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TRANSIENT AIR/FUEL RATIO CONTROL IN SI ENGINES

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ABSTRACT

Mixture strength control system effectiveness depends on its capacity to deal with air and fuel transport processes inside the intake manifold: the prediction of air mass flow to the engine cylinders and the compensation for the fuel lag during engine transients. These issues are all likely to be of extreme importance with the transient air/fuel ratio control strategies.

This paper introduces an innovative model-based air/fuel ratio control strategy for SI engines. It is based on a previously published modeling approach for the air dynamics inside intake manifolds, which is based on the formulation of the mass, momentum and energy conservation equations and named as Method Of Interconnected Capacities. The proposed strategy uses a fuel compensator that is based on a macroscopical modeling of the fuel film dynamical behavior inside the intake manifold, which is derived from the Aquino model.

A wide range of severe transient tests obtained from the experimentation of a single-cylinder research engine (type AVL 5401), equipped with port-fuel injection system, is presented.

The results obtained have proved the effectiveness of the proposed strategy in controlling the air/fuel ratio in SI engines in a better way compared to the traditional control systems.

INTRODUCTION

For a very long time, model-based air/fuel ratio control in SI engines has demonstrated to be an efficient potential alternative. Given the tight requirement of the modern control strategies, model-based control systems need to be better than ever, mainly during transient operation. Unfortunately, practical air/fuel ratio control systems have yet to be transformed in order to implement the model-based control alternative. Actually, a significant part of the related publications stop short of demonstrating experimentally the effectiveness of the such ideas, which leaves a gap between scientific results and industrial application that is needed to be encouraged to adopt this control technique.

The prediction of air mass flow rate to the engine cylinders and the compensation for the lag of the fuel flow during engine transients are issues of extreme importance in the control process. Therefore, air/fuel ratio model-based control system effectiveness depends directly on its capacity to deal precisely with the air and fuel transport processes inside the intake manifolds of the SI engines.

The purpose of this paper is to introduce an innovative and comprehensive model-based air/fuel ratio control strategy for SI engines. The work presented in this paper demonstrates experimentally the effectiveness of this strategy in the fuelling control of the SI engines as it has been applied to a single-cylinder research engine. The basic outlines of this work were presented by Sughayyer [1]. These ideas were extended in this paper to consider many types of more severe engine transients.

The basic concepts behind air dynamics modeling in the intake manifold for the transient air/fuel ratio control in SI engines were the authors previously published approach [2]. The method can be considered as a further evolution of the Mean Value Engine Models, widely used in the literature [3-7], considering its capacity to predict air flow transient effects. Through a suitable formulation of the mass, momentum and energy conservation equations it becomes possible to predict air mass flow rates during engines transients keeping its simplicity to be used in the online control. And more important, it has no need for engine parameters maps such as the well known volumetric efficiency. Therefore, the modeling approach appears to be an intermediate stage between lumped and distributed parameter models. It considers the intake manifold as a collection of small interconnected capacities that exchange mass and energy. Accordingly, it is called Method Of Interconnected Capacities (MOIC).

The proposed control strategy adopts also a fuel compensation technique that is based on the macroscopical modeling of the fuel film dynamical behavior inside the intake manifold, which is a first-order model due to Aquino [8].

This paper presents a wide range of severe transient tests obtained from the experimentation of a single-cylinder research engine (type AVL 5401), which is equipped with a port-fuel injection system. The tests were intended to simulate a wide range of engine transient operation that could happen in normal drive.

NOMENCLATURE

A	Cross-Sectional Area
C_d	Coefficient of Discharge
c_v, c_p	Specific Heats
D	Diameter
E	Internal Energy
f	Friction Factor
i, e	Subscripts for Inlet, Exit
$\dot{m}_{ac}$	Inducted Air Mass Flow Rate
m_{ff}	Fuel-Film Mass
$\dot{m}$	Mass Flow Rate
$\dot{m}_{fc}$	Inducted Fuel Mass Flow Rate
$\dot{m}_{fi}$	Injected Fuel Mass Flow Rate
$\dot{m}_{fv}$	Air Born Fuel Flow Mass Flow Rate
N	Engine Speed (rpm)
P	Pressure
$\dot{Q}$	Heat Transfer
R	Gas Constant
T	Temperature
$\bar{u}$	Mean Flow Velocity
V	Volume
x	Fuel-Film Parameter
λ	Relative Air/Fuel Ratio
τ	Fuel-Film Parameter
γ	Ratio of Specific Heats
AFR	Air/Fuel Ratio
MOIC	Method Of Interconnected Capacities
WOT	Wide Open Throttle

