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The Effect of Hall Current on Magnetohydrodynamics Flow

Haytham Taha Sulieman

Master of Science Thesis

Hebron, Palestine

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Haytham Taha Sulieman

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Supervisor: Dr. Amjad Barham

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The Effect of Hall Current on Magnetohydrodynamics Flow

By

Student Name: Haytham Taha Sulieman

Supervisor: Dr. Amjad Barham

Master thesis submitted and accepted, Date: 30/6/2009.

The names and signatures of the examining committee members are as follows :

1. Dr. Amjad Barham	Head of committee Signature	
2. Dr. Nureddeen Rabie	Internal Examiner Signature	
3. Dr. Naji Qatanani	External Examiner Signature	N. Qatanani

Palestine Polytechnic University

2009

DECLARATION

I, Haytham Sulieman, am fully aware of all the consequences and declare, under oath and penalty of plagiarism that this topic is my own endeavor. It has never been, by any means of the matter, submitted or published in any form ever before.

Signed ……………………………………………..

Date: 30/06/2009

DEDICATION

To my parents,

My wife,

My children,

For their patience and emotional support.

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I am very grateful to all of those who facilitated the procedure used in tackling this rugged important topic, especially my advisor Dr. Amjad Barham. Moreover, I acknowledge of the great assistance which prof. Edriss Titti provided me with.

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ABSTRACT

The importance of this research penetrates in the fact that it sheds light on an unprecedented Mathematical Topic that has been raised for along time, but never utilized modern technology from a certain point of view. The topic of Hall Current is being deeply fathomed under various conditions and circumstances, especially its effects on the MHD flow. Several mathematical equations have been discussed and applied deeply by various techniques. Take the Laplace transform, the "Mathematica 6" program and many other ways of tackling this rugged topic as an example.

A strong example of the personal techniques we used is epitomized in finding out the velocity components, in the tabulated values, and in the illustrated graphics that correspond to each case.

This research deeply tackles the Hall Effect on MHD Rayleigh problem, on the unsteady MHD flow past an oscillating porous plate, and on the MHD flow between two horizontal plates, the lower plate being a stretching sheet.

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NOMENCLATURE

MFD	Magnetic fluid dynamics.
MHD	Magnetic hydrodynamics.
$\overline{E}$	Electric field.
H_o	uniform transverse magnetic field
H	Magnetic field intensity.
J_X, J_Y, J_Z	Components of current density.
m ,N	Hall parameter.
$\overline{q}$	Velocity number.
w_e	Cyclotron frequency.
μ_e	Magnetic permeability.
P_e	Thermo elastic pressure.
P	Pressure
B_0	Applied magnetic field.
e	Charge of electron.
$\vec{J}$	Current density vector (J_X, J_Y, J_Z).

M	Hartmann number.
m_e	Mass density of particles.
n_e	Electron number density.
Re	Reynolds number.
s	Laplace transform variable.
u,v	Fluid velocity components.
U_0	Non-dimensional velocity components.
ρ	Fluid density.
λ	constant with the velocity vector
σ	Electrical conductivity.
μ	Fluid viscosity.
ω_0	Electron cyclotron frequency.
τ	Electron mean free time.
erf	Error function.
erfc	Error function complement.
ODE	Ordinary differential equation
BVP	Boundary value problem
$\delta(t)$	Derac Delta function

Introduction

When the magnetic field diffuses easily through the conducting medium and when the frequency of collision of charge particles is large compared to their frequency of rotation about the magnetic field lines, the current in the medium in controlled by the resistance of the medium and in such a case the generalized Ohm's law is the appropriate law applicable. However, if these conditions are not fulfilled, additional terms will appear on the left hand side of the generalized Ohm's law,

$$\vec{J}=\sigma(\vec{E}+\vec{v}\times\vec{B})$$

The electrical current density $\vec{J}$ represents the relative motion of the charged particles in a fluid and may be derived from the diffusion velocities of the charged particles. Thus, give rise to an electrical current density $\vec{J}_{Hall}$ known as Hall current, given by:

$\vec{J}_{Hall}=\sigma\frac{\vec{J}\times\vec{B}}{n_e.e}$. Where n_e is the electron number density and e is the charge of the electron.

To be very précis and concise, Hall current sustains a major change in the conversation of the flow field to make it change from dimensional into non-dimensional. The Hall effect diminishes the flow of the current in the direction of the electric filed. Therefore the current flows normally in both the electric and the magnetic fields. To illustrate, the flow is induced by an impulsive start of an infinite plane under the action of strong transverse magnetic field. The solution of the equations governing the Hall effects has been obtained in a closed form using calculus techniques.

The solution obtained influences of non-dimensional parameters. Eventually, one notices that Hall current delayed the attainment of steady state. Moreover electrical current density $\bar{J}$ represents the relative motion of charged particles in a fluid. When we apply an electrical field $\bar{E}$,there will be an electrical current in the direction of $\bar{E}$ if the magnetic field $\bar{B}$ is perpendicular to $\bar{E}$, there will be an electromagnetic force $\bar{J}\times\bar{B}$ perpendicular to both $\bar{E}$ and $\bar{B}$, which will cause the charged particles to move in its own direction, known as the Hall effect. It is usually measured by the parameter $\omega\tau$, where $\omega = eB/m$ is the angular velocity and τ is the mean time.

Magnetohydrodynamics (MHD) (magnetofluiddynamics or hydromagnat-ics) is the academic discipline which studies the dynamics of electrically conducting fluids. Examples of such fluids include plasmas, liquid metal-s, and salt water. The word Magnetohydrodynamics (MHD) is derived from magneto- meaning magnetic field, and hydro- meaning liquid, and dynamics meaning movement. The field of MHD was initiated by Hannes Alven, for which he received the Nobel Prize in physics in 1970. The idea of MHD is that magnetic fields can induce currents in a moving conductive fluid, which create forces on the fluid, and also change the magnetic field itself. The set of equations which describe MHD are a combination of the Navier-Stokes equations of fluid dynamics and Maxwell's equations of electromagnetism. These differential equations have to be solved simultaneously, either analytically or numerically. Only in a few special cases exact solutions can be obtained where we have to make certain assumptions about the state of the fluid and certain simple configuration for the pattern is to be considered. One of the most important assumptions is that the compressibility effects of the medium are considered to be negligible. For example, fluid is taken as incompressible and the other fluid properties such as viscosity, thermal conductivity and electrical conductivity are regarded as constants. In such a case the word MHD is used instead of MFD. An additional advantage of MHD flow is that the equations of motion decouple from the equation of energy and therefore, mathematically, it is much easier to handle the problems of MHD flow. In recent years, MHD channel flows and their application to propulsion systems and electric power generation are receiving a considerable amount of scientific attention. In the initial stages, the channel flows were described by assuming Ohm's law without Hall effects. But in the case of very strong magnetic fields, where the electron cyclotron frequency is much greater than the electron collision frequency, Hall effects become significant. Hence, the generalized Ohm's law must be used in solving problems on channel flows. Recently, attempts have been made to describe the channel flows without neglecting Hall effects. Harris and Cobine described the advantages and disadvantages of three types of channel configurations depending upon the arrangement of electrodes. One of the arrangements suggested is the segmentation of electrodes which prevents the flow of axial currents. Sherman and Sutton [16] described the flow of conducting fluids moving in a channel with the segmented electrodes. In their treatment of the problem, the generalized ohm's law, without ion-slip and pressure-diffusion effects, was assumed and the problem was solved with reference to the effects of Hall current on the velocity field, magnetic field, and etc...

Rayleigh's problem, the unsteady flow, due to impulsive motion of an infinite plate in a fluid of infinite extent, is one of the classical but the simplest problems admitting an exact solution for the Navier-Stokes equation, and is well reported in the literature, by Schlichting[15]. For the first time, Rossow[13] extended it to Magnetohydrodynamics for electrically conducting fluid neglecting the induced field. The problem has attracted the attention of many subsequent investigators, namely Lundford[11], Change and Yen and Yang[5] and now this problem is included in standard texts in Magnetofluiddynamics, Bansal[3], However, it is always desirable to reinvestigate or to extend the simple exact solutions, for which they provide an insight into more complex problems, which are more realistic in nature and not an approximate model of real world problem. This has motivated us to reinvestigate this problem for bringing more clearly essential features of magneto fluid-dynamic flows.

Stoke's problem of the unsteady flow, which is due to impulsive motion of an infinite plate in a fluid of infinite extent, is one of the classical but the simplest problems admitting an exact solution of the Navier-stokes equations, and is well reported in the literature Schlichting[15], Rossow[13] who studied the flow of electrically conducting fluid over a flat plate in the presence of transverse magnetic field. Kakutani[10], Ong and Nicholls[12] extended Rossow's[13] work and considered the case of an oscillating flat plate. The problem has attracted attention of many subsequent investigators and now this problem is included in standard texts in Magnetofluiddynamics. Take Bansal[3] as a vivid example of those investigators.

Hall effects on steady MHD flow between two parallel plates have been studied by Yaminishi[17], Sherman and Sutton[2]. These effects in unsteady flows were discussed in Sakhnovskii[14], Borkakati and Bharali[4] have also discussed the hydromagnetic flow between two plates when the lower plate stretches uniformly by introducing two equal and opposite forces.

This thesis consists of three chapters:

Chapter One: discussed the MHD Rayleigh problem including Hall effects, and from the given equation of motion (1.2.3) and the generalized Ohm's law (1.2.6) under the initial and the boundary conditions. With the help of Laplace transform, we find the velocity components (**u** , **w**) , including the influence of Hall effect on these components.

Chapter Two: centered around the unsteady MHD flow past an oscillating porous plate including Hall effect, and from equations (2.2.1), (2.2.4) conditions, (2.2.13) and Laplace transform techniques, we conclude the velocity components (**u, v**) and the effects of Hall Currents on hydromagnetic flow in oscillating porous plate.

Chapter Three: described Hall effects on MHD flow between two horizontal plates, the lower plate being a stretching sheet. From equations (3.2.2), (3.2.3) using the idea of Similar Solution and the boundary conditions (3.3.3) with the help of Mathematica program, we obtain the velocity profiles $(f_0(x) + f_1(x))$ and the influence of Hall parameter on these profiles.

CHAPTER ONE

MHD Rayleigh Problem Including Hall Effects

1.1 Introduction

In this chapter we consider the effects of Hall-Currents on MFD version of classical Rayleigh's problem. Hall-Current causes a major change in character of the problem in the sense that the flow field becomes two dimensional from the non-dimensional.

The Hall effect reduces the current in the direction of the electric field and causes a current to flow normally to both the electric and the magnetic fields. This current interacts with the applied magnetic field to include a transverse motion of the fluid.

The equations governing the flow, when reduced to non-dimensional form, include two dimensionless parameters, Hartmann number and Hall parameter. The equations are coupled, but by making a suitable transformation they are decoupled and are reduced into a single equation. The equations are solved using Laplace transform techniques.

1.2 Problem Formulation

Consider the flow of a viscous incompressible and electrically conducting fluid of infinite extant caused by impulsive motion of an infinite, insulated flat plate with uniform velocity U_0 in its own plane, which is otherwise at rest, in the presence of the uniform transverse magnetic field H_0 fixed in the fluid. In the framework of rectangular Cartesian coordinate (x, y, z) let, the plate be at $y = 0$ and applied magnetic field towards y-axis and the flow is in the positive direction of x-axis. The physical configuration and the nature of the flow suggest the following form of velocity vector $\vec{q}$, magnetic induction vector $\overline{H}$, electrostatic field $\overline{E}$ and pressure P, thus:

$$\left.\begin{aligned} \vec{q} &= (u, 0, w) \\ \overline{H} &= (H_x, H_0, H_z) \\ \overline{E} &= (E_x, 0, E_z) \\ P &= \text{constant} \end{aligned}\right\} \quad (1.2.1)$$

The equations governing the unsteady flow and Maxwell's equations are:

Equation of continuity:

$$\nabla . q = 0 \quad (1.2.2)$$

Equation of motion:

$$\frac{\partial q}{\partial t} + (\nabla . q) . q = -\frac{1}{\rho} \nabla p + \nu \nabla^2 q + \frac{1}{\rho} J \times H \quad (1.2.3)$$

Equation for current:

$$\nabla \times H = \mu J \quad (1.2.4)$$

Faraday's Law:

$$\nabla \times E = -\frac{\partial H}{\partial t}, \nabla . H = 0 \quad (1.2.5)$$

The generalized Ohm's law, taking Hall Effect into account, is

$$\frac{J}{\sigma} = (E + \vec{q} \times H) - \frac{(J \times H)}{n_e . e} \quad (1.2.6)$$

Where $\sigma = \frac{e^2 \tau n}{m}$ (is the electrical conductivity).

Here J is the current density, t is the time, ρ, ν, and μ stand for the density, the kinematics viscosity, and the magnetic permeability, e and m are the electric charge and the mass of an electron, n is the electron number density and τ is the mean collision time. We have ignored the ion-slip effects and electron pressure gradient.

The Lorentz force per unit volume is given by:

$$\vec{J} \times \vec{H} = [\, -J_z H_0, J_z H_x - J_x H_z, J_x H_0] \qquad (1.2.7)$$

moreover:

$$\vec{q} \times \vec{H} = [\, -\omega H_0, \omega H_x - uH_z, uH_0\,] \qquad (1.2.8)$$

where:

$$\vec{J} = [\, J_x,\ 0,\ J_z\,]$$

is given by:

$$J_x = \sigma\, [\, E_x - wH_0 + \frac{J_z H_0}{n_e . e}\,]$$

or,

$$J_x = \frac{\sigma}{1 + \omega^2 \tau^2} [\, E_x - wH_0 + \omega\tau\, (E_z + uH_0)\,] \qquad (1.2.9)$$

$$J_z = \sigma\, [\, E_z + uH_0 - \frac{J_x H_0}{n_e . e}\,]$$

or,

$$J_z = \frac{\sigma}{1 + \omega^2 \tau^2} [\, E_z - uH_0 - \omega\tau\, (E_x - wH_0)\,] \qquad (1.2.10)$$

where $\omega = \frac{eH_0}{m}$ (is the electron Larmor frequency).

The initial and boundary conditions are:

$$\left.\begin{array}{llll} t \le 0: & u = 0, & w = 0 & \text{for } y \ge 0 \\ t > 0: & u = U_0, & w = 0 & \text{for } y = 0 \\ u \to 0: & w = 0 & \text{as } y \to \infty & \\ H_x \to 0 & H_y = H_0, & H_z \to 0 & \text{as } y \to \infty \end{array}\right\} \qquad (1.2.11)$$

The magnetic induction is uniform at infinity with components $(0, H, 0)$, the current density $J = \frac{1}{\mu} (\nabla \times H)$ is identically zero as $y \to \infty$ and since the free stream is at rest, it follows from generalized Ohm's law that $E = 0$ at $y \to \infty$. Assuming small magnetic Reynolds's number for the flow, the induced magnetic field is neglected in comparison to the applied constant field H_0.

In order to put the equations governing the unsteady flow in universal form, we introduce the following non-dimensional quantities:

$$y^* = \frac{U_0 \cdot y}{\nu}, \; u^* = \frac{u}{U_0}, \; w^* = \frac{w}{U_0}, \; t^* = \frac{U_0^2 t}{\nu} \qquad (1.2.12)$$

so, we can write $u(y, t) = u^*(y^*, t^*)$

$$\frac{\partial u}{\partial t}(y,t) = \frac{\partial u^*}{\partial t^*}(y^*,t^*) = \frac{U_0^2}{\nu}\frac{\partial u^*}{\partial t^*} \quad \text{and} \quad \frac{\partial^2 u}{\partial y^2} = \frac{U_0^2}{\nu^2}\frac{\partial^2 u^*}{\partial y^{*2}}$$

in this case:

$$\frac{1}{\rho}(J \times H) = \frac{-1}{\rho}(J_z H_0)$$

$$\frac{1}{\rho}(J \times H) = -\frac{1}{1+N^2}\left[\frac{\sigma H_0^2}{\rho}(u+Nw)\right] \qquad (1.2.13)$$

Similarly, $w(y,t) = w^*(y^*,t^*)$

$$\frac{\partial w}{\partial t} = \frac{U_0^2}{\nu}\frac{\partial w^*}{\partial t^*} \quad \text{and} \quad \frac{\partial^2 w}{\partial y^2} = \frac{U_0^2}{\nu}\frac{\partial^2 w^*}{\partial y^{*2}}$$

in this case:

$$\frac{1}{\rho}(J \times H) = \frac{1}{\rho}(J_x H_0)$$

$$\frac{1}{\rho}(J \times H) = \frac{1}{1+N^2}\left[\frac{\sigma H_0^2}{\rho}(Nu - w)\right] \qquad (1.2.14)$$

The equation of motion (1.2.3) in component term becomes (dropping the stars):

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} - \frac{M^2}{1+N^2}(u + Nw) \qquad (1.2.15)$$

$$\frac{\partial w}{\partial t} = \frac{\partial^2 w}{\partial y^2} + \frac{M^2}{1+N^2}(Nu - w) \qquad (1.2.16)$$

where $M^2 = \dfrac{\sigma H_0^2 \nu}{\rho U_0^2}$ is the Hartmann number and $N = \omega\tau$ is the Hall parameter. The initial and boundary conditions become:

$$\left.\begin{aligned} &u(0, y) = w(0, y) = 0 \\ &u(t, 0) = 1,\ w(t, 0) = 0 \\ &u(t, y) \text{ and } w(t, y) \to 0 \text{ as } y \to \infty \end{aligned}\right\} \qquad (1.2.17)$$

Now, multiply both sides of equation (1.2.15) and (1.2.16) by e^{-st} and integrate from 0 to ∞ with respect to t we get:

$$\frac{d^2 \hat{u}}{d y^2} - \left(\frac{M^2}{1 + N^2} + s\right)\hat{u} = \frac{NM^2}{1 + N^2}\hat{w} \qquad (1.2.18)$$

$$\frac{d^2 \hat{w}}{d y^2} - \left(\frac{M^2}{1 + N^2} + s\right)\hat{w} = -\frac{NM^2}{1 + N^2}\hat{u} \qquad (1.2.19)$$

where:

$$\hat{u}(s, y) = L\{u(t, y)\} = \int_0^\infty u(t, y)\, e^{-st}\, dt$$

$$\hat{w}(s, y) = L\{w(t, y)\} = \int_0^\infty w(t, y)\, e^{-st}\, dt$$

We introduce the variables $\hat{q} = \hat{u} + i\hat{w}$, and then equations (1.2.18) and (1.2.19) can be combined into single equation:

$$\frac{d^2 \hat{q}}{d y^2} - \left\{\frac{M^2}{1 + N^2}(1 - iN) + s\right\}\hat{q} = 0 \qquad (1.2.20)$$

1.3 Analytic Solution

Equations (1.2.15) and (1.2.16) along with boundary conditions (1.2.17) are amenable for solution by usual transform techniques.

