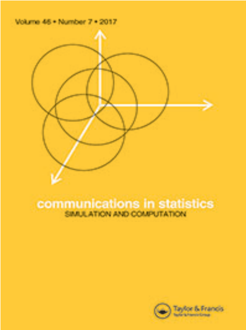

Communications in Statistics - Simulation and Computation

ISSN: 0361-0918 (Print) 1532-4141 (Online) Journal homepage: http://www.tandfonline.com/loi/lssp20

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To cite this article: Lubna Amro & Monjed H. Samuh (2017) More powerful permutation test based on multistage ranked set sampling, Communications in Statistics - Simulation and Computation, 46:7, 5271-5284, DOI: 10.1080/03610918.2016.1152364

To link to this article: http://dx.doi.org/10.1080/03610918.2016.1152364

Accepted author version posted online: 07 Mar 2016.
Published online: 07 Mar 2016.

COMMUNICATIONS IN STATISTICS—SIMULATION AND COMPUTATION®
2017, VOL. 46, NO. 7, 5271–5284
http://dx.doi.org/10.1080/03610918.2016.1152364

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More powerful permutation test based on multistage ranked set sampling

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ABSTRACT
Many researches have used ranked set sampling (RSS) method instead of simple random sampling (SRS) to improve power of some nonparametric tests. In this study, the two-sample permutation test within multistage ranked set sampling (MSRSS) is proposed and investigated. The power of this test is compared with the SRS permutation test for some symmetric and asymmetric distributions through Monte Carlo simulations. It has been found that this test is more powerful than the SRS permutation test; its power increased by set size and/or number of cycles and/or number of stages. Symmetric distributions power increased better than asymmetric distributions power.

ARTICLE HISTORY
Received 23 October 2014
Accepted 2 February 2016

KEYWORDS
Multistage ranked set sampling; Nonparametric tests; Permutation tests; Simulation study

MATHEMATICS SUBJECT CLASSIFICATION
62G09; 62G05; 62D05

1. Introduction

Simple random sampling (SRS) is one of the most popular mechanisms for obtaining data from the population of interest (Cochran, 1988). In 1950s, another sampling method called ranked set sampling (RSS) has been invented for situations where actual measurement of the sample observations is not easy, or it is costly and time consuming, but ordering the sample units by visual inspection or other method without requiring actual measurement is much easier and relatively reliable (Chen et al., 2004).

RSS was first introduced by McIntyre (1952) when he was seeking an effective way to estimate the yield of pasture in Australia. The process of obtaining an RSS of size n, as suggested by McIntyre (1952) can be described as follows: Randomly select m^2 units from the population of interest. Then, allocate the m^2 selected units as randomly as possible into m disjoints subsets, each of size m. Rank the sampling units within each subset visually or by any inexpensive method according to the variable of interest. Then the ith order statistic from the ith subset, $X_{i(i)}$, $i = 1, \ldots, m$, will be quantified (actual measurement). Therefore, $X_{1(1)}, X_{2(2)}, \ldots, X_{m(m)}$ constitutes the RSS. Since these actually measured units are independent, the joint probability density function for $X_{1(1)}, X_{2(2)}, \ldots, X_{m(m)}$ is

$$g_{RSS}(X_{1(1)}, X_{2(2)}, \ldots, X_{m(m)}) = \prod_{i=1}^{m} f_i(x_{i(i)}),$$

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where $f_i(x_{i(i)})$ is the pdf of the ith-order statistic given by

$$f_i\left(x_{i(i)}\right) = \frac{m!}{(i-1)!\,(m-i)!}\left[F\left(x_{i(i)}\right)\right]^{i-1}\left[1-F\left(x_{i(i)}\right)\right]^{m-i} f\left(x_{i(i)}\right)$$

(see Wolfe, 2004).

In order to provide more units for inference, the whole procedure can be repeated h times, thus yielding $n = mh$ measured units of the m^2h selected units. Accordingly, the resulting ranked set sample is: $\{X_{i(i)j};\ i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, h\}$. A complete review of applications and theoretical framework on RSS is available in Kaur et al. (1995), Patil et al. (1994), and Johnson et al. (1996).

Through the past years, many variations of RSS method were developed to come up with more efficient estimators of a population mean, such as extreme ranked set sampling (ERSS) by Samawi et al. (1996), median ranked set sampling (MRSS) by Muttlak (1997), paired ranked set sampling (PRSS) by Hossain and Muttlak (1999), multistage ranked set sampling (MSRSS) by Al-Saleh and Al-Omari (2002), and multistage median ranked set sampling (MSMRSS) by Jemain and Al-Omari (2006). For more other variations of RSS see Al-Nasser (2007), Al-Omari et al. (2011), Jemain et al. (2009), and Syam et al. (2012).

In the 1990s, several researchers expressed interests in studying nonparametric tests based on RSS. For example, RSS versions of Kolmogorov–Smirnov test, two-sample Mann-Whitney-Wilconxon test, signed rank test, and one-sample sign test were studied in Bohn and Wolfe (1992, 1994), Hettmansperger (1995), Kvam and Samaniego (1994), and Stokes and Sager (1988). Koti and Babu (1996) showed that the RSS sign test provides more powerful test than the SRS sign test. For the sign test procedure, it has been found that the median ranked set samples version of the sign test is the best among all the possible sampling schemes in the RSS environment. Also, Liangyong and Xiaofang (2010) proposed the sign test based on RSS for testing hypotheses concerning the quantiles of a population characteristic. Moreover, Samuh (2012) and Samuh and Pesarin (2011) introduced the permutation test within the RSS design. They found that the RSS permutation tests is more powerful than the SRS permutation test. For more work on RSS and its variations see Al-Omari et al. (2011), Jemain et al. (2009), Samuh and Al-Saleh (2011), and Syam et al. (2012).

In this study, we investigate the two-sample permutation design with MSRSS. Also, this proposed method will be compared with their counterpart in SRS in terms of unconditional power. In Sec. 2, the MSRSS method is introduced. The construction of the two-sample MSRSS is described in Sec. 3. The proposed test within permutation framework is investigated in Sec. 4. In Sec. 5, the results from power studies performed through Monte Carlo simulations for a variety of distributions are introduced. Final remarks and conclusions are provided in Sec.6.