MODEL-BASED AFR CONTROL

Model-based control strategies have started to appear in the literature in order to satisfy the long-standing demands of emission reduction and lower fuel consumption. These strategies are based on physically consistent dynamical models, which are intended to go beyond the traditional limits of the control systems that are based on steady-state maps.

The most important feature that characterizes these models is their capacity to predict variables that are difficult to measure with the required accuracy. For example: intake manifold pressure, which is the most important variable to predict the instantaneous air mass flow to the engine's cylinders, especially, during transient operation. This is of crucial importance due to the fact that emissions are highly sensitive to the instantaneous air/fuel ratio as well as the effectiveness of the three-way catalytic converter. Indeed, for any fuel injection control system, precise air mass flow estimation is key element and the most important step toward the precise control in either case, transient or steady operation.

Model-based control systems generally have two main submodels; the principal treats the intake manifold air dynamics, while the other deals with the fuel transport processes (jet, fuel puddle, droplet motion, etc.).

The adoption of model-based control strategies needs serious modifications in the onboard electronic control units with respect to those in use nowadays.

Air Dynamics

The dynamic nature of the air induction process, especially during transient operation (acceleration, deceleration, engine load varying, engine operating conditions varying), needs dynamic models that emphasize the modeling aspect of the problem rather than the control aspect. They can reproduce the main features of the induction process: inertial ramming, resonance, heat transfer, friction, backflow, and in-cylinder processes, moving valves, throttle and junctions. But many models present in the literature [3-7] depend on the quasi-steady assumption that is based on the steady-state maps such as for volumetric efficiency, which are normally obtained using the traditional mapping techniques. In order to overcome the limits of the quasi-steady modeling and to avoid the use of steady maps in the control process, a new method has been developed by the authors [1,2] characterized by its capability to reproduce most aspects of the transient phenomena. This modeling approach is named Method Of Interconnected Capacities (MOIC) and will be described here briefly.

Method Of Interconnected Capacities

This modeling for the air dynamics inside intake manifolds was designed for online implementation; therefore, it suits the case here to form a comprehensive model based controller. A full method derivation can be seen in [1,2], but here just the derived method equations are reported.

Through a suitable formulation of the mass, momentum and energy equations, the model predicts mass flow rates during engine transients keeping its simplicity, which satisfies the limitations of the near future onboard electronic applications. The model appears to be an intermediate stage between lumped and distributed parameter models. The model considers SI engine intake manifold as a collection of small interconnected capacities as shown in Fig. (1), and each one interacts with the surrounding capacities as boundary conditions. Accordingly, modeling of complicated duct geometries, which characterize practical designs of the internal combustion engine's intake manifolds, becomes a very simple task. The application of the conservation equations at the capacity under consideration (j) gives a set of three first-order differential equations – Eq. (1), whose solution describes pressure, temperature in addition to the mean airflow velocity in the capacity. In this way, the modeling does not use any pre mapped engine parameters.

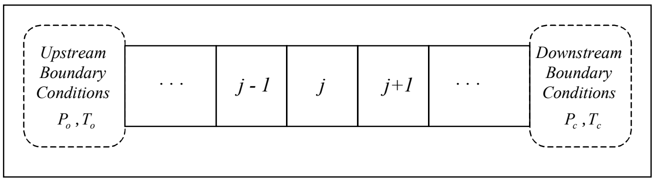

Figure 1: A generic intake manifold duct as it is divided into n capacities.