From (1.2.15) and (2.2.16) we get:

$$\frac{\partial q}{\partial t} = \frac{\partial^2 q}{\partial y^2} - \left(\frac{M^2}{1+N^2}\right)(1 - iN)\, q \qquad (1.3.1)$$

where:

$$q = u + iw$$

and the boundary condition becomes:

$$\left.\begin{aligned} q(0, y) = 0, \quad q(t, 0) = 1 \\ q(t, y) \to 0 \quad \text{as } y \to \infty \end{aligned}\right\} \qquad (1.3.2)$$

let: $-\dfrac{M^2}{1+N^2}(1 - iN) = \alpha$ which is a complex number.

now, equation (1.3.1) can be written as:

$$\frac{\partial q}{\partial t} = \frac{\partial^2 q}{\partial y^2} + \alpha q \qquad (1.3.3)$$

assume:

$$\Phi(t, y) = e^{-\alpha t} q(t, y) \qquad 1.3.4)$$

now multiplying (1.3.3) by ($e^{-\alpha t}$) we get:

$$\frac{\partial \Phi}{\partial t} = \frac{\partial^2 \Phi}{\partial y^2} \qquad (1.3.5)$$

From equations (1.3.2) and (1.3.4) we conclude that:

$$\left.\begin{aligned} \Phi(0, y) = 0, \quad \Phi(t, 0) = e^{-\alpha t} \\ \Phi(t, y) \to 0 \quad \text{as } y \to \infty \end{aligned}\right\} \qquad (1.3.6)$$

Now , by taking the Laplace transform of equation (1.3.5) we have:

$$L\{ \frac{\partial \Phi}{\partial t} \} = L\{ \frac{\partial^2 \Phi}{\partial y^2} \}$$

$$s\hat{\Phi}(s, y) = \frac{d^2 \hat{\Phi}}{dy^2} \tag{1.3.7}$$

where:

$$\hat{\Phi}(s, y) = L\{ \Phi(t, y) \}$$

$$\hat{\Phi}(s, 0) = \frac{1}{s + \alpha}$$

$$\lim_{y \to \infty} \hat{\Phi}(s, y) = 0 \tag{1.3.8}$$

The auxiliary equation for equation (1.3.7) can be written as:

$$\beta^2 - s = 0 \tag{1.3.9}$$

hence,

$$\hat{\Phi}(s, y) = C_1 e^{-\sqrt{s}\, y} + C_2 e^{\sqrt{s}\, y} \tag{1.3.10}$$

Claim $C_2 = 0$

Proof of claim:

divide both sides of equation (1.3.10) by $e^{\sqrt{s}\, y}$

$$e^{-\sqrt{s}\, y} \hat{\Phi}(s, y) = C_1 e^{-2\sqrt{s}\, y} + C_2$$

now ,taking the limit of both sides of the above equation as $y \to \infty$:

$$0.0 = C_2 + C_1 . 0 \quad \text{i.e. } C_2 = 0$$

furthermore:

$$\hat{\Phi}(s, y) = C_1 e^{-\sqrt{s}\, y} \tag{1.3.11}$$

putting $y = 0$ in equation (1.3.11) and from equation (1.3.8), we come up with:

$$\hat{\Phi}(s, 0) = C_1 . 1 = \frac{1}{s + \alpha} \qquad \text{i.e. } C_1 = \frac{1}{s + \alpha}$$

hence,

$$\hat{\Phi}(s, y) = \frac{1}{s+\alpha} e^{-\sqrt{s}\, y} \qquad (1.3.12)$$

$$\Phi(t, y) = L^{-1}\{\hat{\Phi}(s, y)\} = L^{-1}\left\{\frac{s}{s+\alpha} \cdot \frac{e^{-\sqrt{s}\, y}}{s}\right\} \qquad (1.3.13)$$

Now, we use the following fact about Laplace transformation:

$$L\left\{\int_0^t f(t-\tau)\, g(\tau)\, d(\tau)\right\} = L\{f(t)\} . L\{g(t)\} = \hat{f}(s)\, \hat{g}(s)$$

where:

$$f(t) = L^{-1}\left\{1 - \frac{\alpha}{s+\alpha}\right\} = \delta(t) - \alpha\, e^{-\alpha t}$$

$$g(t) = L^{-1}\left\{\frac{e^{-\sqrt{s} y}}{s}\right\} = \operatorname{erfc}\left(\frac{y}{2\sqrt{t}}\right) = \frac{2}{\sqrt{\pi}} \int_{\frac{y}{2\sqrt{t}}}^{\infty} e^{-u^2}\, du$$

thus,

$$\Phi(t, y) = L^{-1}\left\{\frac{s}{s+\alpha} . \frac{e^{-\sqrt{s} y}}{s}\right\} = \int_0^t [\, \delta(t-\tau) - \alpha\, e^{-\alpha(t-\tau)}\,] \times \left[\frac{2}{\sqrt{\pi}} \int_{\frac{y}{2\sqrt{\tau}}}^{\infty} e^{-u^2} du\right] d\tau$$

$$\Phi(t, y) = \frac{2}{\sqrt{\pi}} \int_{\frac{y}{2\sqrt{t}}}^{\infty} e^{-u^2}\, du - \frac{2\alpha\, e^{-\alpha t}}{\sqrt{\pi}} \int_0^t \left(e^{\alpha\tau} \int_{\frac{y}{2\sqrt{\tau}}}^{\infty} e^{-u^2}\, du\right) d\tau \qquad (1.3.14)$$

Recall,

$$q(t, y) = e^{\alpha t}\, \Phi(t, y)$$

therefore:

$$q(t, y) = e^{\alpha t} \operatorname{erfc}\left(\frac{y}{2\sqrt{t}}\right) - \alpha \int_0^t e^{\alpha\tau} \operatorname{erfc}\left(\frac{y}{2\sqrt{\tau}}\right) d\tau \qquad (1.3.15)$$

where:

$$\operatorname{erfc}(x) = 1 - \operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-u^2}\, du$$

now, we want to write $q(t, y)$ as $u + iw$

furthermore,

$$q(t, y) = e^{at} \cos bt \ \mathrm{erfc}(\frac{y}{2\sqrt{t}}) - \int_0^t e^{a\tau} \mathrm{erfc}(\frac{y}{2\sqrt{\tau}})$$

$$[a\cos(b\tau) - b\sin(b\tau)]\, d\tau + i[\, e^{at} \sin(bt)\, \mathrm{erfc}(\frac{y}{2\sqrt{t}})$$

$$- \int_0^t e^{a\tau} \mathrm{erfc}(\frac{y}{2\sqrt{\tau}})\, [a\sin(b\tau) + b\cos(b\tau)]]\, d\tau \qquad (1.3.16)$$

where:

$$\alpha = a + ib$$

and

$$a = -\frac{M^2}{1 + N^2}, \quad b = \frac{M^2 N}{1 + N^2}$$

1.4 Numerical Solution for Second Order BVP

The boundary-value problem in equation (1.2.20) can be stated as:

$$\frac{d^2 \hat{q}}{d y^2} - \omega \hat{q} = 0$$

$$\hat{q}(0, s) = \frac{1}{s}, \quad \hat{q}(\infty, s) = 0$$

where:

$$\omega = (\frac{M^2}{1 + N^2} + s) - i\frac{N M^2}{1 + N^2}.$$

To ensure that the Laplace Transforms are well-defined, it should be assumed that $s > 0$. Thus , $\mathrm{Re}(\omega) = \frac{M^2}{1 + N^2} + s > 0$. This implies that there exists η in the complex number such that $\eta^2 = \omega$ with $\mathrm{Re}(\eta) < 0$ furthermore,

$$\hat{q}(y, s) = \frac{e^{\eta y}}{s}$$

Satisfies the boundary value problem as stated above.

In the case where **y = 0** we have:

$$\hat{q}(0, s) = \frac{1}{s} = \int_0^\infty 1 \,.\, e^{-st}\, dt = \int_0^\infty (1 + 0i)\, e^{-st}\, dt.$$

Thus,

$$u(0, t) \equiv 1 \quad \text{and} \quad w(0, t) \equiv 0 \ \text{for all } t$$

- **Inverse Laplace Transform**

Recall that the inverse Laplace Transform is:

$$q(y, t) = \frac{1}{2\pi i} \int_{\gamma - i\infty}^{\gamma + i\infty} \hat{q}(y, s)\, e^{st}\, ds$$

Where $\gamma > 0$ is chosen so that all the singularities of $\hat{q}(y, s)$ are to the left of γ. The above integral is over the vertical line $z = \gamma$ in the complex plane. Since $\hat{q}(y, s) = \frac{e^{\eta y}}{s}$, we can choose γ to be any positive number. In the calculations below we choose $\gamma = 0.25$. We will define q strictly as a function of t using Mathematics' NIntegrate command. We will approximate the integral above by integrating from $0.25 - 500\, i$ to $0.25 + 500\, i$.

We also define ω as $\left(\frac{M^2}{1 + N^2} + s\right) - i\, \frac{N M^2}{1 + N^2}$,

where $M^2 = \frac{\sigma H_0^2 \nu}{\rho U_0^2}$ is the Hartmann number ,

and $N = \omega\tau$ is the Hall parameter.

Next, we'll define the range of t values as required in cases **(1.a-5.a)**

$t = \{0.4, 0.8, 1.2, 1.6, 2, 3, 4, 5, 6, 7, 8, 9\}$

Case 1.a

In this case, M= 1, N = 1/2, and y = 0.

Next, we obtain the value η such that $\eta^2 = \omega$ with $Re(\eta) < 0$.

t	u(t)	w(t)
0.4	0.9992	0
0.8	1.0005	0
1.2	1.0007	0
1.6	1.0003	0
2	0.9997	0
3	1	0
4	1.0002	0
5	0.9997	0
6	1.0005	0
7	0.9995	0
8	1.0004	0
9	0.9998	0

Table (11): The velocity components u and w for different values of t

As the above table indicates $u(t) = Re(q(t)) \approx 1$ and

$w(t) = Im(q(t)) \approx 0$ for all t.

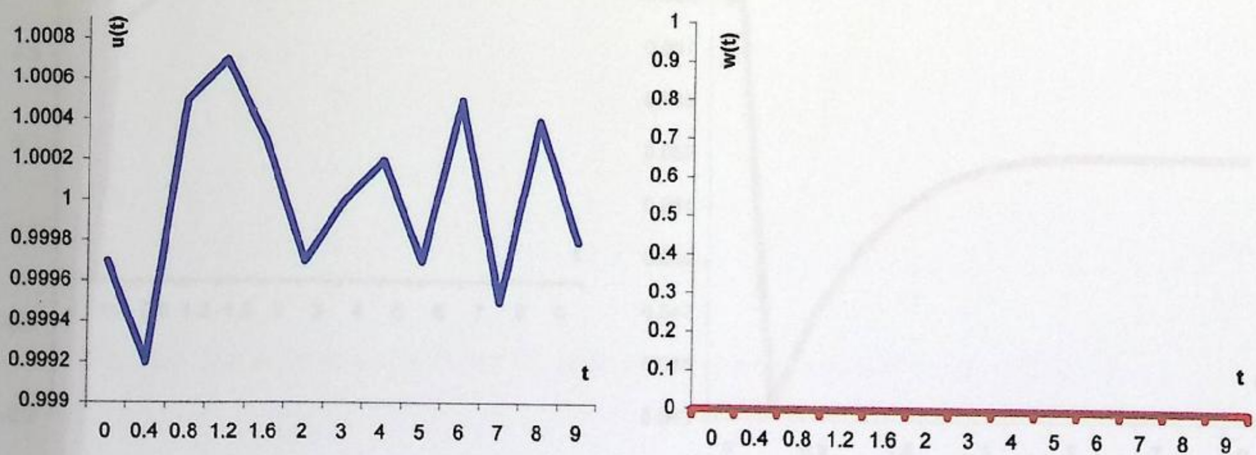


Fig 1.1: The velocity components for different values of t when M=1; N =1/2; y = 0

Case 2.a

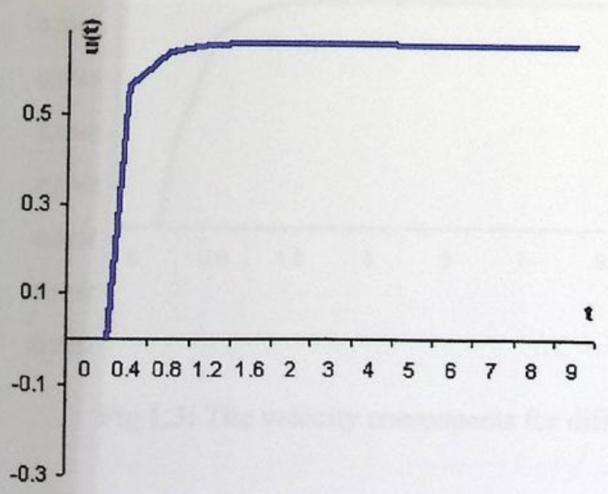
Define the variables:

$M = 1; \quad N = 1/2; \quad y = 1/2;$

Next, we obtain the value η such that $\eta^2 = \omega$ with $\mathrm{Re}(\eta) < 0$.

t	u(t)	w(t)
0.4	0.5622	0.0435
0.8	0.6357	0.0476
1.2	0.6528	0.0468
1.6	0.6578	0.051
2	0.6588	0.0517
3	0.6574	0.0526
4	0.6573	0.0529
5	0.6561	0.053
6	0.6564	0.053
7	0.6562	0.053
8	0.6558	0.053
9	0.6567	0.053

Table (1.2): The velocity components u and w for different values of t



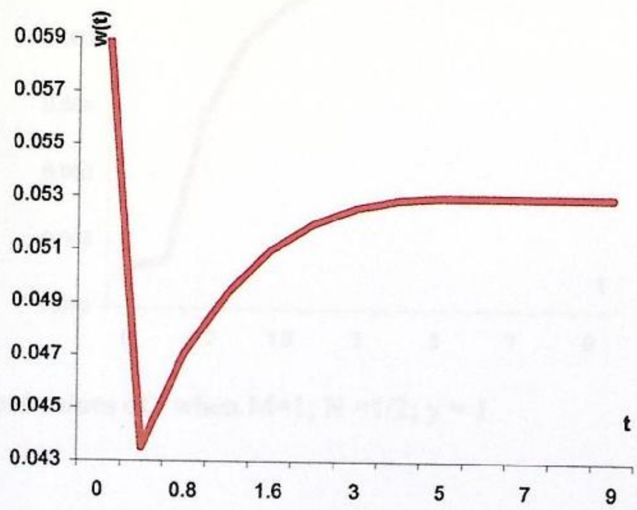


Fig 1.2: The velocity components for different values of t when M=1; N =1/2; y = 1/2

Case 3.a

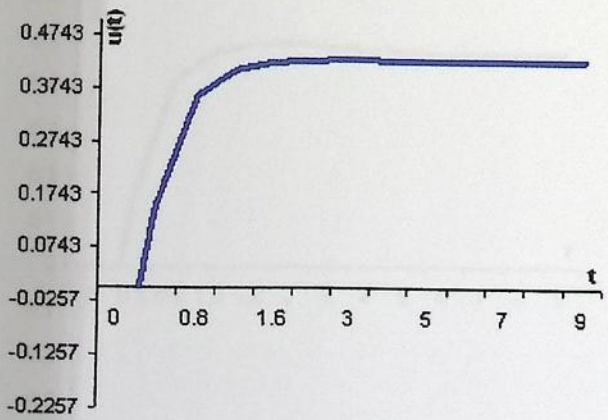
Define the variables:

$M=1; \quad N=1/2; \quad y=1;$

Next, we obtain the value η such that $\eta^2 = \omega$ with $Re(\eta) < 0$.

t	u(t)	w(t)
0.4	0.1525	0.0467
0.8	0.3621	0.0576
1.2	0.4099	0.0628
1.6	0.4251	0.0656
2	0.4302	0.0671
3	0.4303	0.0688
4	0.4291	0.0693
5	0.4285	0.0695
6	0.4275	0.0696
7	0.4283	0.0696
8	0.4272	0.0696
9	0.4281	0.0696

Table (1.3): The velocity components u and w for different values of t



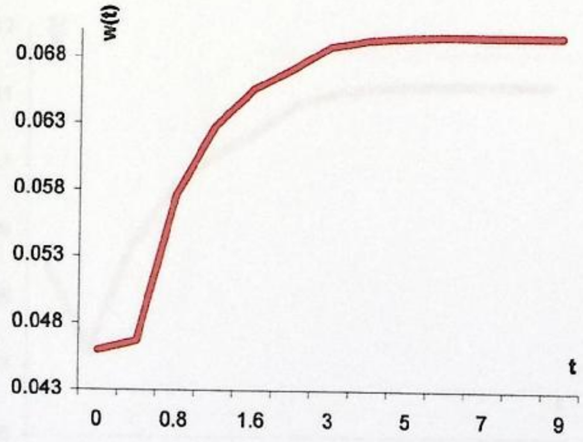


Fig 1.3: The velocity components for different values of t when M=1; N =1/2; y = 1

Case 4.a

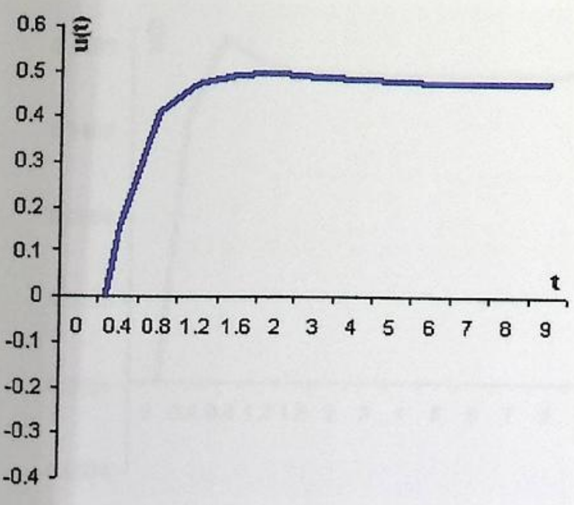
Define the variables:

$M=1; \quad N=1 \; ; y=1;$

Next, we obtain the value η such that $\eta^2 = \omega$ with $Re(\eta) < 0$.

t	u(t)	w(t)
0.4	0.1517	0.0723
0.8	0.4088	0.0879
1.2	0.4726	0.0964
1.6	0.4927	0.1016
2	0.498	0.1049
3	0.4922	0.1092
4	0.4849	0.1108
5	0.4804	0.1115
6	0.4775	0.1118
7	0.4774	0.1119
8	0.4762	0.1119
9	0.4771	0.1119

Table (1.4): The velocity components u and w for different values of t



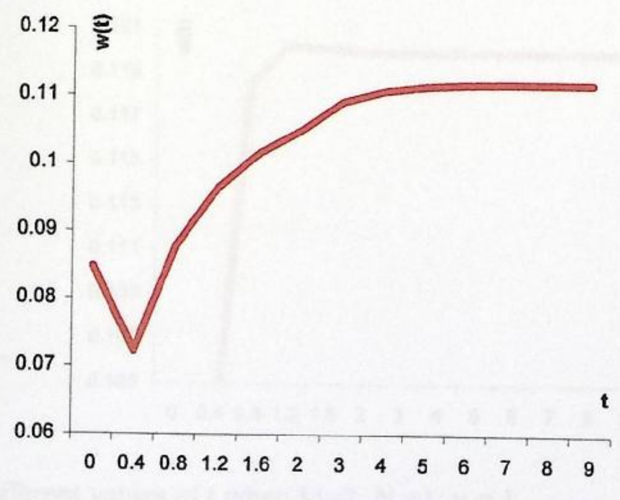


Fig 1.4: The velocity components for different values of t when M=1; N =1; y = 1

Case 5.a

Define the variables:

$M=2$; $N=1$; $y=1$;

Next, we obtain the value η such that $\eta^2 = \omega$ with $Re(\eta) < 0$.

t	u(t)	w(t)
0.4	0.1561	0.1008
0.8	0.2027	0.1185
1.2	0.1896	0.1202
1.6	0.1834	0.1201
2	0.1823	0.1199
3	0.1819	0.1199
4	0.1821	0.1199
5	0.1823	0.1199
6	0.1818	0.1199
7	0.1826	0.1199
8	0.1816	0.1199
9	0.1825	0.1199

