OPTIMAL VECTOR CONTROL OF THREE-PHASE INDUCTION MACHINE

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ABSTRACT

This paper presents the optimal solutions of the induction motor (IM) fluxes and currents that minimize the total energy of the motor. A second order model based on vector control approach relating motor fluxes (states) and currents (controls) is considered. Optimal state and control trajectories of IM are obtained by solving algebraic Riccati equation. In addition, the simulation results for different weight are presented. Also this paper considers and compares between two cases based on the second order model.

KEY WORDS

Induction machine, vector control, optimal control.

1. Introduction

The Induction motors (IM) for many years have been regarded as the workhorse in industry.

Recently, the induction motors were evolved from being a constant speed motors to a variable speed. In addition, the most famous method for controlling induction motor is by varying the stator voltage or frequency. To use this method, the ratio of the motor voltage and frequency should be approximately constant. With the invention of Field Orientated Control [1-12, 17-20], the complex induction motor can be modeled as a DC motor by performing simple transformations.

These transformations are known as the *abc-dq* transformations, where the stator currents are transformed into two dc currents using *Clarke* transformation. The two dc currents are then transformed into a rotating reference frame using *Park* transformation.

Transforming the stator currents allows the motor to be modelled as a DC motor and not a complex three-phase motor.

Using the vector control approach, several methods have been proposed in the literature to control the IM, for example, H. Zidan *et al* successfully applied the estimation method to control the induction motor drives without using speed sensors; they used simple speed estimation method for IM drive at low speed, this method uses the current and the input voltages in closed loop for rotor parameter estimation [1].

While, B. Hovingh, *et al* presented an algorithm to estimate the rotor's speed and torque from the terminal voltage and input current to the motor. They showed that measurement of the stator voltage and currents are sufficient to determine the rotor position, speed and torque of an induction motor during any conditions, whether transient or steady state. Their work is being performed to analyze the response of a Field Orientated Control system when the estimated waveforms are used as an input into the control loop [2].

On the other hand, Jose Ramirez and Carlos Canudas de Wit presented experimental results of a nonlinear torque-flux optimal control for induction motor drives. This controller minimizes the stored magnetic energy and the coil losses, while satisfying torque tracking control objectives [3].

In addition, H. Rasmussen used an adaptive approach leading to a completely new method called Field Angle Adaptation (FAA). The new contribution to the conventional current control system in rotor field oriented dq-coordinates is a signal added to the field angle in the transformation from rotor field coordinates to stator fixed coordinates. This signal adapts the field angle estimate to the correct rotor field angle [4].

In this paper, the optimal control method is applied to induction motor modelled by vector control or field oriented control (FOC). Based on the second order model for induction motor, which presented in [5, 7], two cases have been considered in this paper. The optimal controls and the optimal states of each case are obtained using the algebraic Riccati equation. To show the effectiveness of the proposed method simulations results are shown in this paper.

The paper organized as follows: Section 2 reviews the vector control and induction motor's second order linear model, section 3 formulate the optimal control problem of the induction motor modeled by vector control. Section 4 presents the two cases based on the linear model that have been treated and section 5 shows the simulation of both cases. Finally, section 6 concluded this paper.

2. Vector control and induction motor model

2.1 Direct and Quadrature axis transformation (d-q)

The vector control technique uses the dynamic equivalent circuit of the induction motor, and this technique enables the induction motor to be controlled in a method similar to DC motor [2]. The conversion of three-stator currents into two DC currents enables the speed and torque of the motor to be calculated in manner similar to DC motor (figure 1). The direct and quadrature axis transformation (abc-dq) converts a three-phase signal of three different vectors in two dimensional frame using *Clarke* and *Park* transformations [2, 7].

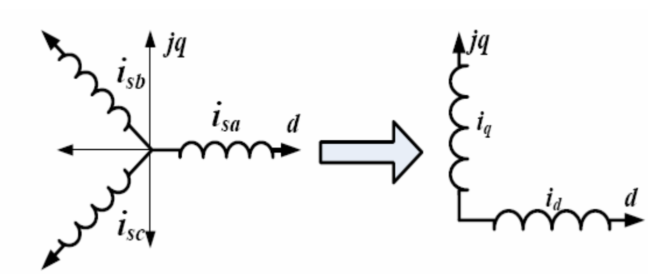


Figure 1: 3-phase to dq equivalents

A d-q axis transformation eliminates the mutual magnetic coupling of the phase winding, making magnetic flux linkage of the current are independent to each other.