$$\begin{bmatrix} \dfrac{dP_j}{dt} \\ \dfrac{dT_j}{dt} \\ \dfrac{d\bar{u}_j}{dt} \end{bmatrix} = \frac{T_j(\gamma-1)}{P_j V_j} \begin{bmatrix} -\left(\dfrac{\bar{u}_j^2 P_j}{2T_j}\right)\dfrac{dm_j}{dt} + \left(\dfrac{P_j}{T_j}\right)\dfrac{dE_j}{dt} - \left(\dfrac{\bar{u}_j P_j}{T_j}\right)\Phi \\ -\left\{\left(\dfrac{R}{\gamma-1}\right) + \left(\dfrac{\bar{u}_j^2}{2T_j}\right)\right\} T_j \dfrac{dm_j}{dt} + \dfrac{dE_j}{dt} - \bar{u}_j \Phi \\ \left(\dfrac{R}{\gamma-1}\right)\Phi \end{bmatrix} \tag{1}$$

For dm_j/dt , dE_j/dt , and Φ the following apply:

$$\frac{dm_j}{dt} = \dot{m}_{i,j} - \dot{m}_{e,j} \tag{2}$$

$$\frac{dE_j}{dt} = \left(\frac{\gamma}{\gamma-1} R T_i + \frac{u_i^2}{2}\right)\dot{m}_{i,j} - \left(\frac{\gamma}{\gamma-1} R T_e + \frac{u_e^2}{2}\right)\dot{m}_{e,j} + \left(\frac{P_j V_j}{R T_j}\right)\left(\frac{2 f C_p |\bar{u}_j|}{D}\right)(T_w - T_j) \tag{3}$$

$$\Phi = P_i A_i - P_e A_e - \bar{u}_j \frac{dm_j}{dt} - sign(\bar{u}) f \frac{\bar{u}_j^2}{2} \frac{4}{D}\left(\frac{P_j V_j}{R T_j}\right) \tag{4}$$

where j refers to the capacity, $\dot{m}_{i,j}$ and $\dot{m}_{e,j}$ are the inflow and the outflow mass rates. P_j and T_j are the pressure and temperature in the capacity of the volume V_j , respectively. dE_j / dt is the time rate of change of internal energy in the capacity, given R is the gas constant of air, h_i and h_e are the specific stagnation enthalpy of the entering and exiting masses, $\dot{Q}$ is the net heat transfer rate into the capacity, γ is ratio of the specific heats, f is friction coefficient, C_p is the constant pressure specific heat, D is the mean duct diameter, and T_w is the mean wall temperature.

In the calculation of Φ , a further helpful simplification in the calculation can be done, by which a sufficiently accurate solution can be obtained. In that, P_i has been replaced by P_j and $\bar{u}_j$ can be considered equal to u_e .

The MOIC allows for a very simple boundary conditions application. For intake manifold modeling, two important boundary cases are reported in the Appendix.

Fuel Dynamics

During the fuel injection process a portion of the injected fuel droplets stays airborne, but significant part of the fuel is deposited on the port surfaces and generates the well-known wall fuel film. Many investigating studies [9,10,11] of the fuel quantity that adheres to the intake port surfaces have revealed that it depends on many parameters; intake port design, spray characteristics, flow field in the intake duct, injection timing relative to the intake valve timing, air and surfaces temperatures, and the engine speed and load. Of course, this highlights the complexity of the fuel transport dynamics.

The transport of the fuel into the engine cylinder occurs as vapor, droplets and liquid streams from the fuel film. This fact means that the fuel dynamics is slower than that of the air. Therefore, under transient engine conditions, the fuel deposition or the discharge of the fuel from the wall film leads to changes in the required air/fuel ratio. Thus, a compensation technique for this phenomenon needs to be included in the control system. Effective modeling of the fuel dynamics can be considered [12,13] for an efficient compensation, but it needs to be extended to consider real gasoline mixture.

For the time being, a simple first-order model that tracks macroscopically the liquid puddle dynamics inside engines intake manifold has been adopted here – the Aquino Model [8].

Aquino Model

For air/fuel ratio control applications, the consideration of the macroscopic modeling of the fuel-film is an acceptable starting point for fuel dynamics compensations.