Table (1.5): The velocity components u and w for different values of t

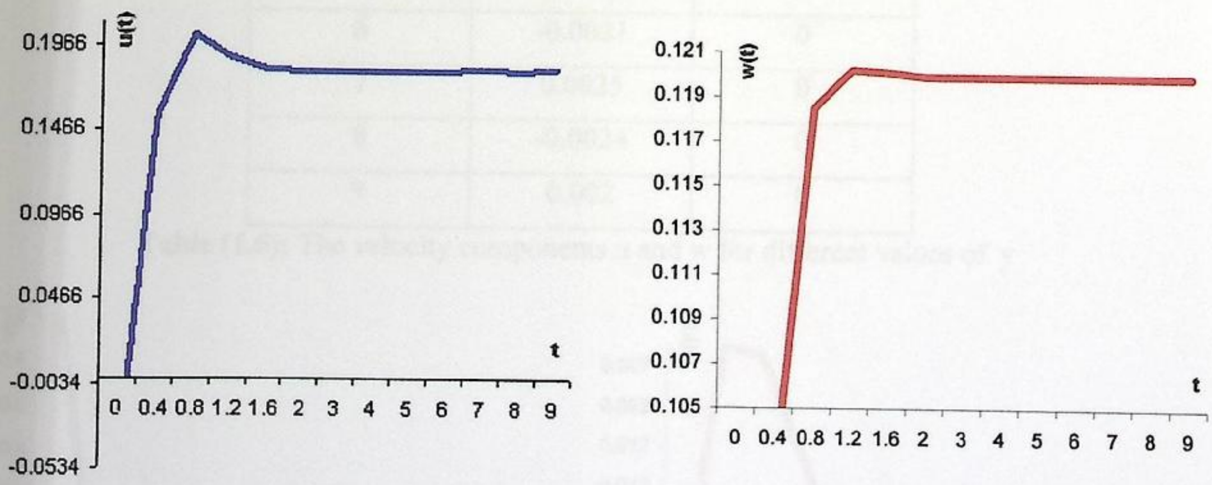


Fig 1.5: The velocity components for different values of t when M=2; N =1; y = 1

For the next table we'll redefine q as a function of y.

$$q(y,t) = \frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} \hat{q}(y,s)\, e^{st}\, ds \quad \text{and} \quad \omega := \left(\frac{M^2}{1+N^2} + s\right) - i\frac{N M^2}{1+N^2}$$

Case 1.b

Define the variables:

$M=1; \quad N=1/2; \quad t=0.05;$

Next, we define ω and obtain the value η such that $\eta^2 = \omega$ with $Re(\eta) < 0$.

y	u(y)	w(y)
0	0.9997	0
0.4	-0.39	0.0298
0.8	-0.1314	0.0286
1.2	-0.0465	0.0167
1.6	-0.0082	0.0059
2	0.0175	0.0001
3	-0.0023	-0.0011
4	0.0003	0
5	0.0012	0
6	-0.0021	0
7	0.0025	0
8	-0.0024	0
9	0.002	0

Table (1.6): The velocity components u and w for different values of y

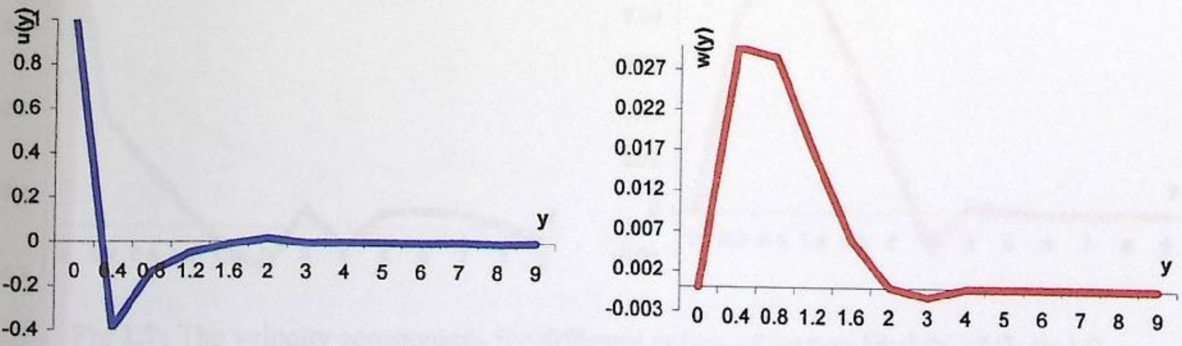


Fig 1.6: The velocity components for different values of t when M=1; N=1/2; t= 0.05

Case 2.b

Define the variables:

$M=1; \quad N=1/2; \quad t=1/2;$

Next, we define ω and obtain the value η such that $\eta^2 = \omega$ with $Re(\eta) < 0$.

y	u(y)	w(y)
0	1.9993	0
0.4	0.6713	0.0398
0.8	0.3791	0.0515
1.2	0.1129	0.0458
1.6	-0.1213	0.0301
2	-0.2506	0.0113
3	0.1163	-0.0079
4	-0.1269	0.0017
5	0.0937	0.0007
6	0.1066	-0.0001
7	0.0883	0
8	0.0165	0.0003
9	-0.0636	-0.0001

Table (1.7): The velocity components u and w for different values of y

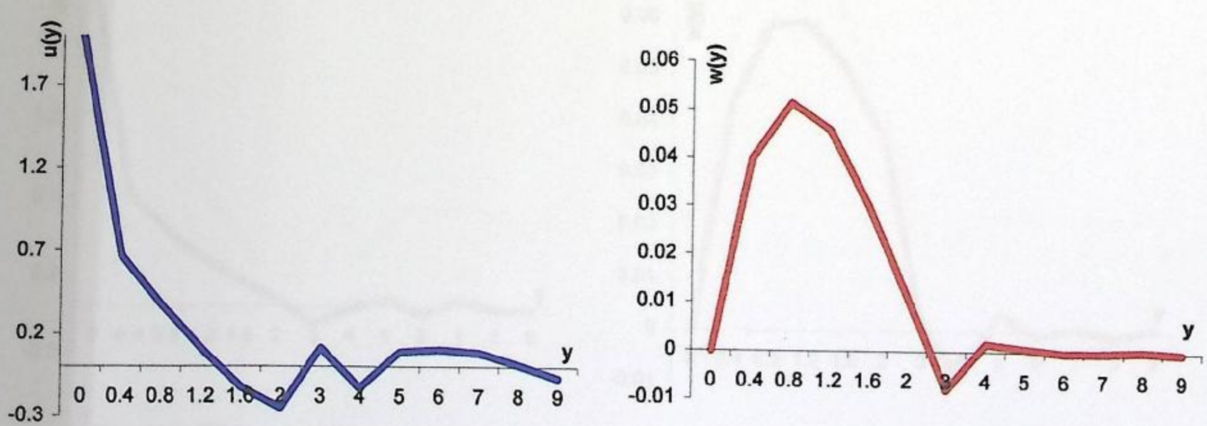


Fig 1.7: The velocity components for different values of t when M=1;N =1/2; t= 1/2

Case 3.b

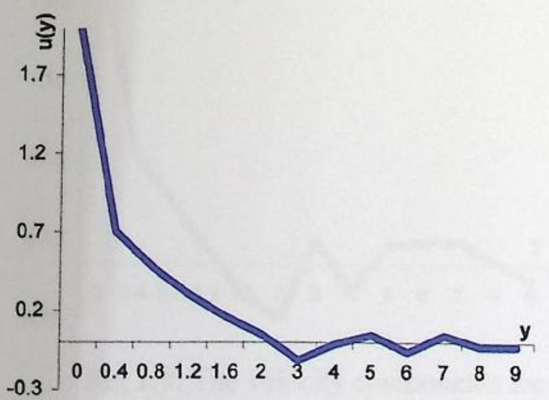
Define the variables:

$M=1; \quad N=1/2; \quad t=1;$

Next, we define ω and obtain the value η such that $\eta^2 = \omega$ with $Re(\eta) < 0$.

y	u(y)	w(y)
0	2.0014	0
0.4	0.7076	0.0428
0.8	0.4862	0.059
1.2	0.3116	0.0593
1.6	0.1685	0.0504
2	0.0528	0.0367
3	-0.108	0.0009
4	-0.0053	-0.0091
5	0.0534	0.0034
6	-0.0555	-0.0008
7	0.46	0.0007
8	-0.0224	-0.001
9	-0.0215	0.0007

Table (1.8): The velocity components u and w for different values of y



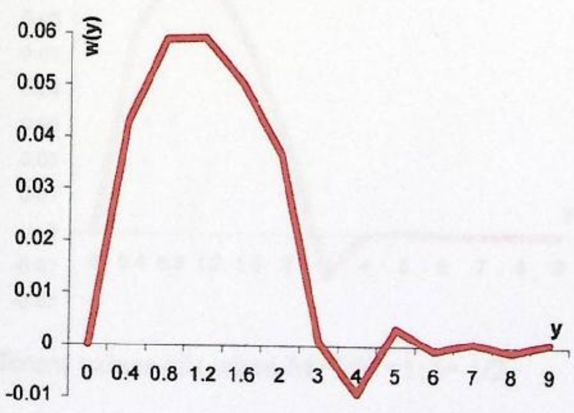


Fig 1.8: The velocity components for different values of t when M=1; N =1/2; t= 1

Case 4.b

Define the variables:

$M = 1; \quad N = 1; \quad t = 1/2;$

Next, we define ω and obtain the value η such that $\eta^2 = \omega$ with $Re(\eta) < 0$.

y	u(y)	w(y)
0	1.9993	0
0.4	0.708	0.0558
0.8	0.4111	0.0766
1.2	0.1076	0.0729
1.6	-0.1789	0.053
2	-0.3431	0.0259
3	0.16	-0.0114
4	-0.1691	-0.0003
5	0.1272	0.0009
6	0.1435	0
7	-0.1195	0
8	0.0224	0.0003
9	-0.0859	-0.0001

Table (1. 9): The velocity components u and w for different values of y

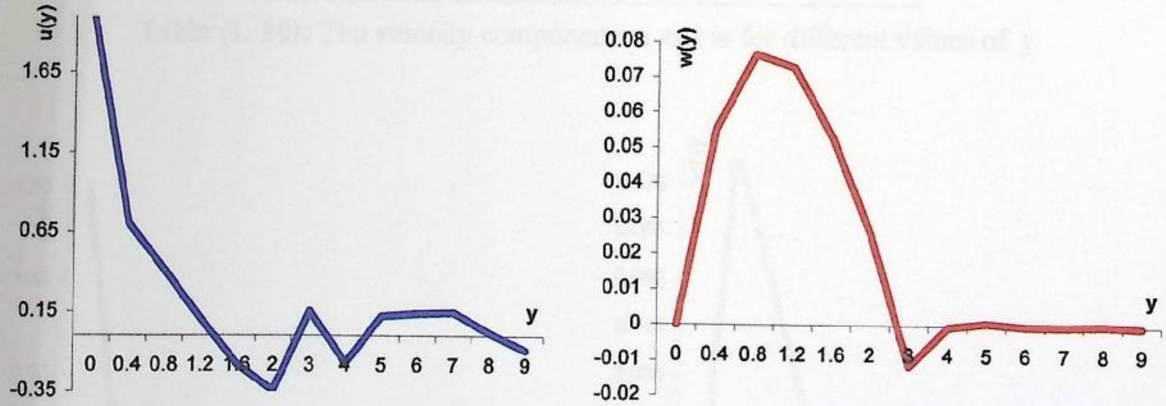


Fig 1.9: The velocity components for different values of t when M=1;N =1; t= 1/2

Case 5.b

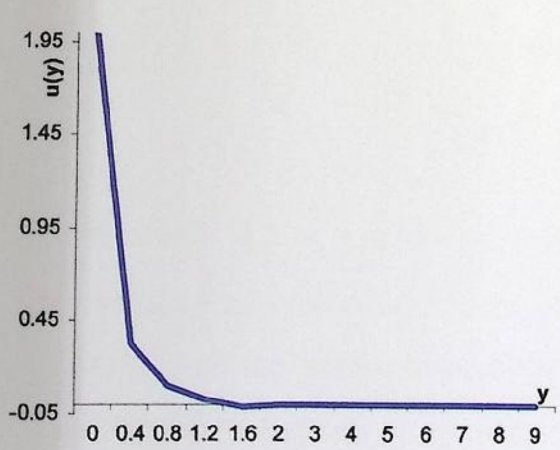
Define the variables:

$M=3$; $N=1/2$; $t=1/2$;

Next, we define ω and obtain the value η such that $\eta^2 = \omega$ with $Re(\eta) < 0$.

y	u(y)	w(y)
0	1.9993	0
0.4	0.3273	0.0829
0.8	0.0995	0.054
1.2	0.0301	0.0257
1.6	-0.0064	0.0103
2	-0.0007	0.0031
3	0.0013	-0.0008
4	-0.0005	0.0002
5	0	0
6	0.0004	0
7	0	0
8	0.0002	0
9	-0.0001	0

Table (1. 10): The velocity components u and w for different values of y



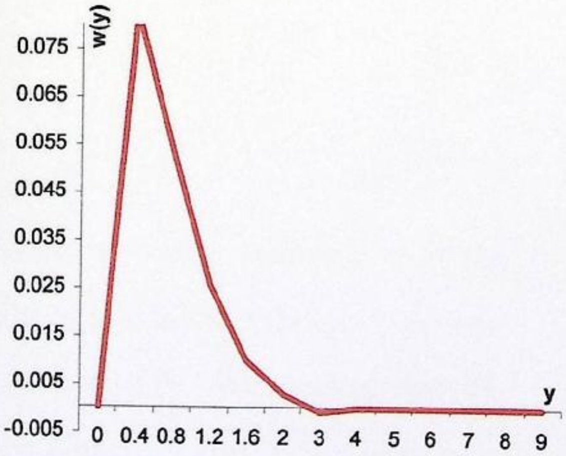


Fig 1.10: The velocity components for different values of t when M=3;N =1/2; t= 1/2

1.5 Discussion

The velocity components are dependent on the parameters **M** and **N**. we have calculated values of **u**, **w** at different times and at different heights for varying values of parameters **M** and **N**.

We observed that the velocity component **u** increases with the increase of **N** at equal heights of **y** and attains a steady state earlier with the increase of **N**, and decreases with the increase of **M**.

The attaining of steady state is delayed as **N** decreases. The velocity component **w** increases with the increase of **M**, also it increases with the increase of parameter **N**.

Moreover, when **y** increases at different values of **t**," **u** "decreases, while **w** increases for stable values of **M** and **N**. In addition the velocity component **u** gets unstable at different values of **N**, and **y** along with the increase of **t**. However, the velocity component **w** often increases as **t** increases.

CHAPTER TOW

Unsteady MHD Flow Past an Oscillating Porous Plate Including Hall Effect

2.1 Introduction

In this chapter we have studied the Hall effects on the hydromantic flow of viscous fluid over an infinite oscillating porous plate. Laplace transform technique has been used for solving the problem.

2.2 Problem Formulation

Consider the flow of a viscous incompressible and electrically conducting fluid in its own plane. In the presence of uniform magnetic filed H_0 normal to the oscillating porous plate. In the frame work of rectangular Cartesian coordinates, (x, y, z) let the porous plate be at $z = 0$ and the applied magnetic filed be along z-axis, and the porous plate be oscillating of frequency ω in time t about a constant mean along x- axis. Fluid of infinite extent caused by oscillating porous plate, oscillating in its plane. The equations describing the unsteady flow are:

$$\frac{\partial \overline{U}}{\partial t} + (\nabla . \overline{U}) V = - \frac{1}{\rho} \text{ grade } \overline{P} + \nu \nabla^2 U + \frac{\vec{J} \times \overline{B}}{\rho} \tag{2.2.1}$$

$$\text{div } \overline{U} = 0 \tag{2.2.2}$$

where $\overline{U}, V, t, \rho, \nu, \vec{J}$ and $\overline{B}$ are the velocity vector, constant with the dimension of velocities, time, density, kinematics viscosity, current density and the total mangneticfield respectively. We assume that the velocity field depends on z and t only, so that,

$$\overline{U} (z, t) = [u(z, t), v(z, t), w(z, t)]. \tag{2.2.3}$$

For the present problem $w(z, t) = 0$.

The generalized Ohm's law, taking Hall effect into account is:

$$\vec{J} = (\sigma)\,[\,\overline{E} + \mu_e\,\vec{q} \times \overline{H} + \frac{1}{\rho_e\, n_e}\nabla P_e] - \frac{w_e \tau_e}{H_0}\,\vec{J} \times \overline{H} \qquad (2.2.4)$$

where $\overline{H}, \vec{q}, \overline{E}, w_e, \tau_e, \sigma, \rho_e, \mu_e$ and n_e are the magnetic filed intensity, velocity vector, electric field, cyclotron frequency, electric collision time ,fluid conductivity, electric pressure ,number density of electron the magnetic permeability and the electron charge respectively. Under the usual assumptions the electron pressure thermoplastic pressure and the ion slip are neglected. We also assume that the electric field $\overline{E} = 0$.

thus:

$$\vec{J} = (J_x, J_y, 0) \qquad \vec{q} = (u, v, 0)$$

$$\overline{H} = (H_x, H_y, H_0) \qquad \overline{B} = (B_x, B_y, B_0)$$

Under these assumptions, we conclude:

$$\left.\begin{aligned} \vec{J} \times \overline{H} &= [\,J_y H_0, -J_x H_0, J_x H_y - J_y H_x\,] \\ \vec{q} \times \overline{H} &= [\,v H_0, -uH_0, uH_y - vH_x] \\ \vec{J} \times \overline{B} &= [\,J_y B_0, -J_x B_0, J_x B_y - J_y B_x\,] \end{aligned}\right\} \qquad (2.2.5)$$

From equation (2.2.4) and (2.2.5) we get:

$$J_x + m\,J_y = \sigma\, vB_0 \qquad (2.2.6)$$

$$J_y - m\,J_x = -\sigma\, u\, B_0 \qquad (2.2.7)$$

where $m = w_e\,\tau_e$ is the Hall parameter and $\overline{B}_0 = \mu_e\,\overline{H}_0$ is the Electro-magnetic induction.

On solving equations (2.2.6) and (2.2.7) we conclude:

$$J_x = \frac{\sigma\, B_0}{1 + m^2}(v + mu) \qquad (2.2.8)$$

$$J_y = \frac{\sigma\, B_0}{1 + m^2}(mv - u) \qquad (2.2.9)$$

In the absence of pressure gradient, equation (2.2.1) along with (2.2.8) and (2.2.9) comprises:

$$\frac{\partial u}{\partial t} + V \frac{\partial u}{\partial z} = \nu \frac{\partial^2 u}{\partial z^2} - \sigma B_0^2 \frac{(u - mv)}{\rho(1 + m^2)} \qquad (2.2.10)$$

$$\frac{\partial v}{\partial t} + V \frac{\partial v}{\partial z} = \nu \frac{\partial^2 v}{\partial z^2} - \sigma B_0^2 \frac{(v + mu)}{\rho(1 + m^2)} \qquad (2.2.11)$$

In order to solve coupled equations (2.2.10) and (2.2.11) we introduce a new variable $q = u + iv$, then the above equations are decoupled and are reduced to a single equation:

$$\frac{\partial q}{\partial t} + V \frac{\partial q}{\partial z} = \nu \frac{\partial^2 q}{\partial z^2} - \sigma B_0^2 \frac{(1 + im)}{\rho(1 + m^2)} q \qquad (2.2.12)$$

The initial and boundary conditions are:

$$\left.\begin{aligned} &q(z,t) = 0 \ \text{ for all } z,\ t \le 0 \\ &q(z,t) = U_0 e^{i\omega t},\ V = -V_0 \ \text{ for } z > 0,\ t > 0 \\ &q(z,t) = 0 \ \text{ or finite as } z \to \infty,\ t > 0 \end{aligned}\right\} \qquad (2.2.13)$$

where U_0 is a uniform velocity .