The transformation from three-phase system to two phase system and vice versa is as given in equations (1) and (2), respectively [6-9]:

$$\begin{bmatrix} i_{sq} \\ i_{sd} \\ i_o \end{bmatrix} = \frac{2}{3}\begin{bmatrix} \cos\theta & \cos\left(\theta-\frac{2\pi}{3}\right) & \cos\left(\theta-\frac{4\pi}{3}\right) \\ \sin\theta & \sin\left(\theta-\frac{2\pi}{3}\right) & \sin\left(\theta-\frac{4\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}\begin{bmatrix} i_{sa} \\ i_{sb} \\ i_{sc} \end{bmatrix} \quad (1)$$

And

$$\begin{bmatrix} i_{sa} \\ i_{sb} \\ i_{sc} \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 1 \\ \cos\left(\theta-\frac{2\pi}{3}\right) & \sin\left(\theta-\frac{2\pi}{3}\right) & 1 \\ \cos\left(\theta-\frac{4\pi}{3}\right) & \sin\left(\theta-\frac{4\pi}{3}\right) & 1 \end{bmatrix}\begin{bmatrix} i_{sq} \\ i_{sd} \\ i_o \end{bmatrix} \quad (2)$$

Moreover, the stator voltages are related to the d-q rotating frame of induction motor (IM) as given in equation (3):

$$\begin{bmatrix} v_{sq} \\ v_{sd} \end{bmatrix} = \frac{2}{3}\begin{bmatrix} \cos\theta & \cos\left(\theta-\frac{2\pi}{3}\right) & \cos\left(\theta-\frac{4\pi}{3}\right) \\ \sin\theta & \sin\left(\theta-\frac{2\pi}{3}\right) & \sin\left(\theta-\frac{4\pi}{3}\right) \end{bmatrix}\begin{bmatrix} v_{sa} \\ v_{sb} \\ v_{sc} \end{bmatrix} \quad (3)$$

Where i_{sa}, i_{sb}, i_{sc} are the stator currents in abc frame, v_{sa}, v_{sb}, v_{sc} are stator voltages in abc frame, i_{sq}, i_{sd} are the stator currents in dq rotating frame, and v_{sq}, v_{sd} are stator voltages in dq rotating frame, θ: the phase angle between rotating frame and stationary frame.

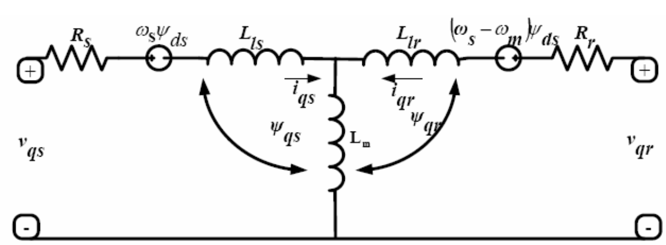
2.2 Induction motor model

Figure 2 shows the d-q equivalent circuits for a three phase symmetrical squirrel cage induction motor in arbitrary rotating frame with zero sequence component neglected [6, 7, 9, 10].

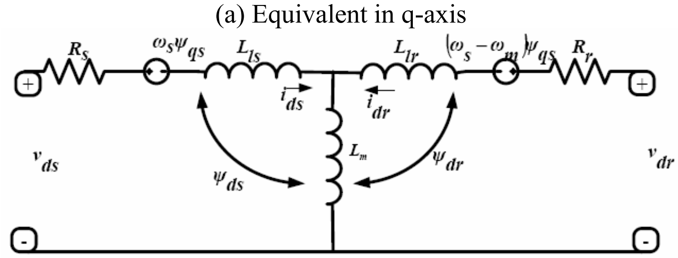
From the dynamic equivalent circuit, the induction motor parameters can be expressed in matrix equation (4), assuming that the rotor bars in squirrel cage induction motor are shorted out and the rotor voltages equal zero.

$$\begin{bmatrix} v_{qs} \\ v_{ds} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} R_s+pL_s & \omega_s L_s & pL_m & \omega_s L_m \\ -\omega_s L_s & R_s+pL_s & -\omega_s L_s & pL_m \\ pL_m & (\omega_s-\omega_m)L_m & R_r+pL_r & (\omega_s-\omega_m)L_r \\ -(\omega_s-\omega_m) & pL_m & -(\omega_s-\omega_m)L_r & R_r+pL_r \end{bmatrix}\begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{qr} \\ i_{dr} \end{bmatrix} \quad (4)$$

Where R_s, R_r are the stator and the rotor resistance per phase respectively, L_s, L_r are the stator, and the rotor inductance per phase, respectively, $p=\frac{d}{dt}$ operator, ω_s, ω_m are synchronous and rotor speeds respectively.



(a) Equivalent in q-axis



(b) Equivalent in d-axis

Figure 2: Dynamic equivalent circuit for induction motor.