The film dynamics is predicted by this model in response to any engine transient (throttle position change, engine load change, etc.). The well-known formulation of this model states that:

$$\frac{dm_{ff}}{dt} = x\dot{m}_{fi} - \frac{m_{ff}}{\tau} \tag{5}$$

$$\dot{m}_{fv} = (1-x)\dot{m}_{fi} \tag{6}$$

$$\dot{m}_{fc} = \dot{m}_{fv} + \frac{m_{ff}}{\tau} \tag{7}$$

where m_{ff} is the mass of the fuel film, $\dot{m}_{fi}$ is the injected fuel mass flow rate, $\dot{m}_{fv}$ is the part of injected fuel mass flow rate that remains airborne, $\dot{m}_{fc}$ is the fuel mass flow rate into engine cylinder, x is the fraction of the injected fuel entering the film, and τ is the characteristic time of the fuel that leaves the film.

AFR CONTROL SYSTEM

For the application of the proposed control strategy, a test bench was built around the single-cylinder research engine and shown in Fig. (2) has been used. The test bench is a platform for the development of innovative model-based air/fuel ratio control strategies for SI engines. Actually, it was designed to function in a very flexible way. The research engine used in this test bench can be controlled either with its traditional electronic control unit or with a different control system that forms the platform on which the new model-based control strategy operates. The two control systems can operate either in parallel or separately using the same sensors and actuators. In both cases, the engine-state observing system provides continuous real time information about the engine during the control process.

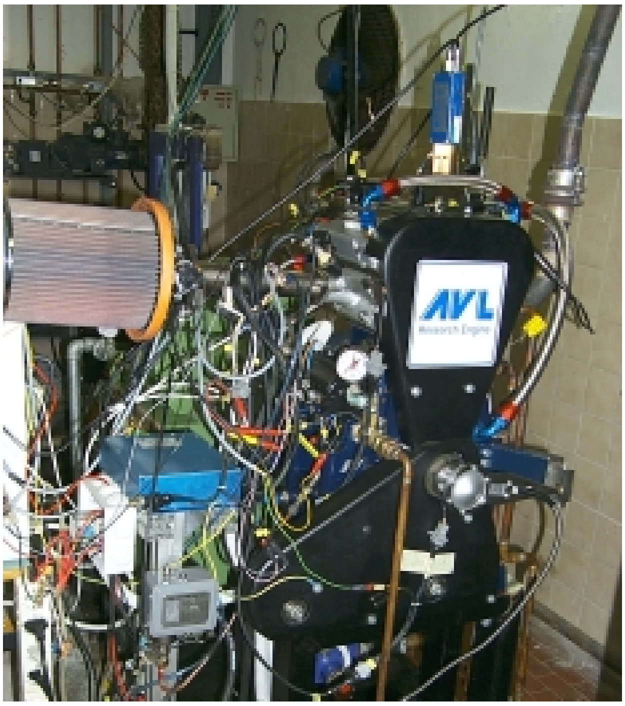

Figure 2: Photo of the research engine (type AVL 5401).

The Proposed-Strategy Platform

A sophisticated and computerized data acquisition system has been built for the application tests. It allows quick and easy recording of all parameters required by the tests including direct readout of engine speed, air/fuel ratio and exhaust emissions in addition to a range of temperatures and pressures.

For signals acquisition, a multi-function National-Instruments data acquisition board (type NI PCI 6034E) was installed in a high speed PC computer. All the codes were written in C++ programming language to reduce to the minimum all time delays, being C++ is the best high level programming language for real time applications. All signals pass through conditioning electronic circuits in order to clean them up from disturbances and reshape them according to type the acquisition board needs.

The control system is based on the real time integration of the equations for air and fuel dynamics. The integration process is based on the input signals, while the result of the process forms the output command to be passed to the fuel injector. The same multi-function board is used to pass the control command to the injector energizing circuit.

EXPERIMENTAL RESULTS

The obtained results have been for a control system that depends exclusively on the proposed strategy without any further compensation for errors of estimation. The aim of that was to provide an insight into the errors, which are related to the over or under estimation of the air mass flow rates. This helps to find out the best way to implement the proposed strategy in a complete control system. Consequently, that could lead to the attainment of maximum efficiency.

The experiments were divided into two categories. During all the tests, throttle position, engine speed, air/fuel ratio and exhaust gas composition were registered over different periods of time. Regarding the oxygen sensor, it was installed very close to the exhaust valve in order to measure the real output of the transient control process before any dilution can take place. (Note: During engine transients, the exhaust gas composition may change inside the exhaust manifold. This occur in the volumes usually present between the exhaust valve and the oxygen sensor, which is due the presence of exhaust gas that has somewhat different composition from previous cycles.)