2.3 Problem Solution

In order to solve equation (2.2.12) we can write it as:

$$\frac{\partial q}{\partial t} + \sigma B_0^2 \frac{(1 + im)}{\rho(1 + m^2)} q = \nu \frac{\partial^2 q}{\partial z^2} + V_0 \frac{\partial q}{\partial z} \qquad (2.3.1)$$

or,

$$\frac{\partial q}{\partial t} - \alpha q = \nu \frac{\partial^2 q}{\partial z^2} + V_0 \frac{\partial q}{\partial z} \qquad (2.3.2)$$

Where:

$$\alpha = - \frac{\sigma B_0^2 (1 + im)}{\rho (1 + m^2)},$$ which is a complex number.

Multiplying both side of equation (2.3.2) by $(e^{-\alpha t})$ to get:

$$\frac{\partial}{\partial t}(e^{-\alpha t} q) = \nu \frac{\partial^2}{\partial z^2}(e^{-\alpha t} q) + V_0 \frac{\partial}{\partial z}(e^{-\alpha t} q) \quad (2.3.3)$$

assume:

$$\Phi(z,t) = e^{-\alpha t} q(z,t) \quad (2.3.4)$$

equation (2.3.3) can be written as:

$$\frac{\partial \Phi}{\partial t} = \nu \frac{\partial^2 \Phi}{\partial z^2} + V_0 \frac{\partial \Phi}{\partial z} \quad (2.3.5)$$

under this assumption the initial conditions in (2.2.13) becomes:

$$\left.\begin{aligned}
&\Phi(z,t) = 0 \quad \forall z, \quad t \le 0 \\
&\Phi(0,t) = e^{-\alpha t} . q(0,t) = e^{-\alpha t} U_0 e^{i\omega t} \\
&\Phi(0,t) = U_0 e^{-(\alpha - i\omega)t} \\
&\lim_{z\to\infty} \Phi(z,t) = \lim_{z\to\infty} e^{-\alpha t} . q(z,t) = 0 \\
&\Phi(z,t) = 0 \text{ as } z\to\infty \\
&\lim_{z\to\infty} \hat{\Phi}(z,s) = 0
\end{aligned}\right\} \quad (2.3.6)$$

Now take Laplace transform to both sides of equation (2.3.5) we get:

$$s\hat{\Phi} = \nu \frac{d^2\hat{\Phi}}{dz^2} + V_0 \frac{d\hat{\Phi}}{dz}$$

$$\nu \frac{d^2\hat{\Phi}}{dz^2} + V_0 \frac{d\hat{\Phi}}{dz} - s\,\hat{\Phi} = 0 \quad (2.3.7)$$

The auxiliary equation is:

$$v\beta^2 + V_0\beta - s = 0$$

where:

$$\beta = \frac{-V_0 \pm \sqrt{V_0^2 + 4vs}}{2v}$$

hence:

$$\beta_1 = \frac{-V_0 + \sqrt{V_0^2 + 4vs}}{2v}$$

$$\beta_2 = \frac{-V_0 - \sqrt{V_0^2 + 4vs}}{2v}$$

$$\hat{\Phi}(z,s) = C_1 e^{\beta_1 z} + C_2 e^{\beta_2 z} \qquad (2.3.8)$$

Claim: $C_1 = 0$

Proof of the claim:

Divide both sides of (2.3.8) by $(e^{\beta_1 z})$ and taking the limit as $z \to \infty$:

$$e^{-\beta_1 z}\,\hat{\Phi}(z,s) = C_1 + C_2\, e^{(\beta_2 - \beta_1) z}$$

that is:

$$e^{-\beta_1 z}\,\hat{\Phi}(z,s) = C_1 + C_2\, e^{-(\sqrt{V_0^2 + 4vs}\,/v).z}$$

now, taking the limit as $z \to \infty$ of both sides of the above equation:

$$\lim_{z\to\infty}(e^{-\beta_1 z}\,\hat{\Phi}(z,s)) = \lim_{z\to\infty}(C_1 + C_2\, e^{-(\sqrt{V_0^2 + 4vs}\,/v).z})$$

hence,

$$0.0 = C_1 + C_2 \,.\, 0$$

thus , we conclude: $C_1 = 0$

The auxiliary equation is:

$$\nu \beta^2 + V_0 \beta - s = 0$$

where:

$$\beta = \frac{-V_0 \pm \sqrt{V_0^2 + 4 \nu s}}{2 \nu}$$

hence:

$$\beta_1 = \frac{-V_0 + \sqrt{V_0^2 + 4 \nu s}}{2 \nu}$$

$$\beta_2 = \frac{-V_0 - \sqrt{V_0^2 + 4 \nu s}}{2 \nu}$$

$$\hat{\Phi}(z, s) = C_1 e^{\beta_1 z} + C_2 e^{\beta_2 z} \qquad (2.3.8)$$

Claim: $C_1 = 0$

Proof of the claim:

Divide both sides of (2.3.8) by $(e^{\beta_1 z})$ and taking the limit as $z \to \infty$:

$$e^{-\beta_1 z} \hat{\Phi}(z, s) = C_1 + C_2 e^{(\beta_2 - \beta_1) z}$$

that is:

$$e^{-\beta_1 z} \hat{\Phi}(z, s) = C_1 + C_2 e^{-(\sqrt{V_0^2 + 4 \nu s} / \nu) . z}$$

now, taking the limit as $z \to \infty$ of both sides of the above equation:

$$\lim_{z \to \infty} (e^{-\beta_1 z} \hat{\Phi}(z, s)) = \lim_{z \to \infty} (C_1 + C_2 e^{-(\sqrt{V_0^2 + 4 \nu s} / \nu) . z})$$

hence,

$$0.0 = C_1 + C_2 . 0$$

thus , we conclude: $C_1 = 0$

therefore:

$$\hat{\Phi}(z, s) = C_2 \, e^{\beta_2 z}$$

$$\hat{\Phi}(0, s) = C_2 \, . \, 1 = \frac{U_0}{s + \alpha - i\omega}$$

$$C_2 = \frac{U_0}{s + \alpha - i\omega}$$

Now equation (2.3.8) becomes:

$$\hat{\Phi}(z, s) = \frac{U_0}{s + \alpha - i\omega} \, . \, e^{\beta_2 z} \qquad (2.3.9)$$

where:

$$\beta_2 = \frac{-V_0 - \sqrt{V_0^2 + 4\nu s}}{2\nu}$$

Now, we have the following fact about Laplace transform:

$$L\{ \int_0^t f(t - \tau) \, . \, g(\tau) \, d\tau \} = L\{ f(t) \} \, . \, L\{ g(t) \} = \hat{f}(s) \, . \, \hat{g}(s)$$

Recall, $\hat{\Phi}(0, s) = L\{ \Phi(0, t) \}$

From the table of Laplace transform we have:

$$L\{ U_0 \, . \, e^{-(\alpha - i\omega) t} \} = \frac{U_0}{s + \alpha - i\omega}$$

$$L\{ \frac{e^{\frac{-(t V_0 + z)^2}{4\nu t}} \, . \, \sqrt{\frac{t^3}{\nu}} \, . \, z}{2t^3 \sqrt{\pi}} \} = e^{\beta_2 z}$$

From the above equations and from equation (2.3.9) we get;

$$\Phi(z, t) = L^{-1}\{ \frac{U_0}{s + \alpha - i\omega} \, . \, e^{\beta_2 z} \}$$

$$\Phi(z, t) = \int_0^t (U_0 \, . \, e^{-(\alpha - i\omega)(t - \tau)} \, . \, \frac{e^{\frac{-(\tau V_0 + z)^2}{4\nu\tau}} \, . \, \sqrt{\frac{\tau^3}{\nu}} \, . \, z}{2\tau^3 \sqrt{\pi}}) \, d\tau$$

Simplify:

$$\Phi(z,t) = \frac{U_0}{2\sqrt{\pi\nu}} \cdot e^{\frac{-V_0 z}{2\nu} - (\alpha - i\omega)t}$$

$$\int_0^t \tau^{\frac{-3}{2}} \cdot e^{\frac{-(V_0^2\tau^2 + z^2 - 4(\alpha - i\omega)\tau^2\nu}{4\nu\tau}} \cdot d\tau \qquad (2.3.10)$$

Recall,

$$q(z,t) = e^{\alpha t}\,\Phi(z,t)$$

$$q(z,t) = \frac{U_0 z}{2\sqrt{\pi\nu}} \cdot e^{\alpha t - \frac{V_0 z}{2\nu} - (\alpha - i\omega)t} \int_0^t \tau^{\frac{-3}{2}} \cdot e^{-\left(\frac{V_0^2\tau^2 + z^2 - 4(\alpha - i\omega)\tau^2\nu}{4\nu\tau}\right)} \cdot d\tau$$

Simplify:

$$q(z,t) = \frac{U_0 z}{2\sqrt{\pi\nu}}\, e^{\frac{-V_0 z}{2\nu} + i\omega t} \int_0^t \tau^{-\frac{3}{2}}\, e^{-\left(\frac{V_0\tau^2 + z^2 - 4(\alpha - i\omega)\tau^2\nu}{4\nu\tau}\right)}\, d\tau \qquad (2.3.11)$$

Now, we want to write $q(z, t)$ as $u + iv$

so, let $\alpha = a + ib$ where $a = \frac{-\sigma B_0^2}{\rho(1+m^2)}$, $b = \frac{-\sigma B_0^2 m}{\rho(1+m^2)}$

therefore:

$$q(z,t) = \frac{U_0 z}{2\sqrt{\pi\nu}} e^{\frac{-V_0 z}{2\nu}} \int_0^t \tau^{-\frac{3}{2}}\, e^{-\left(\frac{V_0^2\tau}{4\nu} + \frac{z^2}{4\nu\tau} - a\tau\right)} .\cos(\omega(t-\tau) + b\tau)\, d\tau \; +$$

$$i\frac{U_0 z}{2\sqrt{\pi\nu}} e^{\frac{-V_0 z}{2\nu}} \int_0^t \tau^{-\frac{3}{2}}\, e^{-\left(\frac{V_0^2\tau}{4\nu} + \frac{z^2}{4\nu\tau} - a\tau\right)} .\sin(\omega(t-\tau) + b\tau)\, d\tau \qquad (2.3.12)$$

The following tables and graphs represent the velocity components for $t = 1$:

z / m	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
0	0.4289	0.3330	0.2536	0.1898	0.1398	0.1014	0.0724	0.0509	0.0353	0.0241
1	0.4683	0.3878	0.3104	0.2415	0.1833	0.1362	0.0992	0.0709	0.0497	0.0343
2	0.4817	0.4089	0.3345	0.2655	0.2053	0.1551	0.1147	0.0832	0.0591	0.0413
3	0.4842	0.4136	0.3405	0.2720	0.2117	0.1610	0.1198	0.0873	0.0625	0.0438

Table (2.1): The velocity component u against z for different values of m.

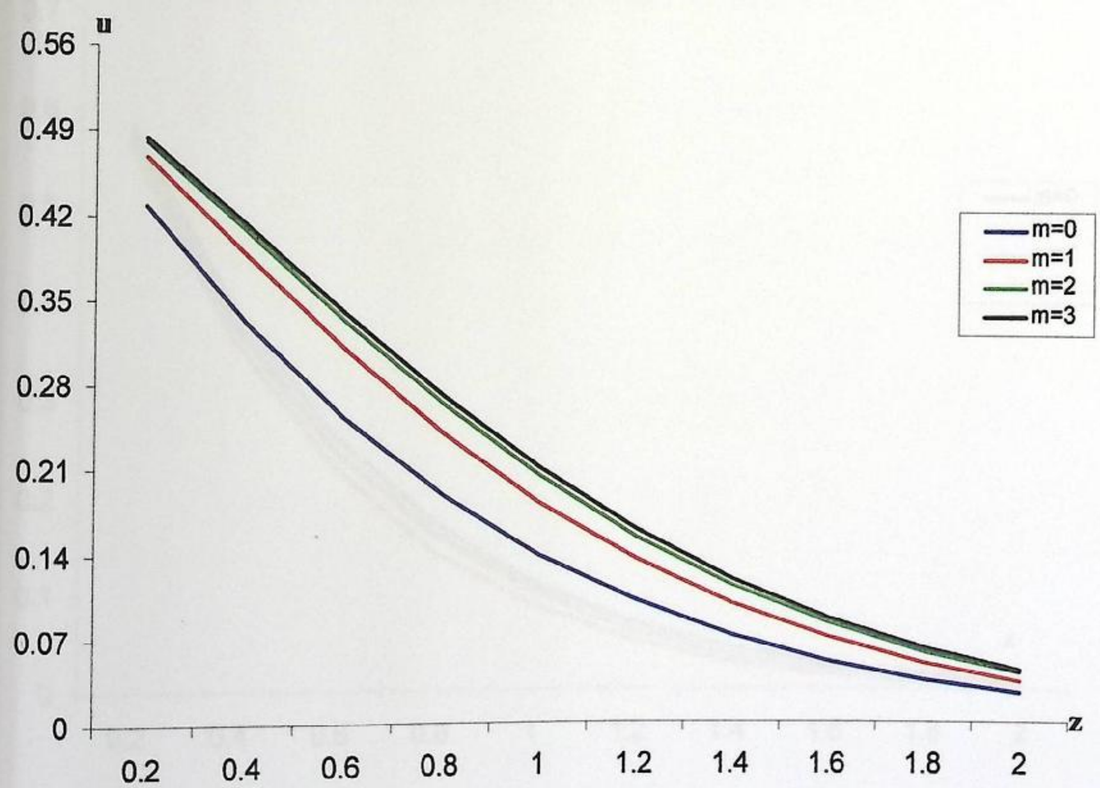


Fig 2.1: Velocity component u against z for different values of m.

z / m	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
0	0.5677	0.3797	0.2518	0.1654	0.1076	0.0693	0.0442	0.0279	0.0174	0.0107
1	0.5607	0.3661	0.2338	0.1456	0.0881	0.0515	0.0289	0.0154	0.0075	0.0032
2	0.5755	0.3859	0.2533	0.1625	0.1017	0.0619	0.0364	0.0206	0.0111	0.0055
3	0.5848	0.3992	0.2674	0.1756	0.1130	0.0711	0.0436	0.0260	0.0151	0.0084

Table (2.2): The velocity component v against z for different values of m.

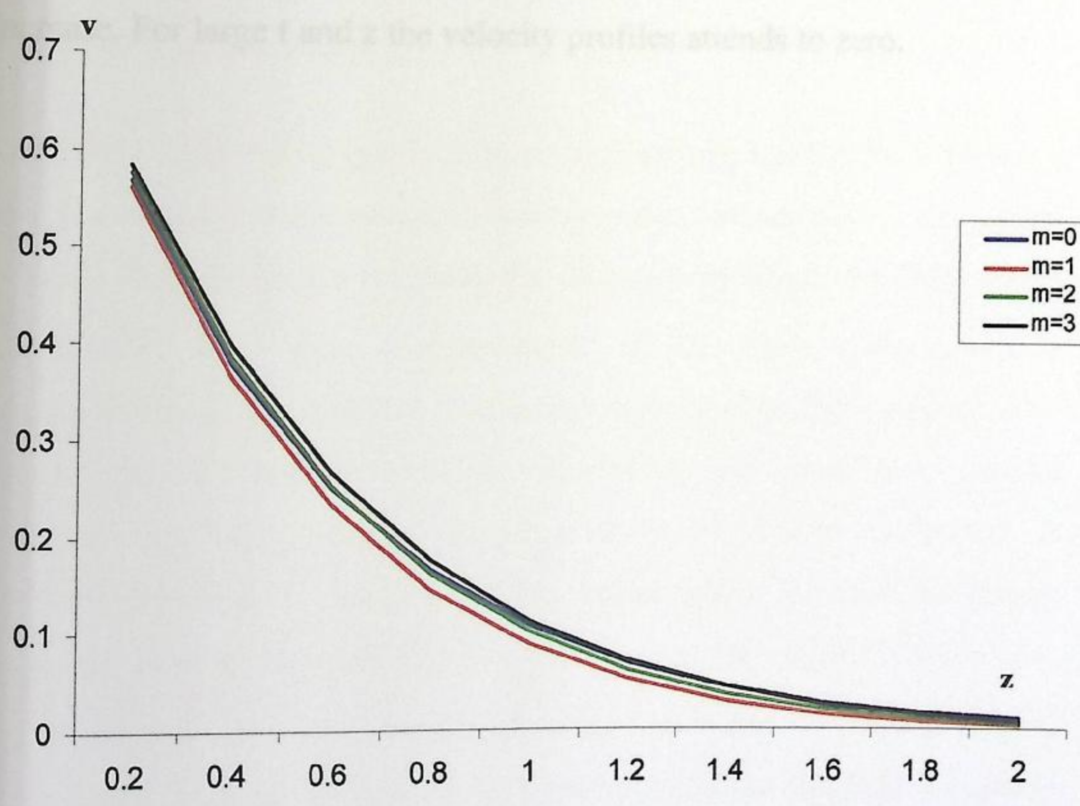


Fig 2.2: Velocity component v against z for different values of m.

2.4 Numerical Discussion

In studying, the effects of Hall Currents on hydromagnetic flow in oscillating porous plate, the non-dimensional velocity profiles are presented versus **z** as shown below in table 1 Fig.1 and table 2 Fig. 2 for various values of parameter **m**, it is evident that the magnitude of the velocity component **u** increases as **m** increases, the results are compared with the case $\mathbf{m}=\mathbf{0}$,while the velocity component **v** have oscillatory character with **z** for different values of **m**.

On the other hand, the magnitude of **u** and **v** decrease very fast as **z** increase. For large t and **z** the velocity profiles attends to zero.

CHAPTER THREE

Hall Effects on MHD Flow Between Two Horizontal Plates The Lower Plate Being a Stretching Sheet

3.1 Introduction

The effects of Hall Currents on MHD flow between tow parallel plates is very important due to the wide applications of such type of problems in MHD generators and MHD flow meters.

3.2 Problem Formulation

Consider the flow of a conducting viscous incompressible fluid between two horizontal parallel non-conducting plates, which lie in the planes $y = \pm b$. Let the x-axis be along the direction in which the fluid enters the channel. The origin is at the centre of the channel. The y-axis is perpendicular to the channel. The z-axis is normal to the xy-plane. The lower plate is being stretched by introducing two equal and opposite forces so that the position of the point (0, -b, 0) remains unchanged. A constant injection is applied at the upper plate. A uniform strong magnetic field of strength H_0 is applied along the z-axis. We consider that there is no flow occurs parallel to the z-axis and we have neglected the induced magnetic field since we have assumed the magnetic Reynolds number to be very small.

The equations describing the flow are:

$$\operatorname{div} \vec{q} = 0 \tag{3.2.1}$$

$$\rho \left[\frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla) \vec{q} \right] = - \operatorname{grad} p + \nu \nabla^2 \vec{q} + \vec{J} \times \vec{B} \tag{3.2.2}$$

where $\vec{q}$ is the velocity vector. ρ, ν, p, B are: respectively the density of the fluid coefficient of viscosity, pressure, and magnetic induction field.

