Operation Sense

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Abstract

The aim of this article is to develop the issue of operation sense of fractions through the conditions in real-world for the students in 7th grade, so this advancing students' mathematical thinking in every day mathematics lessons , which necessary for developing , and improving the higher order thinking skills for the students'.

Key words : Operation sense ,Number sense , Fractions , Addition , Subtraction, Division , Multiplication , Real-world , 7th grad .

INTRODUCTION

Developing operation sense is an important math topic , and it is a basic Principle for understanding and handling operations for Natural Number, Whole Number, Real Number, and Complex. Thus the Curriculum and Evaluation Standards for School Mathematics (NCTM,1989) defined *operation sense* as an important issue that helps the students to develop conceptual knowledge through mental and written computational procedures . This can be developed through the conditions in real-world situations, and an awareness of models and the properties of operations. In this article I'm going to present some views from the 7th class where the teacher (is the author of this article) of math encourages his students to develop operation sense .

Addition of fractions

After he had finished teaching the concept of rational number, magnitude, The size, compare fractions, and adding fractions with like

denominators , the teacher wrote the following question on the board :

$$\frac{2}{5}+\frac{3}{7}=$$

Teacher : Class, look at the question . I'm going to ask you to find the answer .

Nader : [went to the board , took a piece of chalk , and wrote 2/5+3/7=5/12 .] Ok my teacher ,it is easy, the answer is 5/12 .

Teacher : "Are you sure Nader ?". [Nader was hesitant ,and didn't reply]

Teacher : Who agrees or disagrees with Nader ? .

[Ommar went to the board , took a piece of chalk , and wrote 5/12 < 3/7 ; 5/12 ≈ 2/5.]

Ommar : I don't agree with this answer , because 5/12 is a fraction that is less than 3/7 and very close to 2/5. So it is impossible to adding two fractions and get an answer that is smaller than one of them , I…I think the answer will be more than 5/12 .

Teacher : But , how do you know that : 5/12 < 3/7and 5/12 ≈ 2/5 .

Ommar : Recall my teacher ,a week ago, you learn us how to compare fractions like these : 5/12 < 3/7 → : 5×7/12×7 <3×12/7×12 , since 5×7<3×12; hence 5/12 < 3/7 . In the same way I compare 5/12 and 2/5 . But I've done both comparisons mentally and fast .

Teacher : Very good Ommar , is their another answer or explanation ?

Adam : Yes teacher , I'm suspicious about the answer of Nader, because I

may change the question like this. I suppose that my father gave me 2/5 JD (Jordanian Dinar) which is about 40 Q (Qursh), and my mother gave me 3/7 JD which is about 40 Q also , so I have about 80 Q . On the contrast 5/12 (the answer of Nader) is worth about 40 Q, so the answer of 2/5+3/7 is about 80 Q out of 1 JD (i.e. 0.80JD) and not 40 Q .

Teacher : Please Adam , explain to us : 2/5 JD which is about 40 Q ; 3/7 JD which is about 40 Q and 5/12 JD = 40 Q .

Adam : Ok, I know that the 1JD is 5/5JD which is 1/5+1/5+1/5+1/5+1/5, so each 1/5 JD have the same quantity of cents i.e. 20 +20 +20 +20 +20 = 100Q . Hence , 2/5 JD = 40Q. I did the same things to 3/7 JD (each) 1/7 JD ≈13Q ; i.e. 3/7 JD ≈40Q). Finally, I estimated 5/12 JD is about 40Q (or 50Q).

Teacher : This is great Adam . Are there other answers ?

Anaas : Yes teacher , I have another answer, my home is about 500 meters away from here (the school) so …. .

Teacher : Please wait , how do you know that Anaas ?.

Anaas : I've measured the distance many times through walking in the mornings and evenings by counting the number of my footpaces . I have found that it is about 700 footpaces, which is near to 500m according to my estimation … .

Teacher : Please continue your argument Anaas.

Anaas : If I walk 2/5 of the distance this means that I walk 200m , and if

I walk another 3/7 of it I walk about 200m thus the distance I have walked is 400m . But 400m is not the 5/12 of the distance , because 5/12 is about 200m .What I do is estimating 2/5 the distance to 200m, 3/7 the distance to 200m , and 5/12 the distance to 200m . I think the answer of the question will be different from Nader's (i.e.5/12).

Teacher :Thank you very much Anaas , is their another answer children ?

Thaer : I can answer the question like this [he draw on the board the figure.1] .