2. Multistage ranked set sampling method

Al-Saleh and Al-Kadiri (2000) developed double RSS (DRSS), as a procedure for increasing the efficiency of the RSS estimator without increasing the set size. It has been proved that this procedure estimators' of the mean is more efficient than that using RSS. Al-Saleh and Al-Omari (2002) generalized DRSS to MSRSS. It can be described as follows:

$$Y_t = \begin{bmatrix} Y_{t11}^{(r)} & Y_{t12}^{(r)} & \cdots & Y_{t1m}^{(r)} \\ Y_{t21}^{(r)} & Y_{t22}^{(r)} & \cdots & Y_{t2m}^{(r)} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{th_11}^{(r)} & Y_{th_12}^{(r)} & \cdots & Y_{th_1m}^{(r)} \end{bmatrix} \quad Y_c = \begin{bmatrix} Y_{c11}^{(r)} & Y_{c12}^{(r)} & \cdots & Y_{c1m}^{(r)} \\ Y_{c21}^{(r)} & Y_{c22}^{(r)} & \cdots & Y_{c2m}^{(r)} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{ch_21}^{(r)} & Y_{ch_22}^{(r)} & \cdots & Y_{ch_2m}^{(r)} \end{bmatrix}$$

Figure 1. Two-sample MSRSS design, $\boldsymbol{Y}_t$ and $\boldsymbol{Y}_c$, represent the treatment and the control group, respectively.

Step 1: Randomly select m^{r+1} units from the underlying population, where r is the number of stages, and m is the set size and allocate the m^{r+1} selected units as randomly as possible into m^{r-1} sets, each of size m^2.

Step 2: Use the usual RSS procedure on each set to obtain m ranked set samples of size m each. This step yields m^{r-1} (judgment) ranked sets, each of size m.

Step 3: Without doing any actual quantification on these ranked sets, repeat Step 2 on the m^{r-1} ranked sets to obtain m^{r-2} second stage (judgment) ranked sets, of size m each.

Step 4: The process is continued using Step 2, without doing any actual quantification, until we end up with one rth stage ranked set of size m.

Step 5: Independently, repeat Steps 1 through 5 h cycles, if necessary, to obtain an rth stage RSS of size $n = mh$.

3. Two-sample MSRSS

Consider the two samples in which $\mathbf{X}_1 = \{X_{11}, \ldots, X_{1n_1}\}$ are independent and identically distributed (iid) $F(x+\delta)$ and $\mathbf{X}_2 = \{X_{21}, \ldots, X_{2n_2}\}$ are iid $F(x)$ and the two samples are independent. For the MSRSS design, let the treatment sample $\boldsymbol{Y_t}$ of h_1 cycles and m samples be drawn from $F(x+\delta)$ and let the control sample $\boldsymbol{Y_c}$ of h_2 cycles and m samples be drawn from $F(x)$. Note that the two samples, $\boldsymbol{Y_t}$ and $\boldsymbol{Y_c}$, are independent. Fig. 1 shows the measured data for MSRSS. It is worthwhile to notice that the data within each row are independent, but not identically distributed, while the data within each column are iid. That is, for each $i = 1, \ldots, m$, $Y_{t1i}^{(r)}, \ldots, Y_{th_1i}^{(r)}$ are iid $f_{Y_{t,i}^r}(x+\delta)$ and $Y_{c1i}^{(r)}, \ldots, Y_{ch_2i}^{(r)}$ are iid $f_{Y_{c,i}^r}(x)$, where $f_{Y_{t,i}^r}$ and $f_{Y_{c,i}^r}$ are the distributions of the ith-order statistic of $Y_{tji}^{(r-1)}, \ldots, Y_{tji}^{(r-1)}$ and $Y_{chi}^{(r-1)}, \ldots, Y_{cji}^{(r-1)}$ for each j, respectively. Note that $f_{Y_{t,i}^0}(x)$ is the distribution of the ith-order statistic of the iid SRS $X_1, \ldots, X_m$. For each $j = 1, \ldots, h_1$, $Y_{tj1}^{(r)}, \ldots, Y_{tjm}^{(r)}$ are independent and for each $j' = 1, \ldots, h_2$, $Y_{c1}^{(r)}, \ldots, Y_{cm}^{(r)}$ are also independent.

4. The MSRSS permutation test

Permutation tests are considered as a subclass of nonparametric tests. The nonparametric permutation tests have been suggested in the early twentieth century. It has been described as one of the resampling methods for linear statistical models. It is well known that this test is traced back to Fisher (1935) and Pitman (1937). The permutation test is considered as an alternative to parametric tests when one or more than one assumption is/are violated (see Lehmann and Romano, 2005; Pesarin and Salmaso, 2010). For more information about the permutation tests see Edgington (1995), Pesarin (2001), Pesarin and Salmaso (2010), Salmaso (2003), and Samuh (2012).

Table 1. Empirical unconditional power $\alpha = 0.05$, $m = 2$, normal distribution.

		δ					
h	Sampling design	0.00	0.2	0.4	0.6	0.8	1
5	SRS	0.046	0.121	0.200	0.375	0.542	0.677
	MSRSS ($r = 1$)	0.061	0.135	0.272	0.477	0.672	0.838
	MSRSS ($r = 2$)	0.054	0.150	0.324	0.545	0.752	0.896
	MSRSS ($r = 3$)	0.060	0.147	0.357	0.571	0.785	0.919
	MSRSS ($r = 4$)	0.044	0.175	0.364	0.583	0.811	0.935
10	SRS	0.051	0.143	0.348	0.584	0.807	0.927
	MSRSS ($r = 1$)	0.055	0.184	0.441	0.717	0.916	0.982
	MSRSS ($r = 2$)	0.046	0.207	0.493	0.816	0.952	0.993
	MSRSS ($r = 3$)	0.043	0.220	0.566	0.851	0.957	0.999
	MSRSS ($r = 4$)	0.049	0.246	0.574	0.859	0.980	0.998

Table 2. Empirical unconditional power $\alpha = 0.05$, $m = 3$, normal distribution.