The equation of the generalized Ohm's law is:

$$\vec{J} = \sigma \left[\vec{E} + \mu_e \vec{q} \times \vec{H} + \left(\frac{1}{p_e n_e} - \frac{\omega_e \tau_e}{H_0} \right) \vec{J} \times \vec{H} \right] \tag{3.2.3}$$

where σ, E, μ_e, q, H, n_e, p_e, ω_e and τ_e are respectively electric current density, fluid conductivity, electric field, magnetic permeability, velocity vector, magnetic field intensity, the number density of the electron, the electron pressure the cyclotron frequency and the electron collision time .

Under the usual assumption the ion slip, thermo elastic pressure and the electron pressure are neglected. We assume the electric field E = 0.

Let u and v be components of velocity along x and y-axes respectively.

so, we have:

$$\vec{H} = (0, 0, H_0), \qquad \vec{J} = (J_x, J_y, 0), \qquad \vec{q} = (u, v, 0)$$

under these assumption we come up with the following equations:

$$\left.\begin{aligned} \vec{q} \times \vec{H} &= [\, v H_0, - u H_0, 0 \,] \\ \vec{J} \times \vec{H} &= [\, J_y H_0, - J_x H_0, 0 \,] \\ \vec{J} \times \vec{B} &= [\, J_y B_0, - J_x B_0, 0 \,] \end{aligned}\right\} \tag{3.2.4}$$

Furthermore, from equations (3.2.3) and (3.2.4) we conclude:

$$J_x = \sigma [\mu_e v H_0 - \omega_e \tau_e J_y]$$

$$J_x + m J_y = \sigma B_0 v \tag{3.2.5}$$

$$J_y = \sigma [- \mu_e uH_0 + \omega_e \tau_e J_x]$$

$$J_y - m J_x = - \sigma B_0 u \qquad (3.2.6)$$

Thus, from equations (3.2.5) and (3.2.6), we come up with:

$$J_x = \frac{\sigma B_0}{1 + m^2} (v + mu) \qquad (3.2.7)$$

$$J_y = \frac{\sigma B_0}{1 + m^2} (mv - u) \qquad (3.2.8)$$

Where $m = \sigma \omega_e \tau_e$ is the Hall parameters and $B_0 = \mu_e \overline{H}_0$ is the Electromagnetic induction, now:

equations (3.2.7) and (3.2.8) reduce equations (3.2.1) and (3.2.2) to :

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (3.2.9)$$

$$\rho [u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}] = - \frac{\partial p}{\partial x} + \nu [\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}] + B_0[\frac{\sigma B_0}{1 + m^2} (mv - u)]$$

or

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = - \frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{\nu}{\rho} (\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}) -$$

$$\frac{\sigma B_0^2 \Phi}{\rho} (u - mv)] \qquad (3.2.10)$$

and

$$\rho [u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}] = - \frac{\partial p}{\partial y} + \nu [\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}] - B_0[\frac{\sigma B_0}{1 + m^2} (v + mu)]$$

or

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = - \frac{1}{\rho} \frac{\partial p}{\partial y} + \frac{\nu}{\rho} (\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}) -$$

$$\frac{\sigma B_0^2 \Phi}{\rho} (v + mu)] \qquad (3.2.11)$$

where ρ, ν, p and B_0 are respectively the density of the fluid, coefficient of viscosity, pressure and magnetic induction field.

for which $\Phi = \dfrac{1}{1 + m^2}$, and the boundary conditions are:

$$\left.\begin{array}{lll} u = Cx & v = 0, & \text{at } y = -b \\ u = 0 & v = -v_0, & \text{at } y = b \end{array}\right\} \qquad (3.2.12)$$

where C > 0 and Cx stands for lower plate velocity.

3.3 Method of Solution

Velocity Distribution:

We assume the similarity solution:

$$u = Cx\, f'(\eta), \qquad v = -Cb\, f(\eta) \qquad (3.3.1)$$

and the prime denotes differentiation with respect to $\eta = \dfrac{y}{b}$, where f is a function of η.

On substituting (3.3.1) in equations (3.2.9), (3.2.10) and (3.2.11), we get:

$$\overset{(iv)}{f} - Re(f'f'' - f f''') - M^2\, \Phi\, f'' = 0 \qquad (3.3.2)$$

under this assumption the boundary condition in (4.2.12) becomes:

$$\left.\begin{array}{llll} f(\eta) = \lambda & f'(\eta) = 0 & \text{at} & \eta = 1 \text{ and } y = b \\ f(\eta) = 0 & f'(\eta) = 1 & \text{at} & \eta = -1 \text{ and } y = -b \end{array}\right\} \qquad (3.3.3)$$

Where: Re (Reynolds number) $= \dfrac{Cb^2}{\nu}$.

λ(constant with the velocity vector) $= \dfrac{v_0}{Cb}$.

M (Hartmann number) $= \left(\dfrac{\sigma B_0^{\,2}\, b^2}{\rho\, \nu}\right)^{\frac{1}{2}}$.

For small values of (Re), we assume that:

$$f(\eta) = \sum_{n=0}^{\infty} (R_e)^n f_n(\eta) \qquad (3.3.4)$$

$$f = f_0 + R_e f_1 + R_e^2 f_2 + \ldots\ldots\ldots$$

Consider: $f = f_0 + R_e f_1$ (i.e n = 0,1)

On substituting the value of $f(\eta)$ in equation (3.3.4) into equation (3.3.2) and by equating like of (Re), we get:

$$f_0^{(iv)} + R_e f_1^{(iv)} - R_e (f_0' + R_e f_1')(f_0'' + R_e f_1'') +$$

$$R_e (f_0 + R_e f_1)(f_0''' + R_e f_1''') - M^2 \Phi (f_0'' + R_e f''_1) = 0$$

By equating like powers of (R_e), we have:

$$f_0^{(iv)} - M^2 \Phi f_0'' = 0 \qquad (3.3.5)$$

$$f_1^{(iv)} - M^2 \Phi f_1'' = f_0' f_0'' - f_0 f_0''' \qquad (3.3.6)$$

and we neglected $(R_e)^n \quad \forall\, n \geq 2$.

The boundary condition for $f_0, f_1, f_2,$ .... are

$$\left.\begin{aligned} f_0 = \lambda,\ f_0' = 0,\ f_n = f_n' = 0,\ \forall\, n > 0,\ \eta = 1 \\ f_0 = 0,\ f_0' = 1,\ f_n = f_n' = 0,\ \forall\, n > 0,\ \eta = -1 \end{aligned}\right\} \qquad (3.3.7)$$

Consider: $M^2\Phi = \alpha^2$

Though , equations (3.3.5) and (3.3.6)can be written as:

$$f_0^{(iv)} - \alpha^2 f_0'' = 0 \qquad (3.3.8)$$

$$f_1^{(iv)} - \alpha^2 f_1'' = f_0' f_0'' - f_0 f_0''' \qquad (3.3.9)$$

On solving the equations (3.3.8) and (3.3.9) with the help boundary conditions, (replacing η by x) we get:

$$f_0(x) = \Big(e^{-x\alpha}\big(-e^{\alpha}+e^{3\alpha}+e^{4\alpha+x\alpha}-e^{\alpha+2x\alpha}-e^{3\alpha+2x\alpha}-2e^{3\alpha}\alpha+e^{x\alpha}\alpha+2e^{2\alpha+x\alpha}\alpha+e^{4\alpha+x\alpha}\alpha-2e^{\alpha+2x\alpha}\alpha-e^{x\alpha}x\alpha+2e^{2\alpha+x\alpha}x\alpha-e^{4\alpha+x\alpha}x\alpha-e^{\alpha}\alpha\lambda+e^{3\alpha}\alpha\lambda-e^{x\alpha}\alpha\lambda+2e^{2\alpha+x\alpha}\alpha\lambda-e^{4\alpha+x\alpha}\alpha\lambda+e^{\alpha+2x\alpha}\alpha\lambda-e^{3\alpha+2x\alpha}\alpha\lambda-e^{x\alpha}\alpha^2\lambda+e^{4\alpha+x\alpha}\alpha^2\lambda-e^{x\alpha}x\alpha^2\lambda+e^{4\alpha+x\alpha}x\alpha^2\lambda\big)\Big) / \Big(2\big(-1+e^{2\alpha}\big)\alpha\big(1-e^{2\alpha}+\alpha+e^{2\alpha}\alpha\big)\Big)\}\} \tag{3.3.10}$$

Here, let's use Full Simplify to check that the boundary value conditions are satisfied:

we get: $f_0(1)=\lambda$ $\quad f_0(-1)=0$ $\quad f_0'(-1)=0$ and $f_0'(-1)=1$

now, let's check if the function f_0 satieties the fourth order ODE.

we get: $$f_0^{(iv)}-\alpha^2 f_0''=0$$

Let's define a new function, $g=f_0'f_0''-f_0f_0'''$ which appears on the right hand side of the second equation.

$$g(x,\alpha)=\Big(e^{\alpha-x\alpha}\alpha\Big(e^{6\alpha}\big(1+\alpha(-2+\lambda)\big)\big(-2+\alpha-x\alpha+(1+x)\alpha^2\lambda\big)+e^{2(3+x)\alpha}\alpha(-1+\alpha\lambda)(1-x+(-2+\alpha+x\alpha)\lambda)+\alpha(1+\alpha\lambda)(-1+x+(2+\alpha+x\alpha)\lambda)+e^{2x\alpha}\big(-1+\alpha(-2+\lambda)\big)(-2+\alpha(-1+x+(1+x)\alpha\lambda))+e^{4\alpha}(4-(1+x)\alpha^3\lambda^2+3\alpha(-1+x+2\lambda)+2\alpha^2(-2+x(-2+\lambda)+(-2+\lambda)\lambda))-e^{2(2+x)\alpha}\big(-2+(1+x)\alpha^3(-2+\lambda)\lambda+\alpha(1+3x+6\lambda)+2\alpha^2\big(-1+x+\lambda-2x\lambda-2\lambda^2\big)\big)-e^{2\alpha}\big(2+\alpha\big(1+6\lambda+x(3+\alpha(-2+\lambda))(1+\alpha\lambda)+\alpha(2-2(1+\alpha)\lambda+(4+\alpha)\lambda^2)\big)\big)-e^{2(1+x)\alpha}\big(4+\alpha\big(3-6\lambda+\alpha\big(-4+\lambda(-4+(2+\alpha)\lambda)\big)+x\big(-3+\alpha(-4+\lambda(2+\alpha\lambda))\big)\big)\big)\Big)\Big)\Big/\Big(4\big(-1+e^{2\alpha}\big)^2\big(1+e^{2\alpha}(-1+\alpha)+\alpha\big)^2\Big)$$

Just for fun, let's look at $g(\pm 1,\alpha)$.

$$g(-1,\alpha)=\frac{\alpha\left(\text{Cosh}[\alpha]\text{-}\alpha\text{Csch}[\alpha]+\alpha(-2+\lambda)\text{Sinh}[\alpha]\right)}{2\alpha\text{Cosh}[\alpha]\text{-}2\text{sinh}[\alpha]}$$

and

$$g(1,\alpha)=\frac{\alpha^2\lambda\left(\alpha\lambda\text{Cosh}[\alpha]\text{-Sinh}[\alpha]\right)}{2\alpha\text{Cosh}[\alpha]\text{-}2\text{Sinh}[\alpha]}$$

Now, let's compute $\overset{(iv)}{f}_1 - \alpha^2 f_1'' = g(x,\alpha)$ with $f_1(\eta)=f_1'(\eta)=0$ for $\eta=\pm 1$.
we get:

$$f_1(x)=\Big(e^{-x\alpha}\Big(e^{(9+2x)\alpha}\Big(-7+\alpha\Big(11+5\lambda+x^2(-1+\alpha)\alpha(-1+\alpha\lambda)^2+x(-1+\alpha)(5+2\alpha$$

$$+\alpha(-1+\alpha\lambda)\big(-9+2\alpha)\lambda\big)+\alpha\Big(\alpha+\Big(-5+2\big(-6+\alpha)\alpha\big)\lambda+\big(-2+\alpha(2+(8-3\alpha)\alpha)\big)\lambda^2\Big)\Big)\Big)$$

$$-e^{\alpha}\big(7+\alpha(11+5\lambda+\alpha(1+\alpha)(x+x\alpha\lambda)^2+x(1+\alpha)(1+\alpha\lambda)(5-2\alpha+\alpha(9+2\alpha)\lambda)+\alpha$$

$$(\alpha+(5-2\alpha(6+\alpha))\lambda+(2+\alpha(2-\alpha(8+3\alpha)))\lambda^2))\big)+2e^{7\alpha}\big(14+\alpha(-11+16x-5\lambda)$$

$$+\alpha^5\big((1+x)^2-2\lambda\big)\lambda+\alpha^4\big(5+6x+x^2+(5+9x)\lambda-(-6+x(4+x))\lambda^2\big)+\alpha^2+\alpha^3$$

$$\big(2(-15+x)x+13x\lambda+4\lambda^2-4(1+\lambda)\big)\big(7+x^2(-4+\lambda)-2\lambda(7+\lambda)+x(1-\lambda(10+3\lambda))\big)\Big)$$

$$+2e^{(4+x)\alpha}\big(7+\alpha(24+2x-5\lambda)+4\alpha^5(1+x-\lambda)\lambda+\alpha^2 2(-2+\lambda)\lambda)-2\alpha^4\big(17+x+9x\lambda+$$

$$\big(6+(13-3\lambda)\lambda+x(6+\lambda(-1+6\lambda))\big)-\alpha^3\big(18+(19-8\lambda)\lambda+x(-22+\lambda(41+7\lambda))\big)\big)-$$

$$2e^{(6+x)\alpha}\big(-7+\alpha(24+2x-5\lambda)+4\alpha^5(1+x-\lambda)\lambda-\alpha^2(17+x+9x\lambda+2(-2+\lambda)\lambda)+2\alpha^4$$

$$\big(6+(13-3\lambda)\lambda+x(6+\lambda(-1+6\lambda))\big)-\alpha^3\big(18+(19-8\lambda)\lambda+x(-22+\lambda(41+7\lambda))\big)\big)+$$

$$e^{(8+x)\alpha}\big(-21+8\alpha^5\lambda(-1-x+\lambda)+\alpha(46-2x+5\lambda)-\alpha^2(49+2\lambda+6\lambda^2+9x(-1+3\lambda))+\alpha^3$$

$$\big(32+\lambda+12\lambda^2+x(40+\lambda(-29+7\lambda))\big)+2\alpha^4\big(-4+(2-7\lambda)\lambda+x(-4+\lambda(-2+11\lambda))\big)\big)$$

$$+e^{\alpha+2x\alpha}\big(-7+\alpha\big(-11-5\lambda+x(1+\alpha)(-1+\alpha(-2+\lambda))(-9-2\alpha+\alpha(-5+2\alpha)\lambda)+\alpha(4+$$

$$\alpha(19+6\alpha)-5\lambda-2\alpha(2+(-2+\alpha)\alpha)\lambda+(-2+\alpha(-2+(-4+\alpha)\alpha))\lambda^2\big)+x^2\alpha(1+\alpha)$$

$$\left(-1+\alpha\left(-2+\alpha(-2+\lambda)\lambda\right) \right) \Big) \Big) - e^{9\alpha}\left(7+\alpha(-11-5\lambda+x(-1+\alpha)(1+\alpha(-2+\lambda))\right.$$

$$\left(-9+2\alpha+\alpha(5+2\alpha)\lambda\right)+\alpha\left(-4+(19-6\alpha)\alpha+5\lambda-2\alpha\left(2+\alpha(2+\alpha)\right)\lambda+(2+\alpha\right.$$

$$\left.(-2+\alpha(4+\alpha)))\lambda^2\right)+x^2(-1+\alpha)\alpha\left(-1+\alpha\left(2+\alpha(-2+\lambda)\lambda\right)\right)\Big)\Big)+e^{(10+x)\alpha}\left(7+\alpha\left(-18+\right.\right.$$

$$15\alpha-4\alpha^2-5\ (-1+\alpha)^2\ \lambda+2\ (-1+\alpha)^2\ \alpha\lambda^2+x\ \left(2+\alpha\ \left(-11+9\lambda+\alpha\left(4+\lambda+\ (-7+2\right.\right.\right.$$

$$\left.\alpha)\lambda^2\right)\Big)\Big)\Big)\Big)+2e^{(7+2x)\alpha}\ \left(14+\alpha\left(-11-5\lambda+\alpha\left(\alpha\left(-17+3\alpha\right)+\left(-4+\alpha\left(18+\alpha(7+\alpha)\right)\right)\right.\right.\right.$$

$$\lambda-2(2+\alpha)\left(-1+\alpha+\alpha^2\right)\lambda^2\Big)+x^2\alpha(-1+\alpha\lambda)\ \left(-2+\alpha(\alpha+\lambda)\right)+x\left(-12+\alpha(2+23\lambda+\right.$$

$$\alpha\left(1+2\alpha+\left(-2+\alpha(-9+2\alpha)\right)\lambda+\left(-11+4a\right)l^2\right)\Big)\Big)\Big)\Big)+2e^{3a}\left(14+a\left(11+5l-x^2a\right.\right.$$

$$\left(-2+a^2-al\right)(1+al)+\alpha\left(\alpha(17+3\alpha)-\left(4+\alpha\left(18+\left(-7+\alpha\right)\alpha\right)\right)\lambda+2\left(2+\alpha-3\alpha^2+\alpha^3\right)\right.$$

$$e^{(2+x)\alpha}\left(-21+\alpha\left(-46-5\lambda-\alpha\left(49+8\alpha(4+\alpha)+2\lambda+\alpha\ \left(1-4\alpha(1+2\alpha)\right)\lambda+2(1+\alpha)\right.\right.\right.$$

$$\left(3+\alpha\left(3+4\alpha\right)\right)\lambda^2\Big)+x\left(2+\alpha\left(9-27\lambda+\alpha\ \left(-8(5+\alpha)\ +\left(29-4\alpha+8\alpha^2\right)\lambda+\right.\right.\right.$$

$$\left.(-7+22\alpha)\lambda^2\right)\Big)\Big)\Big)\Big)-2e^{(3+2x)\alpha}\left(-14+\alpha\left(-11-5\lambda+\alpha\left(4+7\alpha-5\alpha^2+\left(4+(-7+\alpha)\alpha(2+\alpha)\right)\right.\right.\right.$$

$$\lambda-2\left(2+\alpha+3\alpha^2+\alpha^3\right)\lambda^2\Big)+x\left(16+\alpha\left(30-13\lambda+\alpha\left(1-6\alpha+\left(-10+\alpha(-9+2\alpha)\right)\lambda+\right.\right.\right.$$

$$\left.(-3+4\alpha)\lambda^2\right)\Big)\Big\}+x^2\alpha\left(-2+\alpha\left(-4+\lambda+\alpha\left(-1+\lambda(\alpha+\lambda)\right)\right)\right)\Big)\Big)\Big)+e^{x\alpha}\left(7+\alpha\left(18+5\lambda+\alpha\right.\right.$$

$$\left(15+4\alpha+5(2+\alpha)\lambda+2(1+\alpha)^2\lambda^2\right)+x\left(-2+\alpha\left(-11+9\lambda+\alpha\left(-4+\lambda\left(-1+(7+2\alpha)\right.\right.\right.\right.$$

$$\lambda)\Big)\Big)\Big)\Big)\Big)\Big)+2e^{5\alpha}\left(-21+\alpha\left(-\alpha\left(-2+2\alpha(7-12\lambda)+\alpha^3\lambda(1+\lambda)+\alpha^2(-5+\lambda)\ (1+2\lambda)\right.\right.\right.$$

$$+\lambda(-13+6\lambda)\Big)\Big)+x^2\alpha\left(-3+\alpha\left(3+\alpha-3\lambda+2\alpha\lambda+\alpha^2(-1+\lambda)\lambda\right)\right)+x\left(-21+\alpha\right.$$

$$\left(15-27\lambda+\alpha\left(11+2(8-3\lambda)\lambda+2\alpha^2(-1+\lambda)\lambda+\alpha\left(6+\lambda(-1+7\lambda)\right)\right)\right)\Big)\Big)\Big)\Big)+$$

$$2e^{(5+2x)\alpha}\left(-21+\alpha\left(\alpha\left(2+\alpha(14+5\alpha)+13\lambda+\alpha\left(-24+\alpha(9+\alpha)\right)\lambda+\left(-6+(-2+\alpha)\alpha^2\right)\right.\right.\right.$$

$$\lambda^2\Big)+x^2\alpha\left(-3+\alpha\left(3(-1+\lambda)+\alpha\left(1+\lambda(2+\alpha-\alpha\lambda)\right)\right)\right)+x\left(21+\alpha\left(15-27\lambda+\alpha\left(-11-2\alpha^2\right.\right.\right.$$

$$(-1+\lambda)\lambda+2\lambda(-8+3\lambda)+\alpha\Big(6+\lambda(-1+7\lambda)\Big)\Big)\Big)\Big)\Big)\Big)\Big)\Big)\Big)\Big)\Big/\ 16(-1+e^{2\alpha})^2\alpha^3$$

$$\left(1+e^{2\alpha}(-1+\alpha)+\alpha\right)$$

(3.3.11)

Again, let's check the boundary conditions, we get:

$f_1(-1)=0 \quad f_1'(-1)=0 \quad f_1(1)=0 \quad f_1'(1)=0$

Let's check to see if f_1 satisfies $f_1^{(iv)}-\alpha^2 f_1''=g(x,\alpha)$.