The first category was the constant engine speed in which the research engine was subjected to throttle plate transients. For each engine speed, very severe throttle opening and closing tests have been done. The tests were concentrated more on the range from part-load to the maximum, which is the most severe operating range in the normal driving habits. In addition, the tests were conducted in a very rapid sequence in order to keep the engine in a continuous transient state, which is the most severe range where the wave effects during the induction process become most significant.

The second category of experiments was transient operation that includes throttle and load changes. By this way, the effectiveness of the proposed control strategy during such transient operation can be demonstrated more clearly. During the tests, the research engine was subjected to a very severe transient situations in which the throttle plate was opened from minimum positions to maximum in very rapid way. During this operation the engine was subjected to load changes (from no load to full load) produced by a torque-speed controller using a prefixed maximum engine speed. This was to simulate the very rapid accelerations and decelerations far away from those usually presented in the literature.

To demonstrate the effectiveness of the proposed control strategy in any unpredicted transient situation, the throttle movements were done by hand, which introduces a high degree of randomization in the throttle plate movements.

First Category

Figures (3) to (6) show the results of this category of transient tests for engine speeds 2000, 2500, 3000 and 3500 rpm, respectively. The presented results include the most frequent driving habits. In the considered range of engine speeds, the maximum tuning effect takes place; therefore, the most severe air dynamics transients were reproduced.

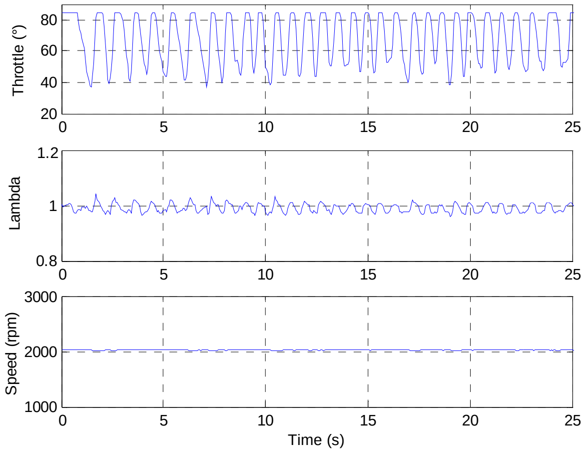

Figure 3: Throttle transients at 2000 rpm.

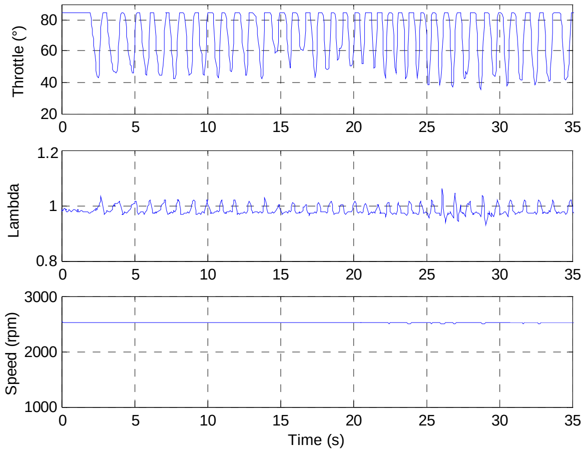

Figure 4: Throttle transients at 2500 rpm.

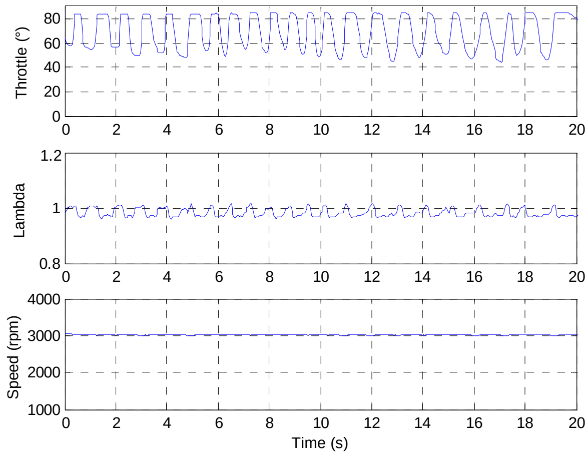

Figure 5: Throttle transients at 3000 rpm.