Moreover, the rotor flux linkages are given by equation (5 & 6):

$$\begin{bmatrix}\psi_{qr}\\ \psi_{dr}\end{bmatrix}=\begin{bmatrix}i_{qr} & i_{qs}\\ i_{dr} & i_{ds}\end{bmatrix}\begin{bmatrix}L_r\\ L_m\end{bmatrix} \tag{5}$$

Solving for i_{qr}, i_{dr} we obtain:

$$\begin{bmatrix}i_{qr}\\ i_{dr}\end{bmatrix}=\begin{bmatrix}\psi_{qr} & i_{qs}\\ \psi_{dr} & i_{ds}\end{bmatrix}\begin{bmatrix}\dfrac{1}{L_r}\\ \dfrac{-L_m}{L_r}\end{bmatrix} \tag{6}$$

Substituting equation (5) in equation (4) and using the equation (6), the following state equation model can be obtained [7]:

$$\begin{bmatrix}\dot{\psi}_{qr}\\ \dot{\psi}_{dr}\end{bmatrix}=\begin{bmatrix}-\dfrac{L_r}{R_r} & -(\omega_s-\omega_m)\\ (\omega_s-\omega_m) & -\dfrac{L_r}{R_r}\end{bmatrix}\begin{bmatrix}\psi_{qr}\\ \psi_{dr}\end{bmatrix}+\frac{L_m}{L_r}R_r\begin{bmatrix}i_{qs}\\ i_{ds}\end{bmatrix} \tag{7}$$

Equation (7) represents a second order model for the induction motor in terms of the flux parameter and the currents. This equation can be rewritten in a compact form as: $\dot{x}=Ax+Bu$.

Where $x=[x_1 \quad x_2]^T=[\psi_{qr} \quad \psi_{dr}]^T$ (Flux components), and $u=[u_1 \quad u_2]^T=[i_{qs} \quad i_{ds}]^T$ (Current components).

3. Optimal Control problem

The standard theory of the optimal control is presented in [14,15,16]. For linear time invariant system with state vector (x) and control vector (u), the optimal control problem is given by:

Find the optimal control vector u^* that minimizes the performance index

$$J=\frac{1}{2}\int_0^{t_f}\left(x^TQx+u^TRu\right)dt \tag{8}$$

Subject to state equation constraints

$$\dot{x}(t)=Ax(t)+Bu(t) \tag{9}$$

Where Q is a positive semi-definite real symmetric state weighting matrix and R is positive definite real symmetric control weighting matrix.

The choice of the element Q and R allows the relative weighting of individual state variables of fluxes and individual control inputs of current components as well as relative weighting state vector (fluxes) and control vector (current components) against each other.

In case of the time invariant system A, the optimal control feedback matrix K results in constant –gain state feedback comparable to multivariable P- controller with currents.

It is well known that the optimal state feedback control that solves this problem is given by

$$u^*(x)=-Kx(t) \tag{10}$$

And matrix gain K is given by

$$K=R^{-1}B^TP \tag{11}$$

For the case of the time invariant system, and $t_f=\infty$ the optimal control feedback gain matrix ***K*** is constant. And ***P*** is the solution of the algebraic Riccati equation

$$A^TP+PA-PBR^{-1}B^TP+Q=0 \tag{12}$$

And $P>0$, $A-BR^{-1}B^TP=(A-BK)$ stable (all eigenvalues in the open left half-plane) [13].

For our problem we consider the following performance index:

$$J=\tfrac{1}{2}\int_0^{\infty}\left(\psi_r^TQ\psi_r+i_s^TRi_s\right)dt \tag{13}$$

which minimizes total energy in the induction motor which is the sum of the stored magnetic energy in the inductance, the dissipated energy in the rotor and stator resistances, the dissipated energy due to core losses (Foucault currents and magnetic hysterics), and mechanical energy [11, 16, 17].

4. Problem Formulation

Based on the second order model of the induction motor shown in equation 6, we will consider the following two cases:

- **Case I:** The frequency difference between the synchronous speed and the mechanical speed [5, 20], i.e. $\omega_s-\omega_m=0$. Therefore, equation (7) become

$$\begin{bmatrix}\dot{\psi}_{qr}\\ \dot{\psi}_{dr}\end{bmatrix}=\begin{bmatrix}-\dfrac{L_r}{R_r} & 0\\ 0 & -\dfrac{L_r}{R_r}\end{bmatrix}\begin{bmatrix}\psi_{qr}\\ \psi_{dr}\end{bmatrix}+\frac{L_m}{L_r}R_r\begin{bmatrix}i_{qs}\\ i_{ds}\end{bmatrix} \tag{14}$$

And the optimal control problem is to determine the optimal feedback control vector [i_{qs} i_{ds}], that minimizes the performance index

$$J=\tfrac{1}{2}\int_0^{\infty}\left(\psi_r^TQ\psi_r+i_s^TRi_s\right)dt \tag{15}$$

Subject to state equation (14).