We have: $f_1^{(iv)}-\alpha^2 f_1''-g(x,\alpha)=0$

Now, let's define F_0 to be a function of x and m. Specifically,

$F_0(x,m)=f_0(x)$ with $\lambda=1$, and $\alpha=5\sqrt{\Phi}$ where $\Phi=\dfrac{1}{1+m^2}$,

$m=0,2,3$, and $M=5$ furthermore, I'll define three separate functions for

$F_1 j(x)=f_1(x)$ with these same values of λ, α, Φ, and with $m=j$ for

$j=0,2$, and 3. I tried defining F_1 to be a two variable function in a

manner similar to F_0 when $\lambda=1$, $\alpha=5$ corresponds to $m=0$, we get:

$$F_{10}(x)=\Big(e^{5x}\Big(6e^{5(6+x)}(1+5x)+e^{5x}(51+20x)+e^{5(2+x)}(-326+370x)-e^{5(4+x)}$$

$$(231+620x)+e^{25+10x}(-68+25x(1+x))+e^{5}(229-75x(3+x))+e^{5+10x}$$

$$(29-152x(1+5x))+e^{15+10x}(357+5x(13+5x))+2e^{25}(82+5x(19+5x))+$$

$$e^{15}(-143+5x(27+5x))\Big)\Big)\Big/\Big(2000\left(-1+e^{10}\right)^2\left(3+2e^{10}\right)\Big)$$

$$F_0(x,0)=\frac{-4-5x+e^{10}(6+5x)-2e^{5}\,\mathrm{Cosh}[5x]}{10(-1+e^{10})}$$

So,

$$F_{10}(x)+F_0(x,0)=\Big(e^{-5x}\Big(-e^{5(4+x)}(631+1620x)-2e^{5(2+x)}(2363+1815x)+$$

$e^{5(6+x)}(2406+2030x)+e^{5x}(2451+3020x)+e^{25+10x}(-536+50x(1+x))+$

$e^{5}(829-75x(3+x))+e^{5+10x}(629-15x(1+5x))+e^{15+10x}(157+5x(13+5x))+$

$2e^{25}(-118+5x(19+5x))+e^{15}(-343+5x(27+5x))))/(2000(-1+e^{10})^{2}(3+2e^{10}))$

When $\lambda=1, m=2, \alpha=\sqrt{5}$ we have:

$F_{12}(x)=\Big(e^{-\sqrt{5}(5+x)}\Big(e^{\sqrt{5}(2+x)}\big(-189+69\sqrt{5}+40x+28\sqrt{5}x\big)+$

$e^{\sqrt{5}(8+x)}\big(-189+69\sqrt{5}+4(10-7\sqrt{5})x\big)-2e^{\sqrt{5}(10+x)}\big(-24+11\sqrt{5}+(-5+\sqrt{5})x\big)+$

$2e^{\sqrt{5}x}\big(24+11\sqrt{5}+(5+\sqrt{5})x\big)e^{\sqrt{5}(6+x)}\big(-356+63\sqrt{5}+2(-125+7\sqrt{5})x\big)-$

$e^{\sqrt{5}(4+x)}\big(359+63\sqrt{5}+2(125+7\sqrt{5})x\big)+6e^{\sqrt{5}(5+2x)}\big(27+x(20-7\sqrt{5}+5x)\big)+$

$6e^{5\sqrt{5}}\big(27+x(20+7\sqrt{5}+5x)\big)+2e^{\sqrt{5}(9+2x)}\big(-13(12+\sqrt{5})+5x(-9+4\sqrt{5}+(-2+\sqrt{5})x)\big)+$

$2e^{9\sqrt{5}}\big(16-3\sqrt{5}+x(25-8\sqrt{5}+5(-2+\sqrt{5})x)\big)+e^{\sqrt{5}(3+2x)}$

$\big(197+43\sqrt{5}+x(95-3\sqrt{5}-5(-1+\sqrt{5})x)\big)+$

$e^{3\sqrt{5}}\big(57+23\sqrt{5}+x(25+11\sqrt{5}-5(-1+\sqrt{5})x)\big)+$

$e^{\sqrt{5}(7+2x)}\big(57-23\sqrt{5}+x(25-11\sqrt{5}+5(1+\sqrt{5})x)\big)+$

$e^{7\sqrt{5}}\big(197-43\sqrt{5}+x(95+3\sqrt{5}+5(1+\sqrt{5})x)\big)+e^{\sqrt{5}(1+2x)}$

$\big(32+6\sqrt{5}+2x(25+8\sqrt{5}-5(2+\sqrt{5})x)\big)-2e^{\sqrt{5}}\big(-13(2+\sqrt{5})+5x(9+4\sqrt{5}+(2+\sqrt{5})x)\big)$

$\mathrm{Csch}[\sqrt{5}]^{2}\Big)/\Big(640\sqrt{5}\big(\sqrt{5}\mathrm{Cosh}[\sqrt{5}]-\mathrm{Sinh}[\sqrt{5}]\big)^{3}\Big)$

and

$F_{0}(x,2)=\Big(e^{-\sqrt{5}(2+x)}\Big((1+\sqrt{5})e^{\sqrt{5}}+(-1+\sqrt{5})e^{3\sqrt{5}}+(1+\sqrt{5})e^{\sqrt{5}(1+2x)}+(-1+\sqrt{5})$

$e^{\sqrt{5}(3+2x)}-2\sqrt{5}e^{\sqrt{5}(2+x)}(2+x)+e^{\sqrt{5}(4+x)}\big(-4(-5+\sqrt{5})x\big)+e^{\sqrt{5}x}\big(4+(5+\sqrt{5})x\big)\Big)$

$\mathrm{Csch}\left[\sqrt{5}\right])/\left(8\left(-5\mathrm{Cosh}\left[\sqrt{5}\right]+\sqrt{5}\mathrm{Sinh}\left[\sqrt{5}\right]\right)\right)$

So, $F_0(x,2)+F_{12}(x)=\frac{1}{640\sqrt{5}}$

$\left(-\left(160e^{-\sqrt{5}x}\left(\left(1+\sqrt{5}\right)e^{\sqrt{5}}+\left(-1+\sqrt{5}\right)e^{3\sqrt{5}}+\left(1+\sqrt{5}\right)e^{\sqrt{5}(1+2x)}+\left(-1+\sqrt{5}\right)e^{\sqrt{5}(3+2x)}-\right.\right.\right.$

$2\sqrt{5}\, e^{\sqrt{5}(2+x)}(2+x)\ +\ e^{\sqrt{5}(4+x)}\left(-4+\left(-5+\sqrt{5}\right)x\right)+e^{\sqrt{5}x}\left(4+\left(5+\sqrt{5}\right)x\right)\Big)$

$\left(-1+\mathrm{Coth}\left[\sqrt{5}\right]\right)\Big)/$

$\left(1+\sqrt{5}+\left(-1+\sqrt{5}\right)e^{2\sqrt{5}}\right)+\left(e^{\sqrt{5}(5+x)}\left(e^{\sqrt{5}(2+x)}\left(-189-69\sqrt{5}+40x+28\sqrt{5}x\right)+\right.\right.$

$e^{\sqrt{5}(8+x)}\left(-189+69\sqrt{5}+4\left(10-7\sqrt{5}\right)x\right)-2e^{\sqrt{5}(10+x)}\left(-24+11\sqrt{5}+\left(-5+\sqrt{5}\right)x\right)+$

$2e^{\sqrt{5}x}\left(24+11\sqrt{5}+\left(5+\sqrt{5}\right)x\right)+e^{\sqrt{5}(6+x)}\left(-359+63\sqrt{5}+2\left(-125+7\sqrt{5}\right)x\right)-$

$e^{\sqrt{5}(4+x)}\left(359+63\sqrt{5}+2\left(125+7\sqrt{5}\right)x\right)+6e^{\sqrt{5}(5+2x)}\left(27+x\left(20-7\sqrt{5}+5x\right)\right)+$

$6e^{5\sqrt{5}}\left(27+x\left(20+\ 7\sqrt{5}+5x\right)\right)\ +\ 2e^{\sqrt{5}(9+2x)}$

$2e^{9\sqrt{5}}\left(16-3\sqrt{5}+x\left(25-8\sqrt{5}+5\left(-2+\sqrt{5}\right)x\right)\right)+$

$e^{\sqrt{5}(3+2x)}\left(197+43\sqrt{5}+x\left(95-3\sqrt{5}-5\left(-1+\sqrt{5}\right)x\right)\right)+$

$e^{3\sqrt{5}}\left(57+23\sqrt{5}+x\left(25+11\sqrt{5}-5\left(-1+\sqrt{5}\right)x\right)\right)+$

$e^{\sqrt{5}(7+2x)}\left(57-23\sqrt{5}+x\left(25-11\sqrt{5}+5\left(1+\sqrt{5}\right)x\right)\right)+$

$e^{7\sqrt{5}}\left(197-43\sqrt{5}+x\left(95+3\sqrt{5}+5\left(1+\sqrt{5}\right)x\right)\right)+$

$e^{\sqrt{5}(1+2x)}\left(32+6\sqrt{5}+2x\left(25+8\sqrt{5}-5\left(2+\sqrt{5}\right)x\right)\right)-$

$2e^{\sqrt{5}}\left(-13\left(2+\sqrt{5}\right)+5x\left(9+4\sqrt{5}+\left(2+\sqrt{5}\right)x\right)\right)\Big)$

$\mathrm{Csch}\left[\sqrt{5}\right]^2\ )\ /\left(\sqrt{5}\mathrm{Cosh}\left[\sqrt{5}\right]-\mathrm{Sinh}\left[\sqrt{5}\right]\right)^3)$

When $\lambda=1, m=3, \alpha=\sqrt{\frac{5}{2}}$

$$F_{13}(x)=\Big(-5+\sqrt{10})x)-6e^{\sqrt{\frac{5}{2}}(10+x)}\left(-116+37\sqrt{10}+2\left(-5+\sqrt{10}\right)x\right)+$$

$$6e^{\sqrt{\frac{5}{2}}x}\left(116+37\sqrt{10}+2\left(5+\sqrt{10}\right)x\right)+2e^{\sqrt{\frac{5}{2}}(2+x)}$$

$$\left(-3\left(368+1\,09\sqrt{10}\right)+14\left(5+\sqrt{10}\right)x\right)+$$

$$4e^{\sqrt{\frac{5}{2}}(6+x)}\left(-622+107\sqrt{10}+\left(-450+76\sqrt{10}\right)x\right)-4e^{\sqrt{\frac{5}{2}}(4+x)}$$

$$\left(622+107\sqrt{10}+\left(450+76\sqrt{10}\right)x\right)-$$

$$2e^{7\sqrt{\frac{5}{2}}}\left(-832+179\sqrt{10}+6\left(-35+\sqrt{10}\right)x+5\left(-8+\sqrt{10}\right)x^2\right)+$$

$$2e^{\sqrt{\frac{5}{2}}(3+2x)}\left(832+179\sqrt{10}+6\left(35+\sqrt{10}\right)x+5\left(8+\sqrt{10}\right)x^2\right)-4e^{\sqrt{\frac{5}{2}}(5+2x)}$$

$$\left(-306++25\sqrt{10}-9x\left(20-7\sqrt{10}+5x\right)\right)+4e^{5\sqrt{\frac{5}{2}}}\left(306+25\sqrt{10}+9x\left(20+7\sqrt{10}+5x\right)\right)-$$

$$2e^{\sqrt{\frac{5}{2}}(7+2x)}\left(-312+99\sqrt{10}+x\left(-350+118\sqrt{10}+5\left(-8+\sqrt{10}\right)x\right)\right)+$$

$$2e^{3\sqrt{\frac{5}{2}}}\left(312+99\sqrt{10}+x\left(350+118\sqrt{10}+5\left(8+\sqrt{10}\right)x\right)\right)+$$

$$e^{\sqrt{\frac{5}{2}}(9+2x)}\left(304-101\sqrt{10}+5x\left(-178+56\sqrt{10}+\left(-34+11\sqrt{10}\right)x\right)\right)+$$

$$e^{9\sqrt{\frac{5}{2}}}\left(184-41\sqrt{10}+x\left(650-196\sqrt{10}+5\left(-34+11\sqrt{10}\right)x\right)\right)+$$

$$e^{\sqrt{\frac{5}{2}}(1+2x)}\left(184+41\sqrt{10}+x\left(650+196\sqrt{10}-5\left(34+11\sqrt{10}\right)x\right)\right)+$$

$$e^{\sqrt{\frac{5}{2}}}\left(304+101\sqrt{10}-5x\left(178+56\sqrt{10}+\left(34+11\sqrt{10}\right)x\right)\right)/$$

$$\left(20\sqrt{10}\left(-1+e^{\sqrt{10}}\right)^2\left(2+\sqrt{10}+\left(-2+\sqrt{10}\right)e^{\sqrt{10}}\right)^3\right)$$

And $F_0(x,3)=$

$$\left(e^{\sqrt{\frac{5}{2}}x}\left(-(2+\sqrt{10})e^{-\sqrt{\frac{5}{2}}}-(-2+\sqrt{10})e^{3\sqrt{\frac{5}{2}}}-(2+\sqrt{10})e^{\sqrt{\frac{5}{2}}(1+2x)}-(-2+\sqrt{10})e^{\sqrt{\frac{5}{2}}(3+2x)}+\right.\right.$$

$$\left.\left.2\sqrt{10}e^{\sqrt{\frac{5}{2}}(2+x)}(2+x)+e^{\sqrt{\frac{5}{2}}(4+x)}\left(3-(-5+\sqrt{10})x\right)-e^{\sqrt{\frac{5}{2}}x}\left(3+\left(5+\sqrt{10}\right)x\right)\right)\right)/$$

$$\left(\sqrt{10}\left(-1+e^{\sqrt{10}}\right)\left(2+\sqrt{10}+\left(-2+\sqrt{10}\right)e^{\sqrt{10}}\right)\right)$$

$$\text{So, } F_0(x,3)+F_{13}(x)=\left(e^{-\sqrt{\frac{5}{2}}x}\left(-e^{\sqrt{\frac{5}{2}}(6+x)}\left(-935+\sqrt{10}+5\left(-176+35\sqrt{10}\right)x\right)-\right.\right.$$

$$4e^{\sqrt{\frac{5}{2}}(4+x)}\left(935+\sqrt{10}+5\left(176+35\sqrt{10}\right)x\right)-$$

$$2e^{\sqrt{\frac{5}{2}}(8+x)}\left(-5035+1382\sqrt{10}+15\left(-162+41\sqrt{10}\right)x\right)-$$

$$2e^{\sqrt{\frac{5}{2}}(2+x)}\left(5035+1382\sqrt{10}+15\left(162+41\sqrt{10}\right)x\right)+2e^{\sqrt{\frac{5}{2}}(10+x)}$$

$$\left(-1155+384\sqrt{10}+5\left(-346+133\sqrt{10}\right)x\right)+2e^{\sqrt{\frac{5}{2}}x}\left(1155+384\sqrt{10}+5\left(346+113\sqrt{10}\right)x\right)+$$

$$2e^{\sqrt{\frac{5}{2}}(5+2x)}\left(-250-54\sqrt{10}+45x\left(-14+4\sqrt{10}+\sqrt{10}x\right)\right)+$$

$$2e^{5\sqrt{\frac{5}{2}}}\left(250-54\sqrt{10}+45x\left(14+4\sqrt{10}+\sqrt{10}x\right)\right)+$$

$$e^{\sqrt{\frac{5}{2}}(9+2x)}\left(-2705+832\sqrt{10}+5x\left(280-89\sqrt{10}+\left(55-17\sqrt{10}\right)x\right)\right)+$$

$$e^{9\sqrt{\frac{5}{2}}}\left(-2405+772\sqrt{10}5x\left(-196+65\sqrt{10}+\left(55-17\sqrt{10}\right)x\right)\right)+$$

$$\left(20\sqrt{10}\left(-1+e^{\sqrt{10}}\right)^2\left(2+\sqrt{10}+\left(-2+\sqrt{10}\right)e^{\sqrt{10}}\right)^3\right)$$

And $F_0(x,3) =$

$$\left(e^{\sqrt{\frac{5}{2}}x}\left(-\left(2+\sqrt{10}\right)e^{-\sqrt{\frac{5}{2}}}-\left(-2+\sqrt{10}\right)e^{3\sqrt{\frac{5}{2}}}-\left(2+\sqrt{10}\right)e^{\sqrt{\frac{5}{2}}(1+2x)}-\left(-2+\sqrt{10}\right)e^{\sqrt{\frac{5}{2}}(3+2x)}+\right.\right.$$

$$\left.\left.2\sqrt{10}e^{\sqrt{\frac{5}{2}}(2+x)}(2+x)+e^{\sqrt{\frac{5}{2}}(4+x)}\left(3-\left(-5+\sqrt{10}\right)x\right)-e^{\sqrt{\frac{5}{2}}x}\left(3+\left(5+\sqrt{10}\right)x\right)\right)\right)/$$

$$\left(\sqrt{10}\left(-1+e^{\sqrt{10}}\right)\left(2+\sqrt{10}+\left(-2+\sqrt{10}\right)e^{\sqrt{10}}\right)\right)$$

