinear third order model for the induction motor which can be rewritten in a compact form: $\dot{x} = A(x,u,t)x(t) + B(t)u(t)$,where $x(t) = \begin{bmatrix} \psi_{qr} & \psi_{dr} & \omega_m \end{bmatrix}^T$ and $u(t) = \begin{bmatrix} i_{qs} & i_{ds} & T_l \end{bmatrix}^T$.

III. Optimal Control problem:

The standard theory of the optimal control is presented in [14], [15], [16]. For nonlinear system described by state space with state vector (*x*) and control vector (*u*):

$$\dot{x}(t) = A(x,u,t).x(t) + B(x,u,t).u(t) \quad (7)$$

The optimal state feedback control can be given by:

$$u^* = f\left(x(t), t\right) \quad (8)$$

And the function *f* can be found by minmizing the quadratic performance index :

$$J = \frac{1}{2} \int_0^{t_f} \left(x^T Q x + u^T R u \right) dt \quad (9)$$

Where *Q* is positive semi definite real symmetric state weighting matrix, and *R* is positive definite real symmetric control weighting matrix.

There are several methods to solve this nonlinear optimal control problem such as direct method, discretization method, parametrization method, and quazilinearization method [17], [18], [19], [20].

To solve this optimal control problem using quasilnearization there are two approaches [16], [21]. The first one is to convert the nonlinear two point boundary value problem (TPBVP) as presented in

[16], into sequence of linear two point boundary value problems. The second approach is to replace the nonlinear optimal control problem by sequence of linear quadratic optimal control problems [21].

In this paper, the second approach is used. By linearizing the nonlinear state equation (7) around nominal trajectories we get:

$$\dot{x}^{(k+1)} = A^{(k)}(t)\, x^{(k+1)}(t) + B^{k}(t)\, u^{(k+1)}(t) + h^{(k)}(t) \tag{10}$$

where $x^{(0)}(t)$, $u^{(0)}(t)$ are the initial guess, and script k is the iteration number. Therefore, the nonlinear optimal control problem become:

Minmize

$$J = \frac{1}{2}\int_{0}^{t_f} \left(x^{(k+1)^T} Q x^{(k+1)} + u^{(k+1)^T} R u^{(k+1)} \right) dt \tag{11}$$

Subject to state equation (10).

And

$$h^{(k)}(t) = \dot{x}^{(k)} - A^{(k)}(t)\, x^{(k)}(t) + B^{k}(t)\, u^{(k)}(t) \tag{12}$$

$$A^{(k)} = \frac{\partial \dot{x}(x,u,t)}{\partial x} |_{x^{(k)},u^{(k)}} \tag{13}$$

$$B^{(k)} = \frac{\partial \dot{x}(x,u,t)}{\partial u} |_{x^{(k)},u^{(k)}} \tag{14}$$

In [15] this problem is solved by considering the term $h^{(k)}$ is a disturbance input for the linear system, so that the optimal control is :

$$u^{*(k+1)} = -K^{(k+1)} x^{(k+1)} + R^{-1} B^{(k)} v^{(k+1)} \tag{15}$$

$$K^{(k+1)} = R^{-1} B^{(k)^T} S^{(k+1)} \tag{16}$$

while $S^{(k+1)}$ is the solution of algebraic Riccati equation:

$$A^{(k)^T} S^{(k+1)} + S^{(k+1)} A^{(k)} - S^{(k+1)} B^{(k)} R^{-1} B^{(k)^T} S^{(k+1)}$$

$$+Q = 0 \tag{17}$$

and

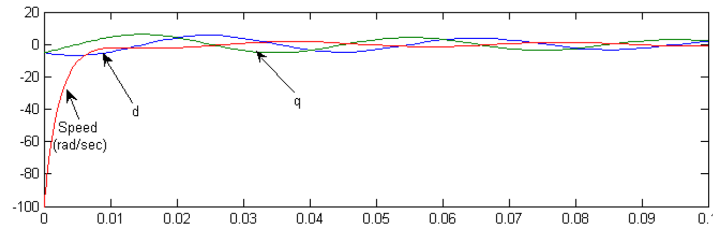
$$v^{(k+1)} = -(A^{(k)} - B^{(k)} K^{(k+1)^{T^{-1}}} S^{(k+1)} h^{(k)} \tag{18}$$

By finding *K* and *v* for several iterations we minimize total energy in the induction motor which is the sum of the stored magnetic energy in the inductance, the dissipated energy in the rotor and stator resistances, the dissipated energy due to core losses (Foucault currents and magnetic hysteresis), and the mechanical energy . [7], [8]

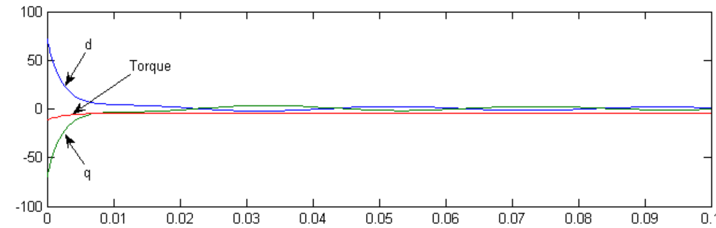
IV. Problem Formulation:

To convert our optimal control problem of induction motor to sequnce of linear quadratic optimal control problems we first linearize the state space equation (6), to get:

$$\begin{bmatrix} \dot{\psi}_{qr}^{(k+1)} \\ \dot{\psi}_{dr}^{(k+1)} \\ \dot{\omega}_{m}^{(k+1)} \end{bmatrix} = A^{(k)}(t) \begin{bmatrix} \psi_{qr}^{(k+1)} \\ \psi_{dr}^{(k+1)} \\ \omega_{m}^{(k+1)} \end{bmatrix}$$



(a) Speed and Fluxes



(b) Torque and currents

Fig. 2. Simulation Result

$$+B^{(k)}(t) \begin{bmatrix} i_{qs}^{(k+1)} \\ i_{ds}^{(k+1)} \\ T_{l}^{(k+1)} \end{bmatrix} + h^{(k)} \tag{19}$$

$$A^{(k)}(t) = \begin{bmatrix} -\frac{L_r}{R_r} & -\omega_s + \omega_m^{(k)} & \psi_{dr}^{(k)} \\ \omega_s + \omega_m^{(k)} & -\frac{L_r}{R_r} & -\psi_{qr}^{(k)} \\ -\dfrac{z^2 L_m}{I L_r} i_{sd}^{(k)} & \dfrac{z^2 L_m}{I L_r} i_{sq}^{(k)} & -\frac{F}{I} \end{bmatrix} \tag{20}$$

$$B^{(k)}(t) = \begin{bmatrix} -\frac{L_m}{L_r} R_r & 0 & 0 \\ 0 & -\frac{L_m}{L_r} R_r & 0 \\ \dfrac{z^2 L_m}{I L_r} \psi_{dr}^{(k)} & -\dfrac{z^2 L_m}{I L_r} \psi_{qr}^{(k)} & -\frac{z}{I} \end{bmatrix} \tag{21}$$

$$h^{(k)}(t) = \begin{bmatrix} -\omega_s \psi_{dr}^{(k)} - (-\omega_s + \omega_m^{(k)}) \psi_{dr}^{(k)} \\ \omega_s \psi_{qr}^{(k)} - (\omega_s - \omega_m^{(k)}) \psi_{qr}^{(k)} \\ -\frac{z^2}{I} \frac{L_m}{L_r} i_{qs}^{(k)} \psi_{dr}^{(k)} + \frac{z^2}{I} \frac{L_m}{L_r} i_{ds}^{(k)} \psi_{qr}^{(k)} \end{bmatrix} \tag{22}$$

Then the problem become: Minmize performance index equation (11) subject to state equation (19) .

V. Simulation

To demonstrate the solution of the problem above, a digital simulation program using MATLAB is implemented.

With weighting matrices: $Q = \begin{bmatrix} q_1 & 0 & 0 \\ 0 & q_2 & 0 \\ 0 & 0 & q_3 \end{bmatrix}$, *and* $R = \begin{bmatrix} r_1 & 0 & 0 \\ 0 & r_2 & 0 \\ 0 & 0 & r_3 \end{bmatrix}$ with their eigenvalues $q_1, q_2, q_3 > 0$ and $r_1, r_2, r_3 > 0$.

The simulation has been performed using five sets of Q and R; $q_1, q_2, q_3 = 0.01, 0.1, 1, 10, 100$, and $r_1, r_2, r_3 = 0.01, 0.1, 1, 10, 100$ respectively, with intial gusse: $\begin{bmatrix} \psi_{qr} & \psi_{dr} & \omega_m \end{bmatrix} = \begin{bmatrix} -5 & -5 & -100 \end{bmatrix}$, $\begin{bmatrix} i_{qs} & i_{ds} & T_l \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$, and with motor parameters: Rated speed =1440 rpm, z= 2 poles, R_s= 1.15 Ω, R_r =1.44 Ω, L_m=0.144 H, L_s=L_r = 0.156 H , $I = 0.013$ kg.m^2, $F = 0.002$ Nm.s/rad.

Figure 2 shows the simulation result for our problem, Figure 2a shows the induction motor states (fluxes and speed), figure 2b shows the induction motor controls (currents and torque) at the second iteration of the presented algorithm

VI. Conclusion:

In this paper we obtained the optimal trajectories of states (IM fluxes, and speed) and controls (IM currents, and torque). These trajectories are obtained by minmizing the quadratic performance measure or total energy of the induction motor.

Since the induction motor was used has nonlinear model, the qusilinearization method is used to solve this problem by solving a sequnce of linear quadratic optimal control problem.

The simulation is carefully done to obtain the controls and states trajectories using matlab program.

Using different state and control weighting matrices (Q and R) didn't affect the response of fluxes, currents, speed, and torque as shown in figures (2c and 2c) , and this shows the stability and robustness for the IM state feedback system.

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