		δ					
h	Sampling design	0.00	0.2	0.4	0.6	0.8	1
5	SRS	0.048	0.129	0.286	0.497	0.690	0.842
	MSRSS ($r = 1$)	0.048	0.172	0.436	0.694	0.901	0.982
	MSRSS ($r = 2$)	0.053	0.225	0.533	0.841	0.969	0.996
	MSRSS ($r = 3$)	0.057	0.234	0.608	0.883	0.982	0.999
	MSRSS ($r = 4$)	0.043	0.280	0.659	0.911	0.985	0.999
10	SRS	0.057	0.182	0.451	0.739	0.932	0.986
	MSRSS ($r = 1$)	0.055	0.273	0.676	0.936	0.994	0.999
	MSRSS ($r = 2$)	0.050	0.356	0.800	0.975	0.999	0.999
	MSRSS ($r = 3$)	0.053	0.378	0.861	0.994	0.999	0.999
	MSRSS ($r = 4$)	0.058	0.439	0.893	0.998	0.999	0.999

Table 3. Empirical unconditional power $\alpha = 0.05$, $m = 4$, normal distribution.

		δ					
h	Sampling design	0.00	0.2	0.4	0.6	0.8	1
5	SRS	0.056	0.137	0.350	0.539	0.793	0.920
	MSRSS ($r = 1$)	0.052	0.247	0.594	0.891	0.981	0.999
	MSRSS ($r = 2$)	0.043	0.323	0.763	0.963	0.998	0.999
	MSRSS ($r = 3$)	0.048	0.360	0.827	0.983	0.999	0.999
	MSRSS ($r = 4$)	0.058	0.391	0.878	0.996	0.999	0.999
10	SRS	0.050	0.217	0.555	0.850	0.971	0.997
	MSRSS ($r = 1$)	0.060	0.389	0.848	0.991	0.999	0.999
	MSRSS ($r = 2$)	0.047	0.526	0.949	0.999	0.999	0.999
	MSRSS ($r = 3$)	0.049	0.585	0.975	0.999	0.999	0.999
	MSRSS ($r = 4$)	0.056	0.642	0.989	0.999	0.999	0.999

Table 4. Empirical unconditional power $\alpha = 0.05$, $m = 5$, normal distribution.

		δ					
h	Sampling design	0.00	0.2	0.4	0.6	0.8	1
5	SRS	0.045	0.172	0.407	0.696	0.879	0.968
	MSRSS ($r = 1$)	0.061	0.319	0.756	0.972	0.999	0.999
	MSRSS ($r = 2$)	0.053	0.437	0.896	0.993	0.999	0.999
	MSRSS ($r = 3$)	0.052	0.511	0.947	0.999	0.999	0.999
	MSRSS ($r = 4$)	0.053	0.595	0.977	0.999	0.999	0.999
10	SRS	0.055	0.259	0.661	0.899	0.993	0.999
	MSRSS ($r = 1$)	0.061	0.523	0.950	0.999	0.999	0.999
	MSRSS ($r = 2$)	0.049	0.665	0.993	0.999	0.999	0.999
	MSRSS ($r = 3$)	0.052	0.772	0.999	0.999	0.999	0.999
	MSRSS ($r = 4$)	0.043	0.823	0.999	0.999	0.999	0.999

Table 5. Empirical unconditional power $\alpha = 0.05$, $m = 2$, exponential distribution.

		δ					
h	Sampling design	0.00	0.2	0.4	0.6	0.8	1
5	SRS	0.055	0.131	0.253	0.445	0.576	0.721
	MSRSS ($r = 1$)	0.049	0.150	0.316	0.495	0.693	0.810
	MSRSS ($r = 2$)	0.046	0.171	0.344	0.582	0.749	0.846
	MSRSS ($r = 3$)	0.058	0.176	0.375	0.578	0.736	0.863
	MSRSS ($r = 4$)	0.046	0.180	0.373	0.607	0.771	0.880
10	SRS	0.046	0.173	0.370	0.623	0.805	0.928
	MSRSS ($r = 1$)	0.049	0.188	0.463	0.716	0.887	0.964
	MSRSS ($r = 2$)	0.049	0.212	0.497	0.758	0.903	0.970
	MSRSS ($r = 3$)	0.048	0.233	0.540	0.783	0.930	0.981
	MSRSS ($r = 4$)	0.040	0.228	0.549	0.796	0.925	0.975

Table 6. Empirical unconditional power $\alpha = 0.05$, $m = 3$, exponential distribution.

		δ					
h	Sampling design	0.00	0.2	0.4	0.6	0.8	1
5	SRS	0.053	0.147	0.315	0.537	0.707	0.869
	MSRSS ($r = 1$)	0.051	0.201	0.430	0.699	0.860	0.940
	MSRSS ($r = 2$)	0.044	0.228	0.509	0.760	0.885	0.969
	MSRSS ($r = 3$)	0.057	0.251	0.561	0.806	0.925	0.973
	MSRSS ($r = 4$)	0.049	0.270	0.583	0.833	0.922	0.974
10	SRS	0.055	0.194	0.479	0.755	0.932	0.977
	MSRSS ($r = 1$)	0.050	0.260	0.656	0.893	0.984	0.998
	MSRSS ($r = 2$)	0.054	0.324	0.712	0.938	0.993	0.999
	MSRSS ($r = 3$)	0.049	0.349	0.780	0.946	0.992	0.999
	MSRSS ($r = 4$)	0.046	0.367	0.779	0.963	0.993	0.999

Table 7. Empirical unconditional power $\alpha = 0.05$, $m = 4$, exponential distribution.

		δ					
h	Sampling design	0.00	0.2	0.4	0.6	0.8	1
5	SRS	0.052	0.174	0.382	0.597	0.812	0.926
	MSRSS ($r = 1$)	0.046	0.229	0.596	0.832	0.946	0.989
	MSRSS ($r = 2$)	0.066	0.320	0.699	0.898	0.968	0.996
	MSRSS ($r = 3$)	0.059	0.328	0.723	0.927	0.984	0.998
	MSRSS ($r = 4$)	0.050	0.385	0.763	0.930	0.985	0.999
10	SRS	0.052	0.229	0.579	0.840	0.962	0.996
	MSRSS ($r = 1$)	0.045	0.355	0.808	0.968	0.998	0.999
	MSRSS ($r = 2$)	0.059	0.445	0.882	0.986	0.999	0.999
	MSRSS ($r = 3$)	0.044	0.491	0.912	0.992	0.999	0.999
	MSRSS ($r = 4$)	0.056	0.518	0.922	0.993	0.999	0.999

Table 8. Empirical unconditional power $\alpha = 0.05$, $m = 5$, exponential distribution.