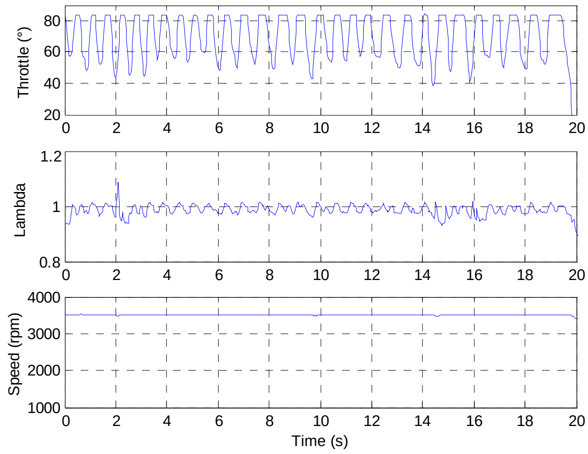

Figure 6: Throttle transients at 3500 rpm.

The results obtained for this category show very small variations in the air/fuel ratio (Lambda) even if the transient situations created by the throttle plate movements are quite severe. In all of the considered situations, the error of AFR set point (Stoichiometric) lies within the range of 1-2 percent, despite the severity of the tests. The authors believe that this error is mainly due to the difference between the actual coefficient of discharge and the assumed one obtained from steady measurements. The difference between the real and the mean values of the wall temperatures used by the method and the delay time of one engine cycle adds to this error.

Disturbances were also an evident source of error as it is impossible to completely filter the incoming signals. The disturbances can reshape slightly the real signal, which is to be used by the control system and consequently contribute to the total error.

If one considers the severe transients produced on the mixture (air and fuel) transport processes due to the throttle plate movement, it can be concluded that the control strategy appears to be very effective and the modeling of the processes is suitable for air/fuel ratio control in SI engines.

Similar results to those presented can be obtained at higher engine speeds, which therefore do not introduce a speed limit.

Second Category

Figures (7) to (11) show the obtained results for second category of experiments that regarded the effectiveness of the proposed control strategy during more complex transients, which include throttle and speed changes.

In the tests, the research engine was subjected to five different transient cases (A to E) as shown in Fig. (7) to Fig. (11), respectively. The figures show the movement of the throttle plate (from minimum positions to WOT or vice versa) that produces a quite rapid accelerations and decelerations. Accordingly, very critical transients are therefore produced that go beyond what normally happen onboard.

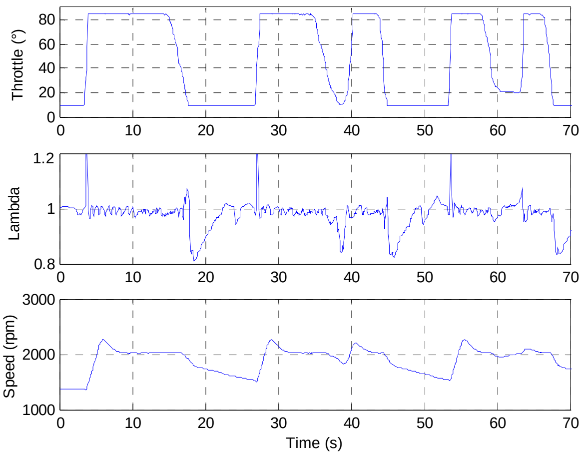

Figure 7: Throttle and speed transients – Case A.

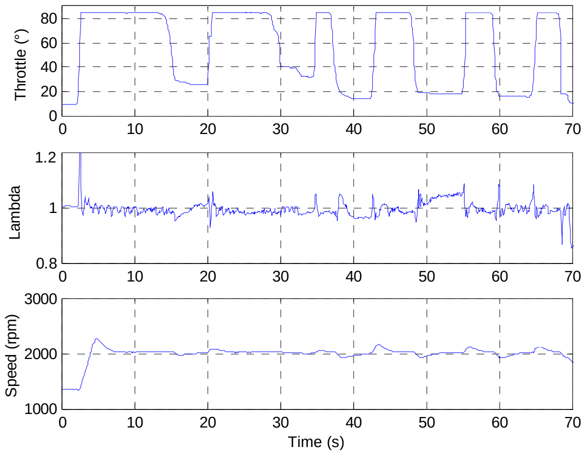

Figure 8: Throttle and speed transients – Case B.