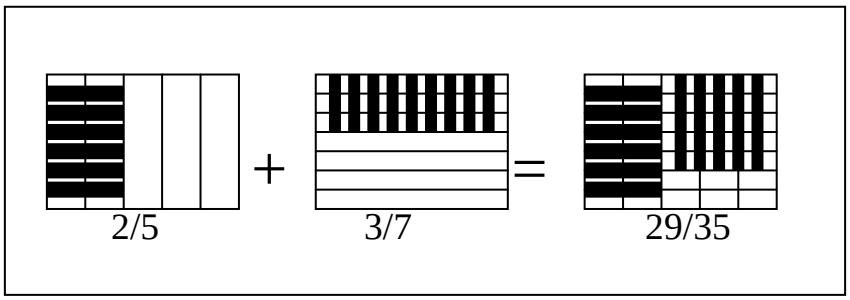

Fig. 1 Thaer method

Teacher : How can you find the answer this way ? .

Thaer: I know this is difficult because the units are not identical …

Teacher : Please , what do you mean by identical ? .

Thaer : I mean both the width and length which are 2/5 and 3/7 are not the same so I need to make them equal, because if I don't do this , I will have the same result that Nader had , which is wrong , hence the correct answer will be 29/35.

Teacher : Ok , but I don’t know what you mean by " I need to make them equal" .

Thaer : I mean to make the number of pieces on each rectangle the same (i.e. 35 parts) in order to count the shadowed parts well , so 2/5 is equivalent to 14/35 and 3/7 is equivalent to 15/35 .

Teacher : Very good Thaer . Ok , we have just 5 minuets more .

and for the final answer we take Kamees .

Kamees : When Anaas represents his answer , I remember my mug that I always drink milk with [Kamees drew a mug on board and marked it with fractions as in fig.2] ,so if the mug has 2/5 of it milk and I added 3/7 of it milk.

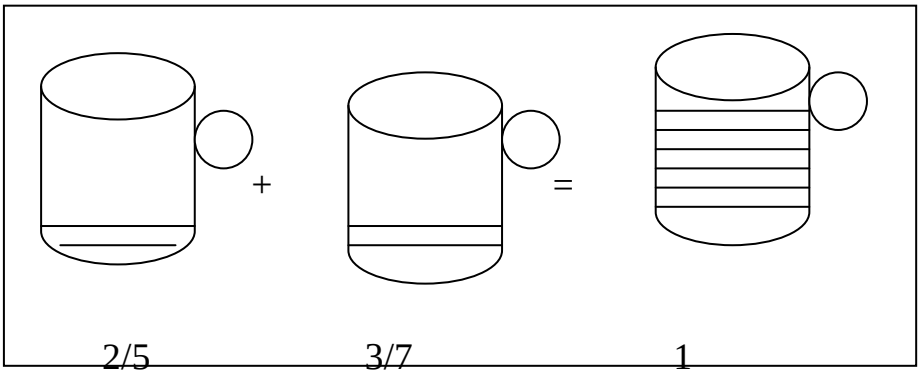

fig.2 . Kamees method

I think the mug will become nearly full . I like the amount 2/5+3/7 of milk, rather than 5/12 to be in my mug .

Discussion

In this real session for the 7^{th} grade the teacher led the students struggle to solve the question ; listened to all the answers ; doesn't tell the students that the answer is wrong or right but the students themselves

judged , and the students used their own modeling and multiple representation for handling the problem . Nader answered the question as in the previous lesson (e.g. 2/5+1/5 = 3/5) so he tried the question 2/5+3/7 as if the denominators were the same , in addition he added denominators (as many students did) without reflecting on the answer or converting the question to a real situation .

This led Nader and the other students to misunderstanding, and there was no logical nor psychological meaning of the concept and the size of fractions .I can say Nader and the other students lack a number sense , since number sense performance is low in many countries in the world (e.g. see McIntosh et al (1997) ; Reys &Yang (1998) ; Jebreel (1999)) .So the weakness in number sense causes Nader and other students wrong application for procedures like this 2/5+3/7 =5/12 .

In the contrary of this , Ommar ; Anaas ; Adam; Thaer ,and Kamees ; show a good operations, and number sense .

These students, displaying understanding of the meaning of the addition of fractions ; the size of number ; multiple representations for number , and the students are encouraged by the teacher to use their own benchmarks , strategies for reasoning through conditions in a real-world situation, which means that the teacher believes in supporting students reasoning (1999 Yearbook ; NCTM) .Finally , this type of

lessons highlights the students critical thinking .

Multiplication

Teaching students multiplication and other operations by word problem encouraged them to invent procedures for handling the operations. Also the student makes sense and meaning of the operation , after which the student transfers this learning to abstract problems (e.g. 1/2× 2/3=?) . In this manner the teacher of the 7^{th} grade uses this approach to teach his students how to multiply fractions .

Teacher presents the word problem by the overhead projector :

" there is a wall , in which is located a door . The width of the door is half (1/2) of the wall, and the length is two third (2/3) of the wall .What is the fraction that represents the area of the door " .

Three students volunteered to answer this problem (Basel, Adam, and Anwer).