		δ					
h	Sampling design	0.00	0.2	0.4	0.6	0.8	1
5	SRS	0.046	0.169	0.418	0.680	0.875	0.965
	MSRSS ($r = 1$)	0.052	0.300	0.690	0.919	0.985	0.998
	MSRSS ($r = 2$)	0.053	0.374	0.803	0.959	0.993	0.997
	MSRSS ($r = 3$)	0.058	0.442	0.837	0.968	0.997	0.999
	MSRSS ($r = 4$)	0.045	0.472	0.858	0.970	0.998	0.999
10	SRS	0.052	0.269	0.625	0.899	0.988	0.999
	MSRSS ($r = 1$)	0.046	0.438	0.894	0.993	0.999	0.999
	MSRSS ($r = 2$)	0.066	0.574	0.954	0.999	0.999	0.999
	MSRSS ($r = 3$)	0.059	0.633	0.964	0.999	0.999	0.999
	MSRSS ($r = 4$)	0.050	0.650	0.971	0.999	0.999	0.999

Table 9. Empirical unconditional power $\alpha = 0.05$, $m = 2$, uniform distribution.

		δ					
h	Sampling design	0.00	0.2	0.4	0.6	0.8	1
5	SRS (permutation)	0.057	0.109	0.209	0.361	0.526	0.687
	MSRSS ($r = 1$)	0.046	0.127	0.258	0.472	0.660	0.834
	MSRSS ($r = 2$)	0.049	0.154	0.33	0.555	0.764	0.909
	MSRSS ($r = 3$)	0.047	0.158	0.342	0.642	0.819	0.945
	MSRSS ($r = 4$)	0.043	0.178	0.393	0.649	0.875	0.968
10	SRS	0.053	0.152	0.332	0.577	0.800	0.922
	MSRSS ($r = 1$)	0.050	0.186	0.450	0.737	0.916	0.989
	MSRSS ($r = 2$)	0.046	0.200	0.513	0.828	0.964	0.999
	MSRSS ($r = 3$)	0.043	0.247	0.614	0.887	0.985	0.999
	MSRSS ($r = 4$)	0.044	0.259	0.629	0.902	0.993	0.999

Table 10. Empirical unconditional power $\alpha = 0.05$, $m = 3$, uniform distribution.

		δ					
h	Sampling design	0.00	0.2	0.4	0.6	0.8	1
5	SRS	0.056	0.135	0.262	0.476	0.688	0.852
	MSRSS ($r = 1$)	0.057	0.205	0.449	0.721	0.927	0.985
	MSRSS ($r = 2$)	0.055	0.220	0.581	0.871	0.982	0.999
	MSRSS ($r = 3$)	0.049	0.264	0.679	0.939	0.995	0.999
	MSRSS ($r = 4$)	0.057	0.308	0.745	0.967	0.999	0.999
10	SRS	0.056	0.196	0.458	0.741	0.919	0.987
	MSRSS ($r = 1$)	0.042	0.283	0.713	0.952	0.997	0.999
	MSRSS ($r = 2$)	0.051	0.387	0.841	0.990	0.999	0.999
	MSRSS ($r = 3$)	0.056	0.458	0.909	0.996	0.999	0.999
	MSRSS ($r = 4$)	0.054	0.499	0.952	0.999	0.999	0.999

Table 11. Empirical unconditional power $\alpha = 0.05$, $m = 4$, uniform distribution.

		δ					
h	Sampling design	0.00	0.2	0.4	0.6	0.8	1
5	SRS	0.054	0.151	0.333	0.593	0.792	0.935
	MSRSS ($r = 1$)	0.056	0.271	0.633	0.901	0.986	0.999
	MSRSS ($r = 2$)	0.055	0.351	0.816	0.991	0.999	0.999
	MSRSS ($r = 3$)	0.057	0.437	0.918	0.999	0.999	0.999
	MSRSS ($r = 4$)	0.039	0.533	0.960	0.999	0.999	0.999
10	SRS	0.055	0.218	0.537	0.835	0.969	0.998
	MSRSS ($r = 1$)	0.056	0.393	0.882	0.998	0.999	0.999
	MSRSS ($r = 2$)	0.046	0.592	0.982	0.999	0.999	0.999
	MSRSS ($r = 3$)	0.058	0.695	0.997	0.999	0.999	0.999
	MSRSS ($r = 4$)	0.060	0.779	0.999	0.999	0.999	0.999

Table 12. Empirical unconditional power $\alpha = 0.05$, $m = 5$, uniform distribution.

		δ					
h	Sampling design	0.00	0.2	0.4	0.6	0.8	1
5	SRS	0.054	0.163	0.409	0.662	0.871	0.967
	MSRSS ($r = 1$)	0.051	0.309	0.778	0.973	0.999	0.999
	MSRSS ($r = 2$)	0.053	0.492	0.945	0.999	0.999	0.999
	MSRSS ($r = 3$)	0.053	0.639	0.992	0.999	0.999	0.999
	MSRSS ($r = 4$)	0.050	0.741	0.998	0.999	0.999	0.999
10	SRS	0.045	0.237	0.626	0.901	0.992	0.999
	MSRSS ($r = 1$)	0.047	0.536	0.966	0.999	0.999	0.999
	MSRSS ($r = 2$)	0.047	0.757	0.999	0.999	0.999	0.999
	MSRSS ($r = 3$)	0.055	0.889	0.999	0.999	0.999	0.999
	MSRSS ($r = 4$)	0.051	0.941	0.999	0.999	0.999	0.999

For the two-sample design in this study, we considered testing problems for one-sided alternatives as generated by symbolic treatment with a fixed shift effect δ (nonnegative). The hypothesis of interest are $H_0 : \{\delta = 0\}$ versus $H_1 : \{\delta > 0\}$.

As we mentioned above, the data within columns are iid, so under the null hypothesis, the exchangeability assumption holds within columns, which means that an exact permutation solution may exist. From that, due to independence of columns, we conclude that the permutation should be applied to the data column by column, so the 1st column from Y_t by the 1st column from $\boldsymbol{Y}_c$, then the 2nd column from $\boldsymbol{Y}_t$ by the 2nd column from $\boldsymbol{Y}_c$, and so on. From this procedure, we get a new matrix $\boldsymbol{Y}$ of size $(h_1 + h_2) \times m$. Now let $\boldsymbol{Y}^*$ represents the permutation of $\boldsymbol{Y}$. In order to preserve diversity of distributions, the permutation $\boldsymbol{Y}^*$ is obtained

Table 13. Empirical unconditional power $\alpha = 0.05$, $m = 2$, skew normal distribution.