So, $F_0(x,3)+F_{13}(x) = \left(e^{-\sqrt{\frac{5}{2}}x}\left(-e^{\sqrt{\frac{5}{2}}(6+x)}\left(-935+\sqrt{10}+5\left(-176+35\sqrt{10}\right)x\right)-\right.\right.$

$$4e^{\sqrt{\frac{5}{2}}(4+x)}\left(935+\sqrt{10}+5\left(176+35\sqrt{10}\right)x\right)-$$

$$2e^{\sqrt{\frac{5}{2}}(8+x)}\left(-5035+1382\sqrt{10}+15\left(-162+41\sqrt{10}\right)x\right)-$$

$$2e^{\sqrt{\frac{5}{2}}(2+x)}\left(5035+1382\sqrt{10}+15\left(162+41\sqrt{10}\right)x\right)+2e^{\sqrt{\frac{5}{2}}(10+x)}$$

$$\left(-1155+384\sqrt{10}+5\left(-346+133\sqrt{10}\right)x\right)+2e^{\sqrt{\frac{5}{2}}x}\left(1155+384\sqrt{10}+5\left(346+113\sqrt{10}\right)x\right)+$$

$$2e^{\sqrt{\frac{5}{2}}(5+2x)}\left(-250-54\sqrt{10}+45x\left(-14+4\sqrt{10}+\sqrt{10}x\right)\right)+$$

$$2e^{5\sqrt{\frac{5}{2}}}\left(250-54\sqrt{10}+45x\left(14+4\sqrt{10}+\sqrt{10}x\right)\right)+$$

$$e^{\sqrt{\frac{5}{2}}(9+2x)}\left(-2705+832\sqrt{10}+5x\left(280-89\sqrt{10}+\left(55-17\sqrt{10}\right)x\right)\right)+$$

$$e^{9\sqrt{\frac{5}{2}}}\left(-2405+772\sqrt{10}5x\left(-196+65\sqrt{10}+\left(55-17\sqrt{10}\right)x\right)\right)+$$

$$\left(20\sqrt{10}\left(-1+e^{\sqrt{10}}\right)^2\left(2+\sqrt{10}+\left(-2+\sqrt{10}\right)e^{\sqrt{10}}\right)^3\right)$$

And $F_0(x,3)=$

$$\Bigg(e^{\sqrt{\frac{5}{2}}x}\Bigg(-\left(2+\sqrt{10}\right)e^{-\sqrt{\frac{5}{2}}}-\left(-2+\sqrt{10}\right)e^{3\sqrt{\frac{5}{2}}}-\left(2+\sqrt{10}\right)e^{\sqrt{\frac{5}{2}}(1+2x)}-\left(-2+\sqrt{10}\right)e^{\sqrt{\frac{5}{2}}(3+2x)}+$$

$$2\sqrt{10}e^{\sqrt{\frac{5}{2}}(2+x)}(2+x)+e^{\sqrt{\frac{5}{2}}(4+x)}\left(3-\left(-5+\sqrt{10}\right)x\right)-e^{\sqrt{\frac{5}{2}}x}\left(3+\left(5+\sqrt{10}\right)x\right)\Bigg)\Bigg)/$$

$$\left(\sqrt{10}\left(-1+e^{\sqrt{10}}\right)\left(2+\sqrt{10}+\left(-2+\sqrt{10}\right)e^{\sqrt{10}}\right)\right)$$

$$\text{So, } F_0(x,3)+F_{13}(x)=\Bigg(e^{-\sqrt{\frac{5}{2}}x}\Bigg(-e^{\sqrt{\frac{5}{2}}(6+x)}\left(-935+\sqrt{10}+5\left(-176+35\sqrt{10}\right)x\right)-$$

$$4e^{\sqrt{\frac{5}{2}}(4+x)}\left(935+\sqrt{10}+5\left(176+35\sqrt{10}\right)x\right)-$$

$$2e^{\sqrt{\frac{5}{2}}(8+x)}\left(-5035+1382\sqrt{10}+15\left(-162+41\sqrt{10}\right)x\right)-$$

$$2e^{\sqrt{\frac{5}{2}}(2+x)}\left(5035+1382\sqrt{10}+15\left(162+41\sqrt{10}\right)x\right)+2e^{\sqrt{\frac{5}{2}}(10+x)}$$

$$\left(-1155+384\sqrt{10}+5\left(-346+133\sqrt{10}\right)x\right)+2e^{\sqrt{\frac{5}{2}}x}\left(1155+384\sqrt{10}+5\left(346+113\sqrt{10}\right)x\right)+$$

$$2e^{\sqrt{\frac{5}{2}}(5+2x)}\left(-250-54\sqrt{10}+45x\left(-14+4\sqrt{10}+\sqrt{10}x\right)\right)+$$

$$2e^{5\sqrt{\frac{5}{2}}}\left(250-54\sqrt{10}+45x\left(14+4\sqrt{10}+\sqrt{10}x\right)\right)+$$

$$e^{\sqrt{\frac{5}{2}}(9+2x)}\left(-2705+832\sqrt{10}+5x\left(280-89\sqrt{10}+\left(55-17\sqrt{10}\right)x\right)\right)+$$

$$e^{9\sqrt{\frac{5}{2}}}\left(-2405+772\sqrt{10}5x\left(-196+65\sqrt{10}+\left(55-17\sqrt{10}\right)x\right)\right)+$$

$$2e^{7\sqrt{\frac{5}{2}}}\left(-695+256\sqrt{10}+5x\left(-6+21\sqrt{10}+\left(-5+4\sqrt{10}\right)x\right)\right)+$$

$$2e^{\sqrt{\frac{5}{2}}(7+2x)}\left(-295-4\sqrt{10}+5x\left(-118+35\sqrt{10}+\left(-5+4\sqrt{10}\right)x\right)\right)+$$

$$2e^{\sqrt{\frac{5}{2}}(3+2x)}\left(695+256\sqrt{10}+5x\left(6+21\sqrt{10}+\left(5+4\sqrt{10}\right)x\right)\right)+$$

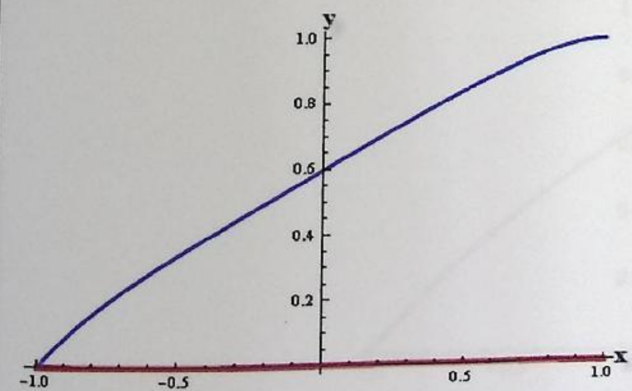
$$2e^{3\sqrt{\frac{5}{2}}}\left(295-4\sqrt{10}+5x\left(118+35\sqrt{10}+\left(5+4\sqrt{10}\right)x\right)\right)+$$

$$e^{\sqrt{\frac{5}{2}}(1+2x)}\left(2405+772\sqrt{10}+5x\left(196+65\sqrt{10}-\left(55+17\sqrt{10}\right)x\right)\right)+$$

$$e^{\sqrt{\frac{5}{2}}}\left(2705+832\sqrt{10}-5x\left(280+89\sqrt{10}+\left(55+17\sqrt{10}\right)x\right)\right)\Bigg)\Bigg)/$$

$$\left(100\left(-1+e^{\sqrt{10}}\right)^{2}\left(2+\sqrt{10}+\left(-2+\sqrt{10}\right)e^{\sqrt{10}}\right)^{3}\right)$$

The following graphs represent the velocity distribution for different values of m and $\boldsymbol{\lambda}$.



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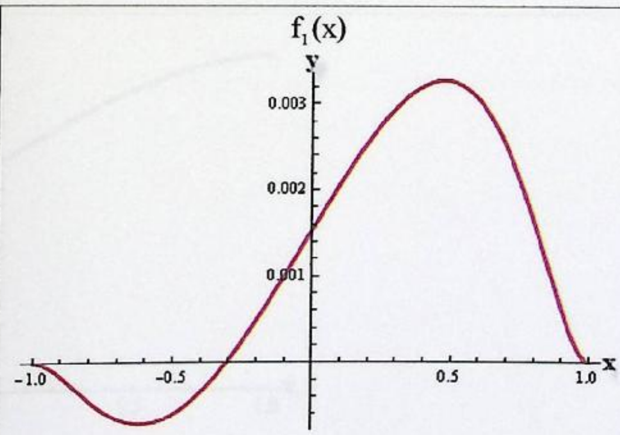
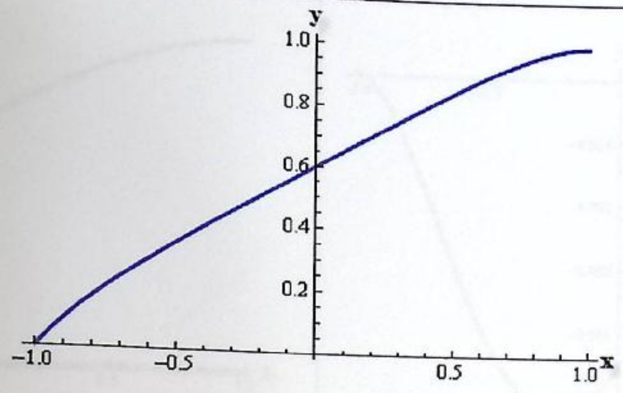
The blue graph is $f_0(x)$ and the red graph is $f_1(x)$ corresponding to $m = 0,\ \lambda = 1$

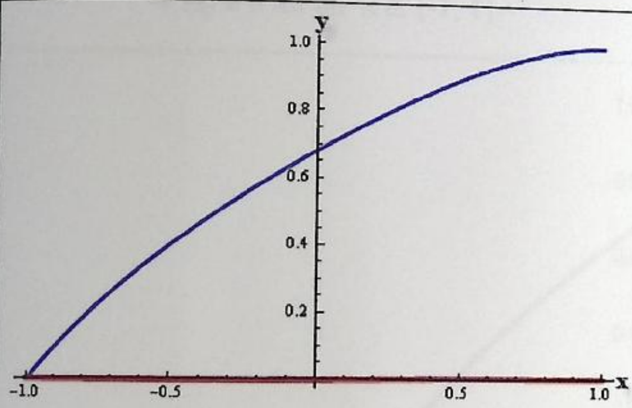


$m = 0,\ \lambda = 1$

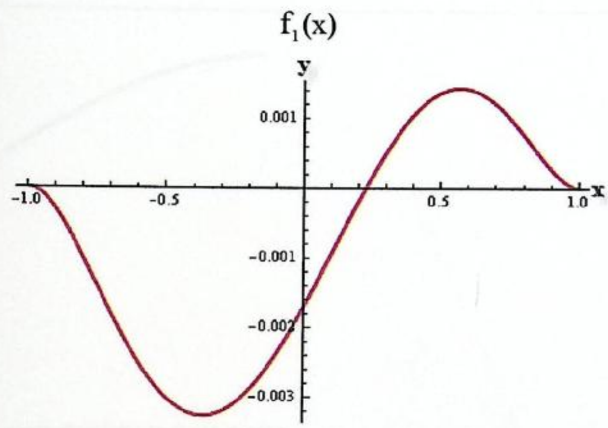
The above plot indicate that $f_1(x) \approx 0$ on [-1, 1]



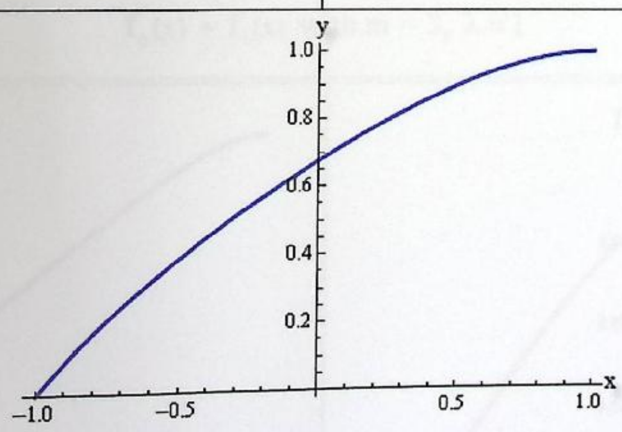
$f_0(x) + f_1(x)$ corresponding to $m = 0,\ \lambda = 1$



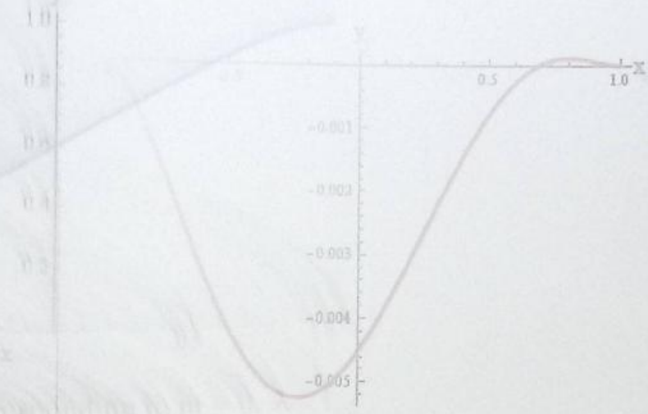
The blue graph is $f_0(x)$ and the red graph is $f_1(x)$ corresponding to $m = 2,\ \lambda = 1$



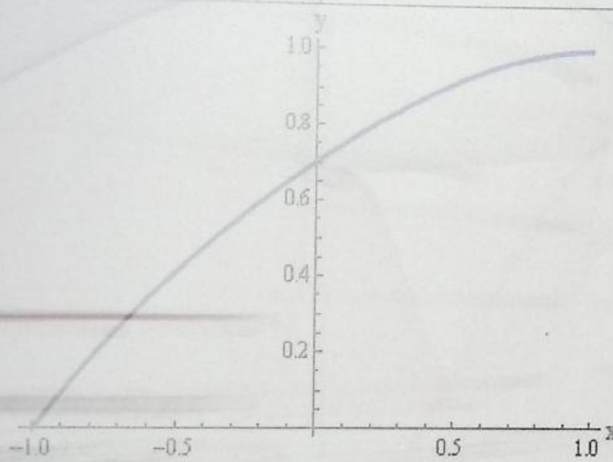
$m = 2,\ \lambda = 1$



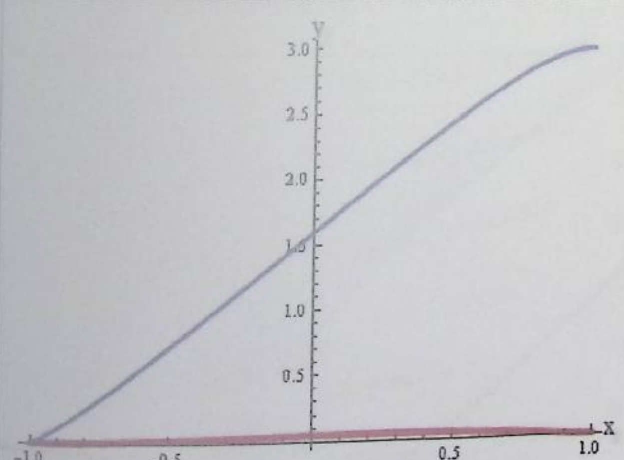
$f_0(x) + f_1(x)$ corresponding to $m = 2,\ \lambda = 1$



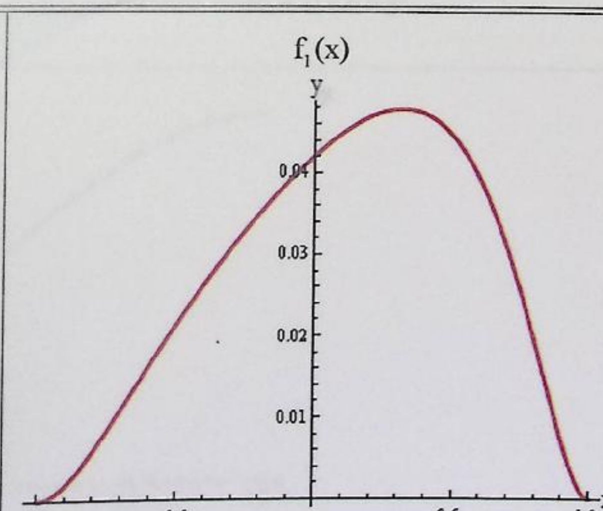
$f_1(x)$ corresponding to $m = 3, \lambda = 1$



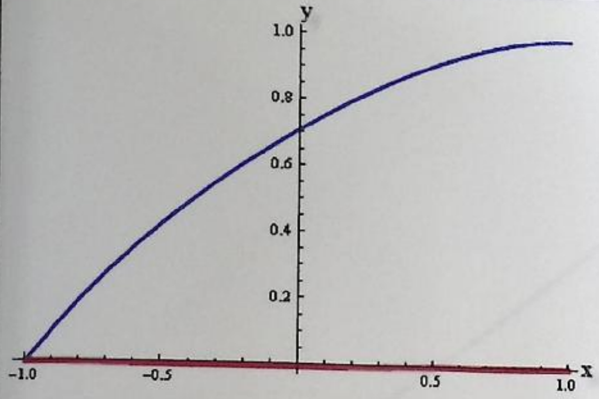
$f_0(x) + f_1(x)$ with $m = 3,\ \lambda = 1$



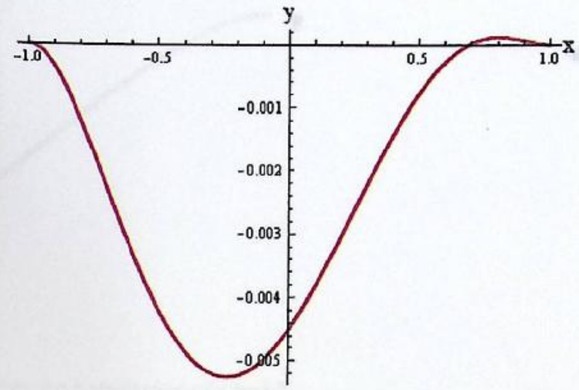
The blue graph is $f_0(x)$ and the red graph is $f_1(x)$ corresponding to $m = 0,\ \lambda = 3$



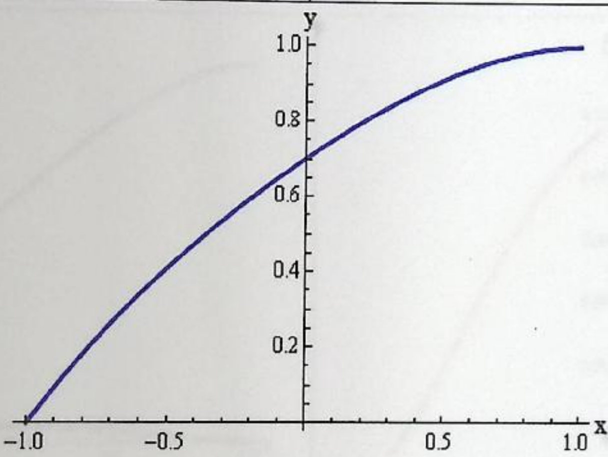
$m = 0,\ \lambda = 3$



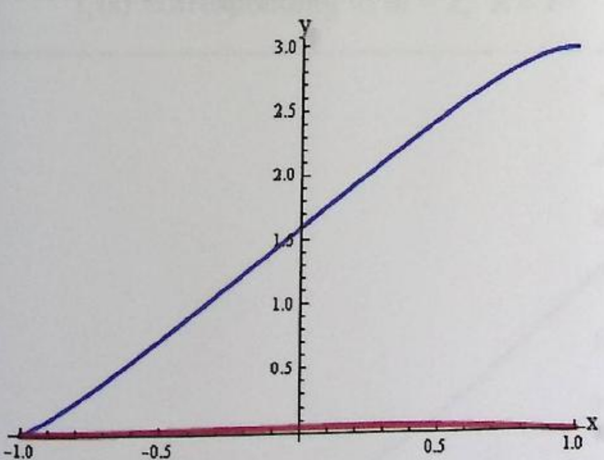
$f_0(x)$ together with $f_1(x)$ with $m = 3,\ \lambda = 1$
$f_1(x) \approx 0$ for all $x \in [-1, 1]$



