Modes of Motion as an Alternative Approach for Mobile Robot Kinematic Constraint Representation

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Abstract— **This paper presents original concepts to represent and deal with kinematic constraints imposed on mobile robots. This is based on formulations derived originally for the dynamics of multibody systems and applied to robotic manipulator arms. This representation has several advantages such as its ability to deal with kinematically excited modes, derive a physically meaningful state-space representation, and find corresponding constraining forces. The paper shows how to apply these concepts to different types of mobile robots such as those modelled as unicycle or bicycle.**

I. Introduction

Most of the time, robotic manipulator arms are free to move without constraints except when their end effectors come in contact with external objects or surfaces. This is the case when such robots are employed to assemble objects, machine certain surfaces, or cut pieces. On the other side, mobile robots which move on the ground are always subject to motion constraints. These constraints are of different types depending on the locomotion method used. Wheeled mobile robots are subject to constraints making it possible for the wheel-ground contact point to move along the direction of the wheel only and not having any motion component orthogonal to it. Such constraints are classified mainly as holonomic and nonholonomic. Most mobile robots are subject to nonholonomic constraints limiting their motion at the velocity level, while leaving them capable of reaching any desired posture. This requires usually some sort of maneuvering similar to car parallel parking.

The mainstream approach for modeling the kinematics of mobile robots and specifically the representation of constraints has origins in differential geometry. Lie algebra is often used to check the maneuverability and controllability of those robots. This approach derives kinematic models with easy to interpret variables most of the time. However, this is not always the case. This becomes more profound when looking at constraining forces and even reduced dynamic models. Sometimes, the variables used in those reduced models do not have physical meanings.

Kinematics with motion constraints is well studied within classical mechanics. For example, multibody dynamics offer nice formulations and tools to deal neatly and systematically with various types of constraints including holonomic, nonholonomic, kinematically excited motions, and others. In this paper, we will show how to apply such formulations to mobile robots.

The paper is organized as follows. In section 2, the so called "modes of motion" formulation is presented. It is shown how different types of constraints are dealt with. Further, it is shown how to derive the kinematic model and how to find the constraining forces. This will be applied to mobile robots in Section 2. Two examples will be done in detail. These are the popular unicycle- and bicycle-like mobile robots.

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II. Modes of Motion

The analysis presented here is based on the mathematical tools and notations of Roberson and Schwertassek [1] and developed for constrained robots by Tahboub [2]. Most of the notations will be adopted, however some formulas and results will be rewritten to make the derivation more docile and aim oriented. The aim of tackling the issue of constraints in the case of robotic arms is to control contact forces within the framework of force control or what is classically known as hybrid position-force control. However, for the case of mobile robots, the aim is different and is usually limited to motion planning and control in addition to obtaining reduced state-space models without any explicit force control. Still, the presented modeling methodology is applicable as will be shown.

First, the general kinematic constraint equation on the motion of the end effector in the task frame can be given by

$$W^T \begin{bmatrix} V \\ \boldsymbol{\Omega} \end{bmatrix} = \boldsymbol{\zeta}(t), \tag{1}$$

where $\boldsymbol{V}$and $\boldsymbol{\Omega}$ denote the linear and angular velocities respectively, $\boldsymbol{W} \in R^{6\times m}$, $\boldsymbol{\zeta} \in R^m$, and m denotes the number of constraints. The general solution of (1) in terms of a fewer number of velocity state variables $\dot{\boldsymbol{P}}$ is

$$\begin{bmatrix} \boldsymbol{V} \\ \boldsymbol{\Omega} \end{bmatrix} = \boldsymbol{\Phi}\dot{\boldsymbol{P}} + \boldsymbol{\xi}, \tag{2}$$

with $\boldsymbol{\Phi} \in R^{6\times(6-m)}$, $\dot{\boldsymbol{P}} \in R^{6-m}$, and $\boldsymbol{\xi} \in R^6$. The interpretation of (1) and (2) depends strongly on the method used for the reduction of the dynamical equations. Here, the interpretation is given in terms of modes of motion which keep geometrical characteristics of the constrained motion clearly in view.

The velocity variables $[\boldsymbol{V}^T \quad \boldsymbol{\Omega}^T]^T$ form an element of the six-dimensional vector space R^6. A set of spanning vectors for this space is collected as columns of a matrix

$$\widehat{\boldsymbol{\Phi}} = [\widehat{\boldsymbol{\Phi}}_i] \quad i = 1,2,\ldots,6. \tag{3}$$

This set of vectors is chosen according to the imposed constraints and types of motions allowed. This will be demonstrated by two examples. Any vector in the space can be