		δ					
h	Sampling design	0.00	0.2	0.4	0.6	0.8	1
5	SRS	0.046	0.172	0.420	0.699	0.865	0.954
	MSRSS ($r = 1$)	0.046	0.227	0.519	0.793	0.950	0.993
	MSRSS ($r = 2$)	0.054	0.241	0.591	0.860	0.976	0.995
	MSRSS ($r = 3$)	0.052	0.254	0.639	0.883	0.980	0.999
	MSRSS ($r = 4$)	0.050	0.278	0.653	0.899	0.985	0.999
10	SRS	0.043	0.246	0.648	0.903	0.994	0.994
	MSRSS ($r = 1$)	0.062	0.331	0.773	0.977	0.999	0.999
	MSRSS ($r = 2$)	0.055	0.374	0.846	0.984	0.999	0.999
	MSRSS ($r = 3$)	0.051	0.415	0.861	0.989	0.999	0.999
	MSRSS ($r = 4$)	0.049	0.427	0.896	0.992	0.999	0.999

Table 14. Empirical unconditional power $\alpha = 0.05$, $m = 3$, skew normal distribution.

		δ					
h	Sampling design	0.00	0.2	0.4	0.6	0.8	1
5	SRS	0.051	0.210	0.543	0.819	0.959	0.991
	MSRSS ($r = 1$)	0.051	0.335	0.754	0.960	0.998	0.999
	MSRSS ($r = 2$)	0.053	0.399	0.853	0.986	0.999	0.999
	MSRSS ($r = 3$)	0.051	0.456	0.892	0.996	0.999	0.999
	MSRSS ($r = 4$)	0.045	0.482	0.919	0.997	0.999	0.999
10	SRS	0.050	0.348	0.791	0.982	0.999	0.999
	MSRSS ($r = 1$)	0.057	0.530	0.952	0.998	0.999	0.999
	MSRSS ($r = 2$)	0.051	0.637	0.986	0.999	0.999	0.999
	MSRSS ($r = 3$)	0.054	0.698	0.992	0.999	0.999	0.999
	MSRSS ($r = 4$)	0.052	0.721	0.997	0.999	0.999	0.999

Table 15. Empirical unconditional power $\alpha = 0.05$, $m = 4$, skew normal distribution.

		δ					
h	Sampling design	0.00	0.2	0.4	0.6	0.8	1
5	SRS	0.050	0.255	0.656	0.913	0.990	0.999
	MSRSS ($r = 1$)	0.049	0.448	0.908	0.998	0.999	0.999
	MSRSS ($r = 2$)	0.048	0.598	0.973	0.999	0.999	0.999
	MSRSS ($r = 3$)	0.057	0.667	0.986	0.999	0.999	0.999
	MSRSS ($r = 4$)	0.055	0.714	0.991	0.999	0.999	0.999
10	SRS	0.063	0.410	0.890	0.995	0.999	0.999
	MSRSS ($r = 1$)	0.057	0.705	0.995	0.999	0.999	0.999
	MSRSS ($r = 2$)	0.054	0.847	0.999	0.999	0.999	0.999
	MSRSS ($r = 3$)	0.049	0.897	0.999	0.999	0.999	0.999
	MSRSS ($r = 4$)	0.053	0.923	0.999	0.999	0.999	0.999

Table 16. Empirical unconditional power $\alpha = 0.05$, $m = 5$, skew normal distribution.

		δ					
h	Sampling design	0.00	0.2	0.4	0.6	0.8	1
5	SRS	0.060	0.308	0.737	0.955	0.998	0.999
	MSRSS ($r = 1$)	0.055	0.576	0.973	0.999	0.999	0.999
	MSRSS ($r = 2$)	0.049	0.756	0.995	0.999	0.999	0.999
	MSRSS ($r = 3$)	0.041	0.835	0.999	0.999	0.999	0.999
	MSRSS ($r = 4$)	0.046	0.854	0.999	0.999	0.999	0.999
10	SRS	0.061	0.481	0.953	0.999	0.999	0.999
	MSRSS ($r = 1$)	0.056	0.827	0.999	0.999	0.999	0.999
	MSRSS ($r = 2$)	0.056	0.499	0.999	0.999	0.999	0.999
	MSRSS ($r = 3$)	0.048	0.475	0.999	0.999	0.999	0.999
	MSRSS ($r = 4$)	0.054	0.992	0.999	0.999	0.999	0.999