Basel : Ok, .. I drew a wall like this [he draws on the board fig.3]

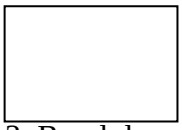

Fig.3. Basel drawing

Adam : Then I drew the length and width of the wall like these

[he drew on the board fig.4]

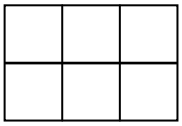

Fig.4. Adam drawing

Anwer : I count the parts of the door , I found it two out of six i.e. 2/6 which is the answer you want . [he wrote on the board, fig.5]

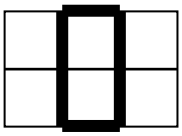

Fig.4. Anwer solution .

Teacher : Thank you very much Basel, Adam,and Anwer. Now I want to ask a question. How much is 1/2 of 2/3 ?

Hader : It is the same answer of Anwer, which is 2/6 . Also , I conclude that 1/2 of 2/3 = 2/6 the same as if I multiply 1 by 2 on nominators, and 2 by 3 on denominators i.e. 1/2× 2/3= 2/6 .

Subtraction

The teacher of the 7^{th} grade gave the students the midterm exam . One of The items is to solve the question 3/4- 2/3 =? . While he was correcting the papers of the students, he found about the half of the students who were able to solve the question like this : 3/4- 2/3 = (3-2)/ (4-3) = 1/1=1 . The students subtract 2 from 3 on numerator and 3 from 4 on denominator .

Next day , the teacher wrote the problem and the solution on the board : 3/4- 2/3 = 1. Ok , what do you think about this solution? the teacher said . The students waited a few minutes , and Basel raised his hand to reply.

Basel : I think 3/4- 2/3 = 1 is wrong , because 1 is bigger than 3/4 and also bigger than 2/3 and 3/4 is bigger than 2/3 . So how can you subtract 2/3 from 3/4 and get a result bigger than 3/4 ? , I think the result must be smaller than 3/4 .

Here the teacher made the following things to apply and develop the operation sense for his students . First of all , he gave the students the time (waiting time for answering) . Second he , let the student detect the error by reflecting the problem and result . Finally , the teacher had developed number sense for this the students before , since Basel had made sense of fractions . On the other hand the student who solved the problem 3/4- 2/3 = 1, lacked number sense, or operation sense .

Developing Division of Fraction

In this view , the teacher of the 7^{th} grade wanted to teach the students the division of fractions for the first time . He used a problem solving approach and learning by doing .

Teacher came to school with one melon (about 10 kg) and a knife. The class contains 31 students . The teacher said : " Hello students , good

morning , today we are going to learn and eat . We want to divide this melon into equal parts for of all to us and eat it ."

Sameer : [take the knife and cut the melon into two equal parts from the sectional cross of it]

Shade : [take the knife and cut the two parts to 4 equal parts]
Jasem : [take the knife and cut the 4 parts to 8 equal parts]

Rashed: [take the knife and cut the 8 parts to 16 equal parts]

Rabh : [take the knife and cut the 16 parts to 32 equal parts]

Salem: We have one part over.

Hadee: But, there is no more, because we are 31students and the Teacher, so we are 32 persons …Yh. huh .

Teacher: Thank you all, Hadee give the students the melon to eat it, and I want you to describe what happens mathematically , i.e. to write on the board all the steps using number and fractions .

Karm : What Sameer has done was $1 \div 2 = 1/2$.

Saffee : What Shade has done was $1/2 \div 2 = 1/4$.

Waleed : What Jasem has done was $1/4 \div 2 = 1/8$.

kaled : What Rashed has done was $1/8 \div 2 = 1/16$.

Yacob : What Rabh has done was $1/16 \div 2 = 1/32$.

Teacher : All right , I want you to compare your answer with the following questions that you knew before:

$1 \times 1/2 =$, $1/2 \times 1/2 =$, $1/4 \times 1/2 =$, $1/8 \times 1/2 =$, $1/16 \times 1/2 =$

After the students solved these questions in the copybook, Saed (and Many students) concluded:

Saed: I found this result:

$1 \times 1/2 = 1 \div 2 = 1/2$,

$1/2 \times 1/2 = 1/2 \div 2 = 1/4$,

$1/4 \times 1/2 = 1/4 \div 2 = 1/8$,

$1/8 \times 1/2 = 1/8 \div 2 = 1/16$,

$1/16 \times 1/2 = 1/16 \div 2 = 1/32$.

Teacher : Ok , you learn how to divide fractions, and now we can eat our melon .

Conclusion

In this article we have seen that the teacher of the 7^{th} grade taught his Students how and why we need the four operations in real-world situations . Some students have alternative methods
and some students appear to display good understanding for the meaning of operations . The role of the teacher as an organizer of the Lesson and reinforcement of the students and sometime make probe. Some students learn form each other and bring similar methods.

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