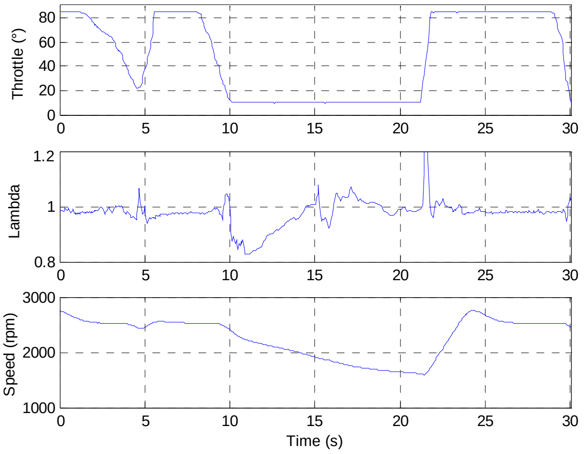

Figure 9: Throttle and speed transients – Case C.

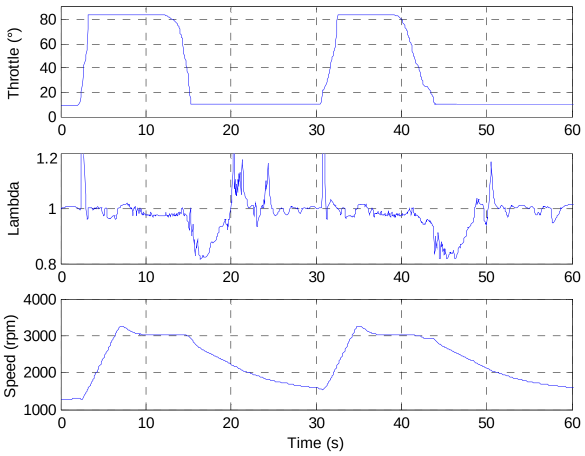

Figure 10: Throttle and speed transients – Case D.

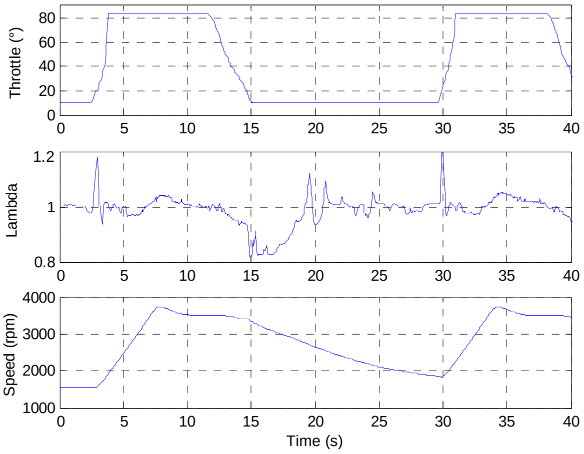

Figure 11: Throttle and speed transients – Case E.

Obviously, all sources of error explained in the previous category have their effects in these tests also. But, in such transients some sources of error may give more significant contributions to the total error of the air/fuel ratio control process. During throttle tip-outs engine speed remains in transient for more than 5 seconds and more than 15 seconds during tip-ins. This means two important things: the first is that during this period the control system response is always in retard of one engine cycle, which accumulates the error for a significant part of the transient period. In fact, it is evident as can be seen in the figures. But the error, however, is extremely limited in time with respect to that of the transient situation and can be compensated by a derivative feedforward technique. Also, using the drive-by-wire throttle would be very helpful; given the fact that, throttle opening or closing command comes from the control system and consequently eliminates the acquisition-response delay time.

The second important thing that more evident error zone comes as a result of a severe deceleration caused by the throttle plate closure (seen in the figure till to the minimum position). The main source of error in this case is the over estimation of the airflow through the throttle plate, which is due to the fact that a steady-state coefficient of discharge has been used, and given the fact that engine speed changes significantly in these tests. In such situations, the modeling of the fuel film dynamics used here shows a limited capability in describing the real phenomena, so significantly contributes to the total error. In fact, the fuel film parameters in this case are mean values, which are not sufficiently specified neither as a function of engine speed nor throttle position. Though, it will be helpful to consider more effective modeling for the fuel film dynamics with suitable action on the fuel injection.