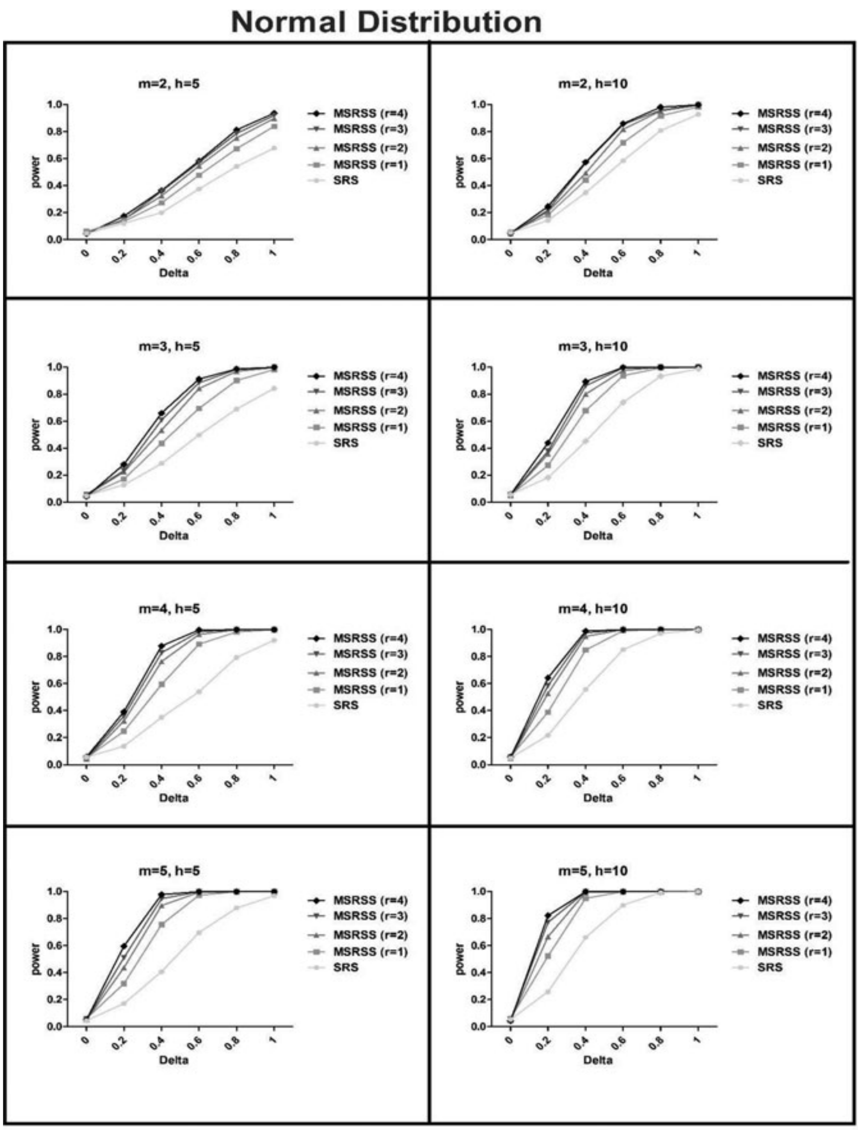

Figure 2. Comparison between the empirical unconditional power of permutation test based on SRS and MSRSS $\alpha = 0.05$, normal distribution.

by permuting the data within each column of $\boldsymbol{Y}$. All permutations of $\boldsymbol{Y}$ are contained in the permutation sample space $\mathcal{Y}_{/\boldsymbol{Y}}$.

In order to solve the testing problem, assuming that large values are significant, we can refer to a test statistic of the form

$$\boldsymbol{T}_{MSRSS} = \bar{\boldsymbol{Y}}_{\boldsymbol{ti}} - \bar{\boldsymbol{Y}}_{\boldsymbol{ci}},$$

where $\bar{Y}_{ti} = \sum_{j=1}^{h_1} Y_{tji}/h_1$, $\bar{Y}_{ci} = \sum_{j=1}^{h_2} Y_{cji}/h_2$. Conditional Monte Carlo algorithm is usually used to estimate the p-value for testing H_0, since getting the exact p-value is hard as

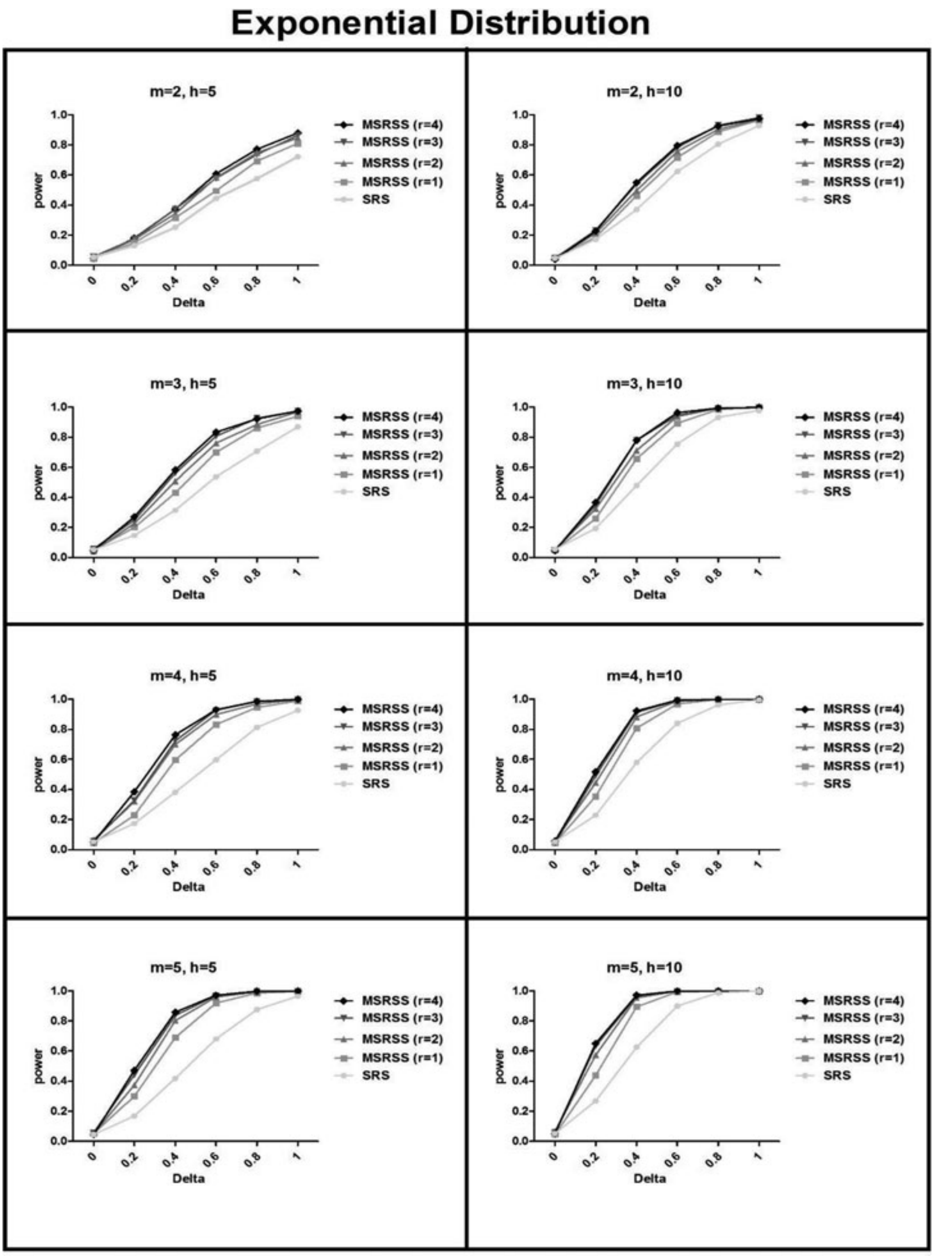

Figure 3. Comparison between the empirical unconditional power of permutation test based on SRS and MSRSS $\alpha = 0.05$, exponential distribution.

writing down the whole permutation space is tedious or practically impossible. Now, to obtain the p-value for testing H_0, the two-sample MSRSS permutation algorithm is described as follows.