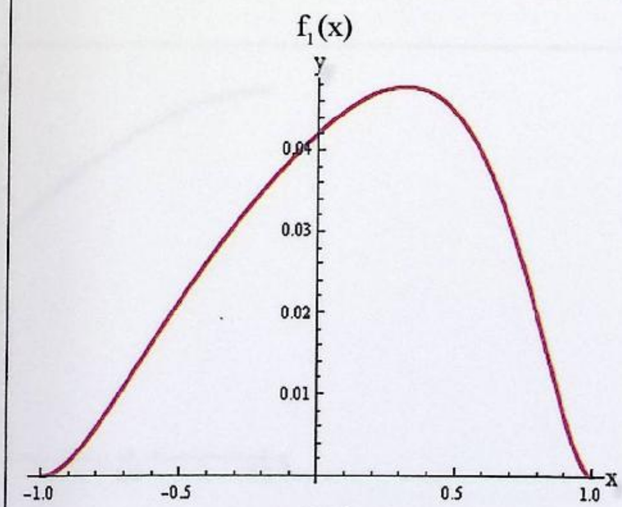
$f_1(x)$ corresponding to $m = 3, \lambda = 1$



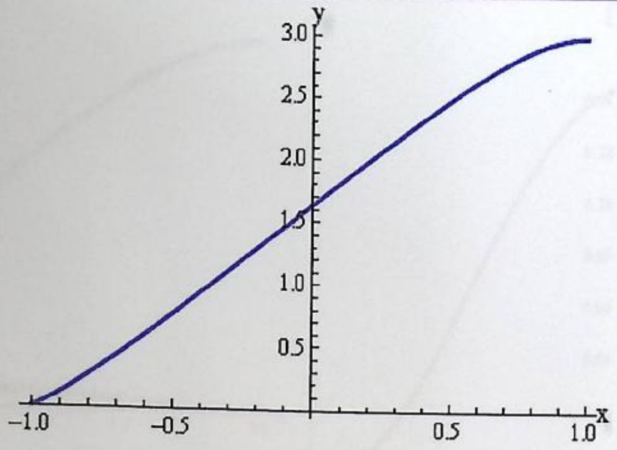
$f_0(x) + f_1(x)$ with $m = 3,\ \lambda = 1$



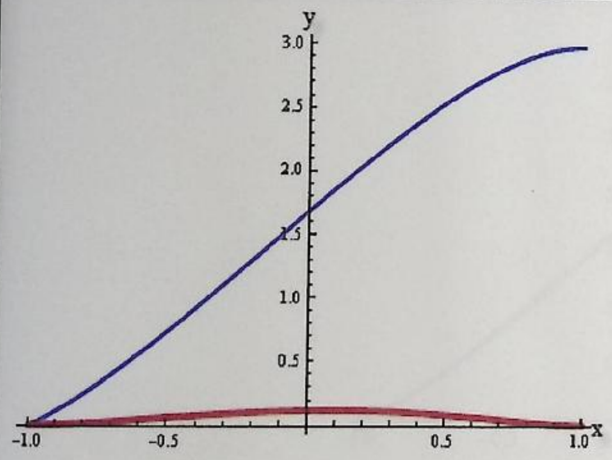
The blue graph is $f_0(x)$ and the red graph is $f_1(x)$ corresponding to $m = 0,\ \lambda = 3$



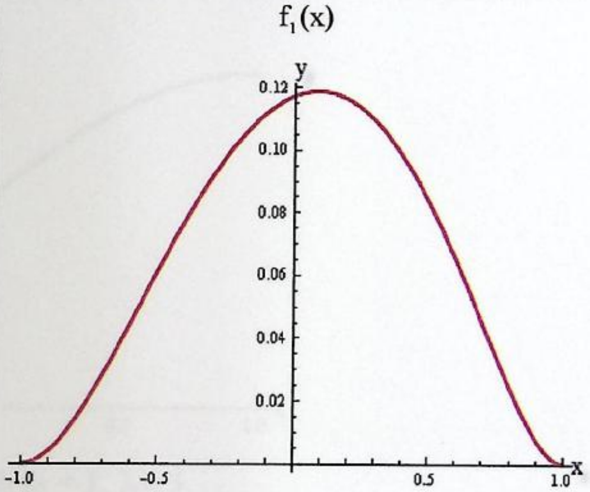
$m = 0,\ \lambda = 3$



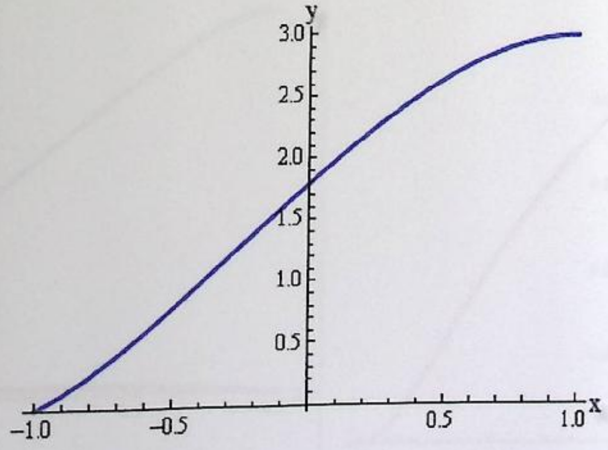
$f_0(x) + f_1(x)$ corresponding to $m = 0,\ \lambda = 3$



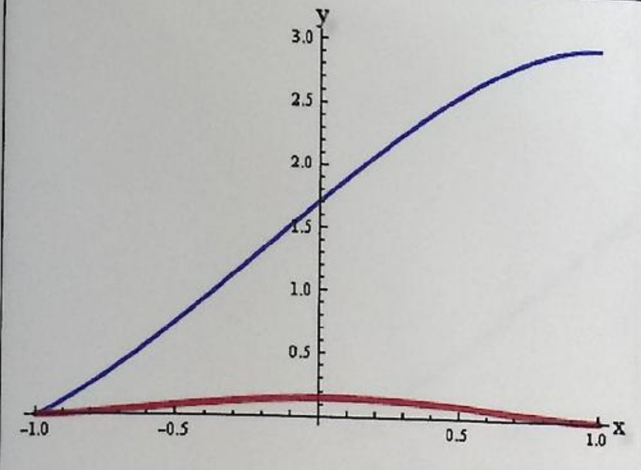
The blue graph is $f_0(x)$ and the red graph is $f_1(x)$ corresponding to $m = 2,\ \lambda = 3$



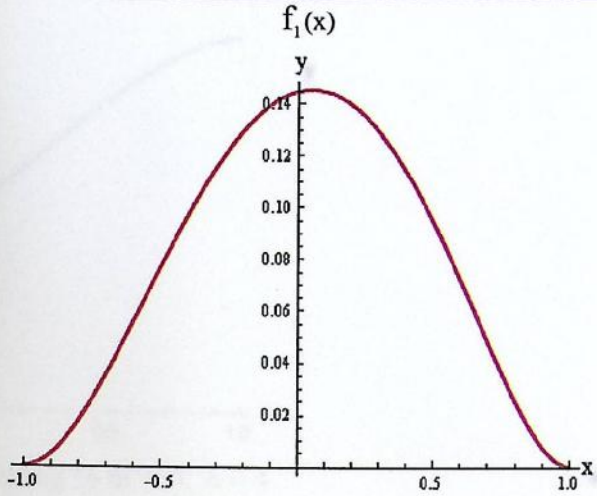
$m = 2,\ \lambda = 3$



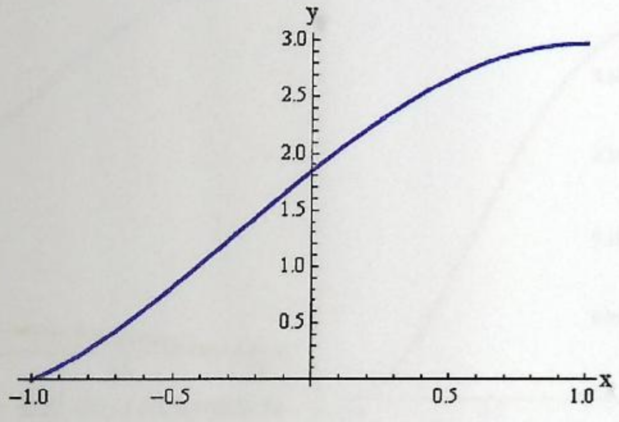
$f_0(x) + f_1(x)$ corresponding to $m = 2,\ \lambda = 3$



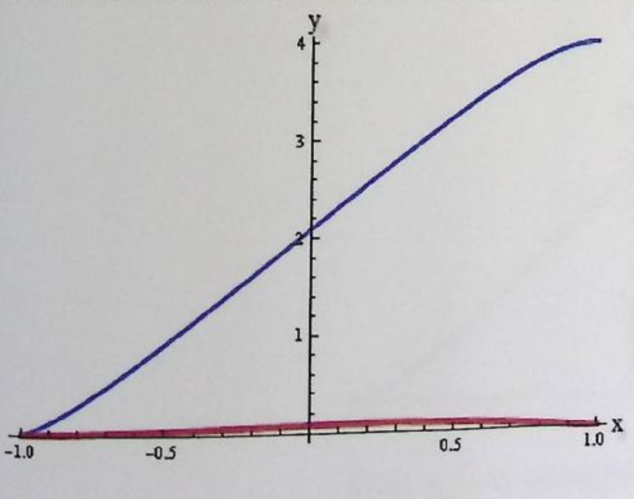
$f_0(x)$ together with $f_1(x)$ with $m = 3$, $\lambda = 3$



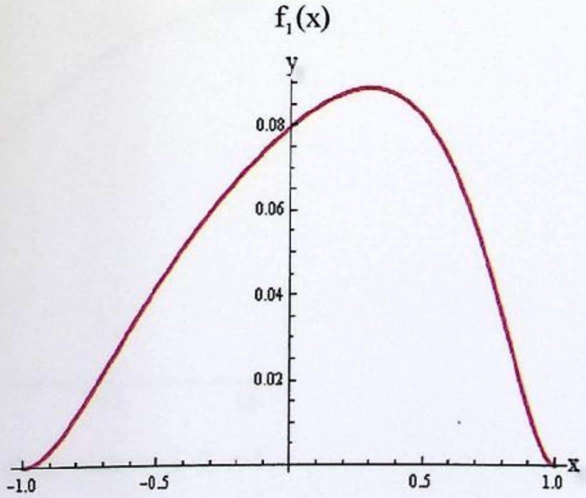
$m = 3, \lambda = 3$



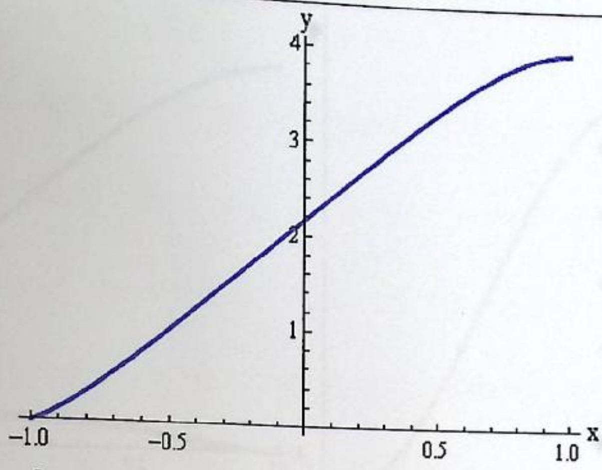
$f_0(x) + f_1(x)$ with $m = 3$, $\lambda = 3$



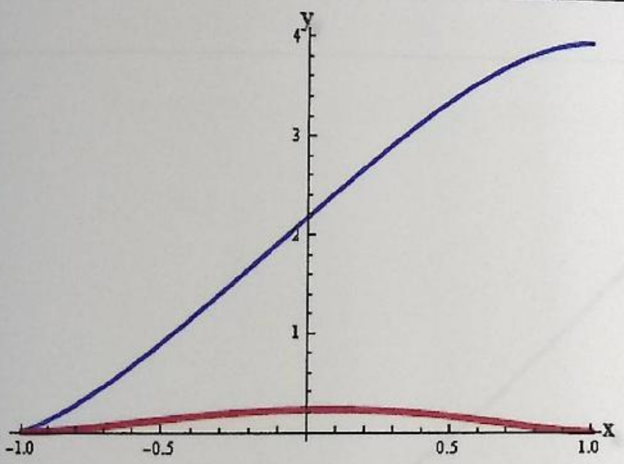
The blue graph is $f_0(x)$ and the red graph is $f_1(x)$ corresponding to $m = 0$, $\lambda = 4$



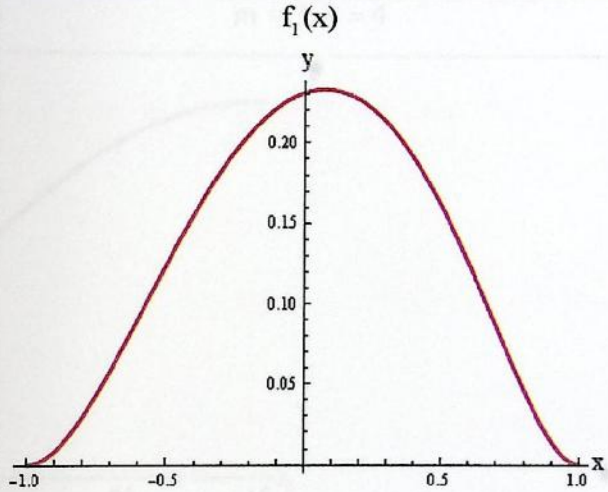
$m = 0$, $\lambda = 4$



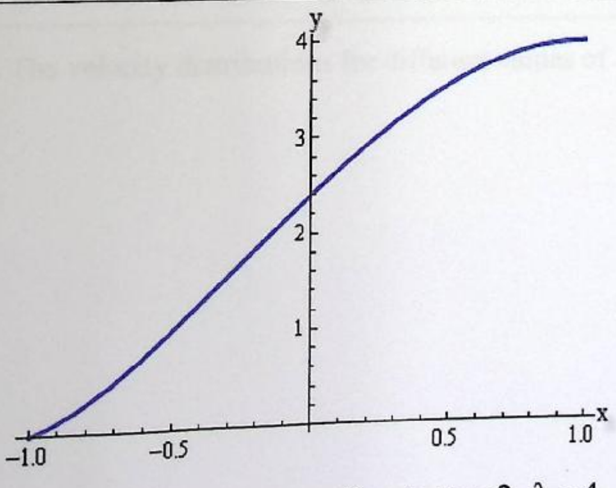
$f_0(x) + f_1(x)$ corresponding to $m = 0$, $\lambda = 4$



The blue graph is $f_0(x)$ and the red graph is $f_1(x)$ corresponding to $m = 2$, $\lambda = 4$



$m = 2$, $\lambda = 4$



$f_0(x) + f_1(x)$ corresponding to $m = 2$, $\lambda = 4$

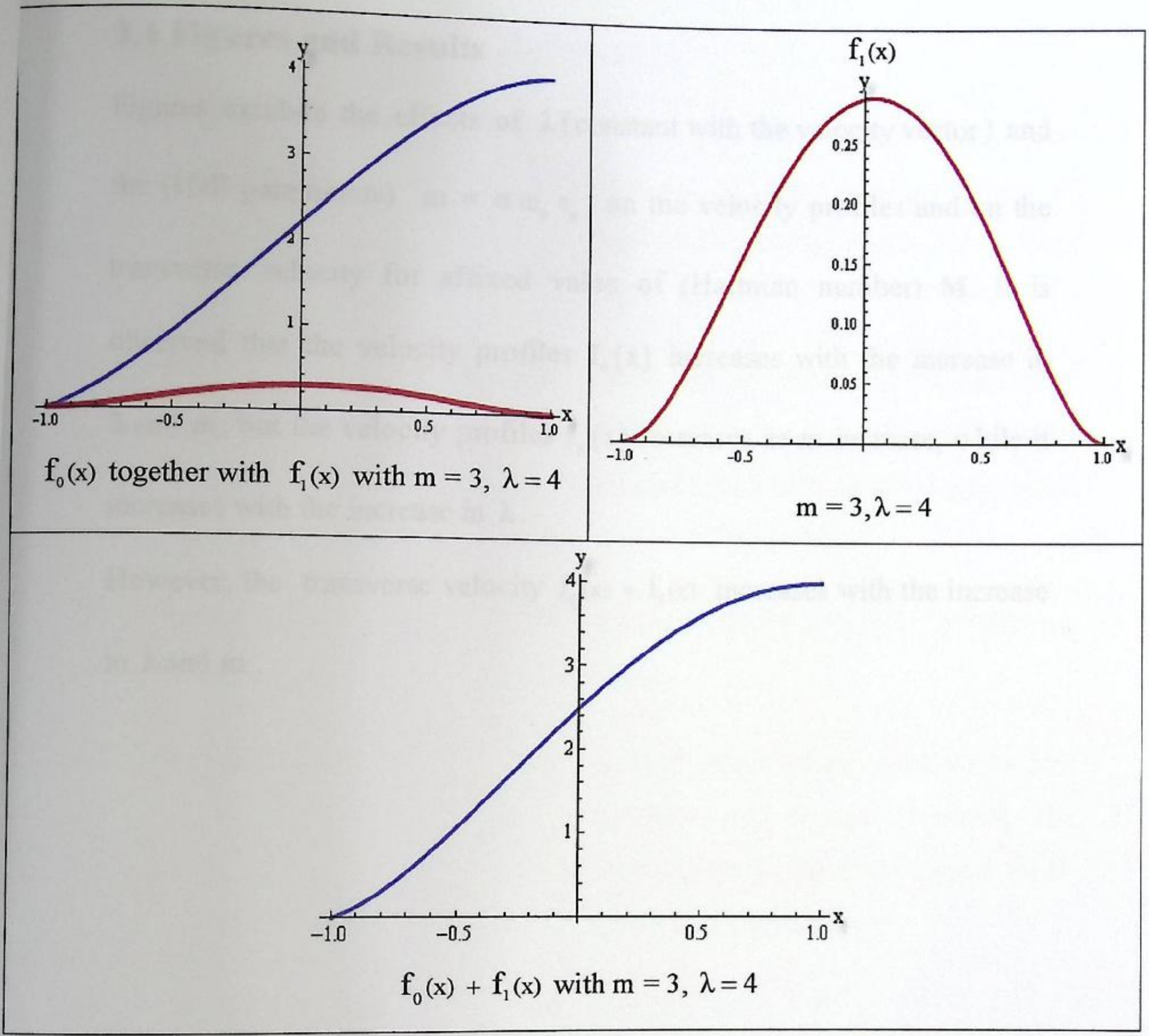


Fig 3.1: The velocity distributions for different values of m and λ

3.4 Figures and Results

Figures exhibits the effects of λ(constant with the velocity vector) and the (Hall parameters) $m = \sigma \omega_e \tau_e$ on the velocity profiles and on the transverse velocity for affixed value of (Hartman number) **M**. It is observed that the velocity profiles $f_0(x)$ increases with the increase in λ and **m**, but the velocity profiles $f_1(x)$ decreases as **m** increase, while it increases with the increase in λ.

However, the transverse velocity $f_0(x) + f_1(x)$ increases with the increase in λ and m .

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