The results of Case B presented in Fig. (8) confirm all the previous discussion: In this case, inspite of the severity of the throttle movement (does not reach the minimum position) the engine speed was not allowed to vary too much; the previously explained effects related to the throttle coefficient of discharge and engine speed are less significant.

Moreover, the examination of the results also highlights the fact that, the control process can keep the peek of the air/fuel ratio excursions small enough in time, which helps not to deteriorate the drivability; it is rapid to return to the steady-state conditions even before the transients situation ends.

CONCLUSIONS

An innovative model-based air/fuel ratio control strategy for SI engines has been presented to be an effective alternative to the technology in use nowadays.

The strategy is based on a new air dynamics modeling that is able to represent the air transient specially developed for the online AFR control applications. The well-known Aquino model has been adopted for the fuel transient compensation.

The proposed strategy was applied successfully to a port-fuel-injected single-cylinder research engine (type AVL 5401).

Results of a wide range of extremely severe engine transient situations have been presented and analyzed to give an insight to the strategy capability.

All the presented analysis and results have proved the effectiveness of proposed control strategy. Therefore, it will be suitable to be used in the upcoming generation of the air/fuel ratio control systems.

Finally, the results obtained highlight the necessity of introducing some improvements on the fuel film dynamics and the throttle discharge coefficients. This is to allow for more precise airflow estimation when severe transient occurs, especially during throttle plate tip-ins.

ACKNOWLEDGMENTS

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APPENDIX

A. Intake Pipe Section

The modeling is different according to the flow direction. In case of direct flow, the application of the energy conservation for isentropic flow gives for subsonic flow

$$\dot{m}_t = C_{d_t} A_t P_o \beta \Big/ \sqrt{RT_o} \tag{A.1}$$

and for sonic flow

$$\dot{m}_t = C_{d_t} A_t P_o \alpha \Big/ \sqrt{RT_o} \tag{A.2}$$

being

$$\beta = \left\{ \frac{2\gamma}{\gamma - 1} \right\}^{1/2} P_r^{1/\gamma} \sqrt{1 - P_r^{(\gamma-1)/\gamma}} \tag{A.3}$$

$$\alpha = \gamma^{1/2} \left\{ \frac{2\gamma}{\gamma + 1} \right\}^{(\gamma+1)/(2(\gamma-1))} \tag{A.4}$$

$$P_r = P \big/ P_o \tag{A.5}$$

In case of reverse flow, the pressure can not be fully recovered through a sudden enlargement; considering all the kinetic energy lost, the following conditions applies

$$P = P_o \tag{A.6}$$

B. Intake Valve Section

In case of direct flow, from pipe to cylinder through the valve, for subsonic flow applies

$$\dot{m}_v = \rho C_{d_v} A_P \left\{ \frac{2\gamma R T_c}{\gamma - 1} \left[P_r^{(\gamma-1)/\gamma} - 1 \right] \Big/ \left[\frac{1}{\varphi^2} P_r^{2/\gamma} - 1 \right] \right\}^{1/2} \tag{B.1}$$

being

$$P_r = P \big/ P_c \tag{B.2}$$

$$\varphi = A_v \big/ A_P \tag{B.3}$$

while for sonic flow

$$\dot{m}_v = \rho_o C_{d_v} A_v \varphi \sqrt{\gamma R T_o} \Big/ K^{2/(\gamma-1)} \tag{B.4}$$

where K is given by the solution of

$$\varphi^2 = \left\{ \frac{\gamma + 1}{\gamma - 1} - \frac{2}{\gamma - 1} K^2 \right\} K^{4/(\gamma-1)} \tag{B.5}$$

In case of reverse flow, from cylinder to pipe through the valve, two processes occur. The first from cylinder to valve is similar to direct flow in the intake pipe section previously discussed, considering cylinder pressure and temperature as the stagnation conditions.

The second after the valve is referred to a sudden enlargement till to the wall-pipe-section where the kinetic energy can be considered lost.