1. For the given two-sample $\boldsymbol{Y_t}$ and $\boldsymbol{Y_c}$, calculate the observed test statistic T°_{MSRSS}
2. Concatenate $\boldsymbol{Y_t}$ and $\boldsymbol{Y_c}$ row-wise to get $\boldsymbol{Y} = \boldsymbol{Y_t} \cup \boldsymbol{Y_c}$
3. Take a random permutation $\boldsymbol{Y}^* \in \mathcal{Y}_{/\boldsymbol{Y}}$ of $\boldsymbol{Y}$.
4. Split $\boldsymbol{Y}^*$ into two matrices, $\boldsymbol{Y}^*_t$ containing the first h_1 rows, and $\boldsymbol{Y}^*_c$ containing the rest.

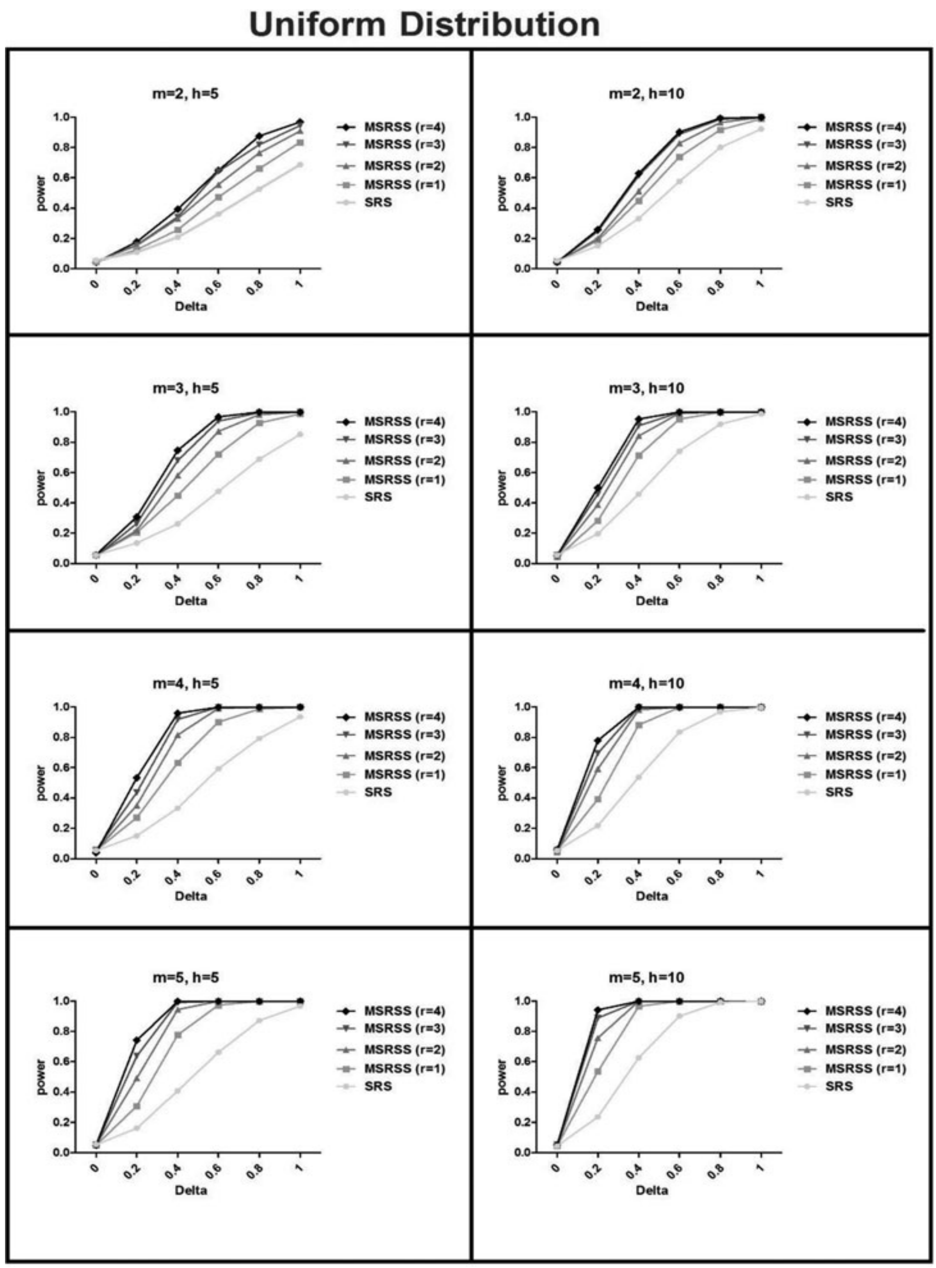

Figure 4. Comparison between the empirical unconditional power of permutation test based on SRS and MSRSS $\alpha = 0.05$, uniform distribution.

5. Calculate the corresponding test statistic $T^*_{MSRSS} = T_{MSRSS}(\boldsymbol{Y}^*)$.
6. Independently, repeat Steps 3 to 5 a large number, say B, of times, so we are having $\{T^*_{MSRSSb}; b = 1, \ldots, B\}$.
7. Finally, we estimate the permutation p-value through the following equation:

$$\hat{\lambda}_T(\boldsymbol{X}) = \frac{\sum_{b=1}^{B} \mathbb{I}\left(T^*_{MSRSSb} \geq T^\circ_{MSRSS}\right)}{B}.$$

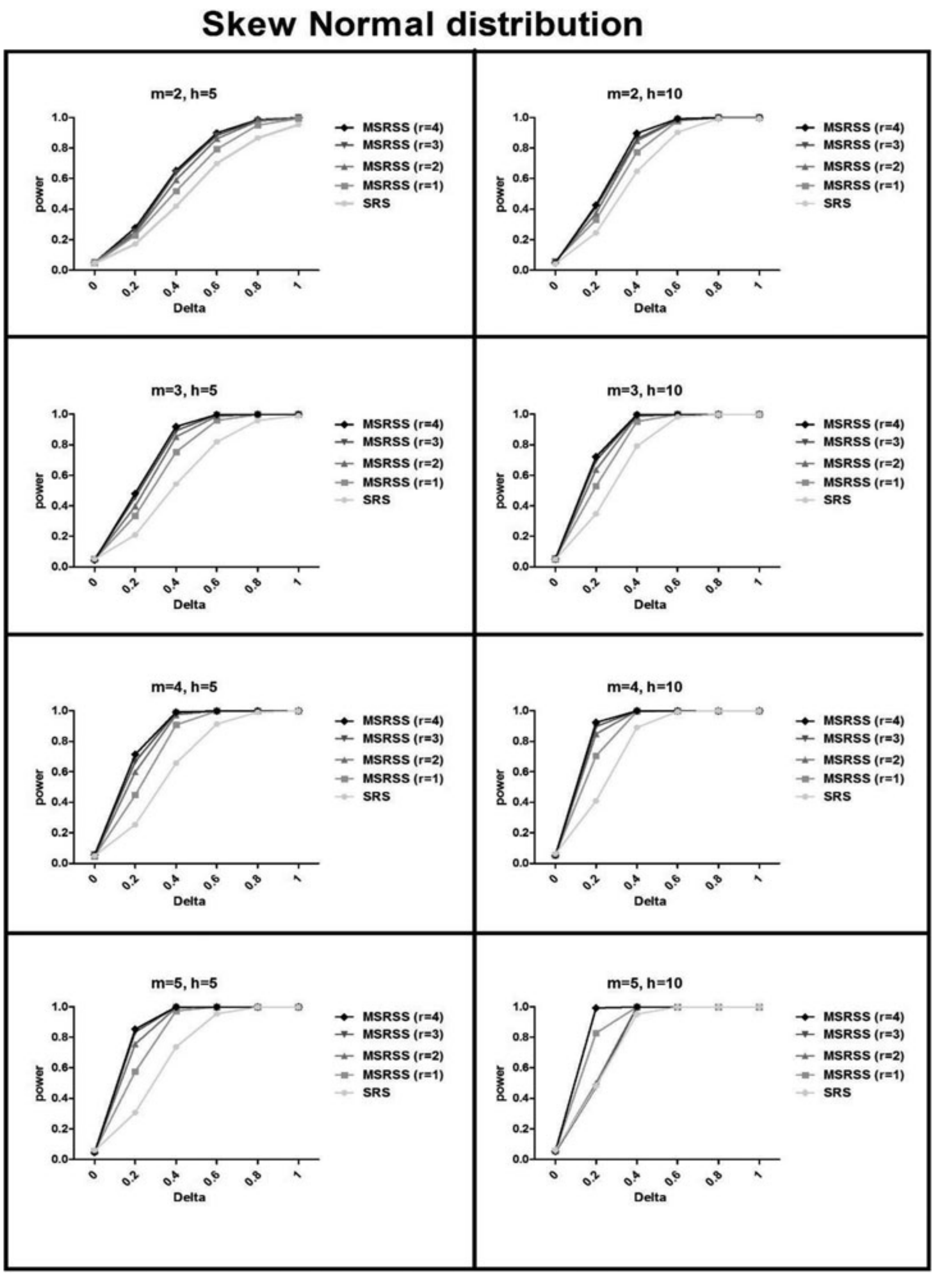

Figure 5. Comparison between the empirical unconditional power of permutation test based on SRS and MSRSS $\alpha = 0.05$, skew normal distribution.

5. Simulation study

The empirical unconditional power of the permutation test under MSRSS and SRS is investigated by simulation. The power of permutation test based on two-sample MSRSS of size $m \times h$, with set size m, number of stages r, and number of cycles h, in each sample is computed and compared with the power of permutation test based on another two-sample SRS of size $m \times h$ in each sample. So the comparisons are made considering the same numbers of actually measured data, since in this way we make sure that the costs of the two sampling schemes are the same.

In the simulation, four different distributions were considered: normal $N(0, 1)$, uniform $\mathrm{U}(-\sqrt{3}, \sqrt{3})$, skew normal $SN(0, 1, -5)$, and exponential $\mathrm{Exp}(1)$. The adopted set sizes are $m = \{2, 3, 4, 5\}$, the number of cycles with balanced designs are taken as $h_1 = h_2 = \{5, 10\}$, and the number of stages $r = \{1, 2, 3, 4\}$. The nominal significance level is taken as 0.05. In order to evaluate the empirical power of the test, the treatment groups are shifted by adding the shift parameters $\delta = \{0, 0.2, 0.4, 0.6, 0.8, 1\}$. We chose $\delta = 0$ to check the empirical level of significance. A simulation based on $R = 2{,}000$ data sets is performed. The considered permutations are $B = 1{,}000$ on each data set.

Tables 1–16 and Figs. 2–5 show the results about the empirical unconditional power of the permutation test under MSRSS and SRS based on the four considered distributions, where the simulations have been accomplished in R software (R Development Core Team, 2011). For more details about the simulation codes, contact the authors.

We noticed that all considered probability distributions, the MSRSS permutation test control the type I error rate at $\alpha = .05$. Also, it is strictly more powerful than the SRS permutation test for all given δ holding the sample size fixed. Moreover, the power of the permutation test based on MSRSS increases as δ, r, m, and h increase. For fixed total sample size, the power increases by m much better than by h. Moreover, the power of the symmetric distributions (normal and uniform), in general increased better than the asymmetric distribution (exponential and skew normal) power.

6. Conclusion

As a summary, simulation results assure that the permutation test based on MSRSS estimator is more powerful than the SRS permutation test. The comparison of the power has been based on the number of actually measured units in the SRS and the MSRSS. The power of the symmetric distributions increases better than the asymmetric distribution power. Also, simulation study (not provided here) showed that the SRS permutation tests is more powerful than the SRS t-test and the SRS Mann–Whitney test, which means that the MSRSS permutation test is more powerful than all of the mentioned tests. Consequently, the MSRSS permutation test is recommended to be used instead of the SRS permutation test. Finally, it is worth to observe that our approach could be extended to some other designs such as paired-sample design, one- and two-way ANOVA designs (Basso et al., 2007; Corain and Salmaso, 2007), and split plot design (Corain et al., 2013). It could also be extended to multivariate setting where each observation is actually a vector, also in case of nonnumerical responses (Arboretti et al., 2012).

Acknowledgments

The authors are very thankful to the Associate Editor and the anonymous referees for the comments, which led to a considerable improvement of an earlier version of this paper.

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