This problem is solved using the optimal control technique presented in section 3.

- **Case II:** The difference between the synchronous speed and the mechanical speed is given by:

$$\omega_s - \omega_m = sl.\omega_s \tag{16}$$

Where sl: represent the induction motor slip [7, 18].

By substituting (16) into equation (7) will get:

$$\begin{bmatrix} \dot{\psi}_{qr} \\ \dot{\psi}_{dr} \end{bmatrix} = \begin{bmatrix} -\frac{L_r}{R_r} & -sl\,\omega_s \\ sl\,\omega_s & -\frac{L_r}{R_r} \end{bmatrix} \begin{bmatrix} \psi_{qr} \\ \psi_{dr} \end{bmatrix} + \frac{L_m}{L_r} R_r \begin{bmatrix} i_{ds} \\ i_{ds} \end{bmatrix} \tag{17}$$

And the optimal control problem is to find the optimal feedback control vector that minimizes (15) subject to (17).

Also, this problem is solved using the method presented in section 3.

5. Simulation

To show the solution of the two problems given in the previous section, a simulation program using MATLAB and SIMULINK is implemented. Different weighting matrices have been considered.

$Q = \begin{bmatrix} q_1 & 0 \\ 0 & q_2 \end{bmatrix}$ and $R = \begin{bmatrix} r_1 & 0 \\ 0 & r_2 \end{bmatrix}$ with their eigenvalues $q_1, q_2 \geq 0$, and $r_1, r_2 > 0$.

The simulation has been performed using five sets of Q and R; $q_1, q_2 = 0.01, 0.1, 1, 10, 100$, and $r_1, r_2 = 0.01, 0.1, 1, 10, 100$. respectively.

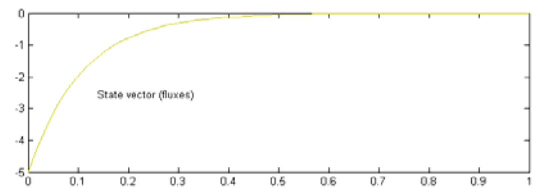
In addition, we show the simulation of the rotor speed given by [19]:

$$\dot{\omega}_m = \frac{z^2 L_m}{I L_r}\left(i_{sq}\psi_{rd} - i_{sd}\psi_{rq}\right) - \frac{F}{I}\omega_m - \frac{z}{I}T_l \tag{18}$$

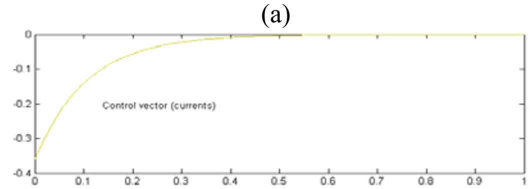
Where z is the number of the poles of the induction motor, I is the moment of inertia, F is the viscous friction coefficient, T_l is the torque load.

Figure 3 shows the simulation result of (case I) presented in section 4. Figure 3.a shows the state feedback control vector, figure 3.b shows the states, and figure 3.c for the motor speed under load torque 1.5 Nm, rated speed =1440 rpm, z= 2 poles, R_s=1.15 Ω, R_r=1.44 Ω, L_m=0.144 H, L_s=L_r= 0.156 H, I= 0.013 kg.m², F=0.002Nm.s/rad and initial states [-5 -5] weber.

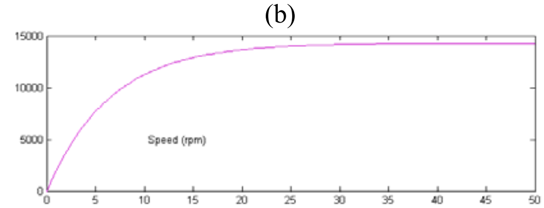
While Figure 4 shows the simulation result of the (case II) of section 4. Where figure 4.a shows the state feedback control vector, figure 4.b shows the states, and figure 4.c for the motor speed under load torque 1.5 Nm and with same previous motor parameters.



(a)

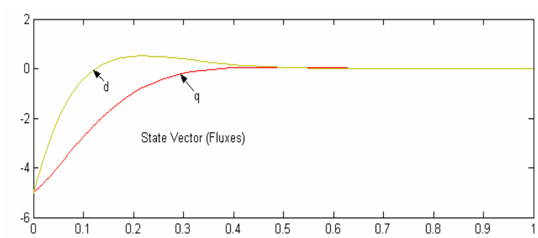


(b)

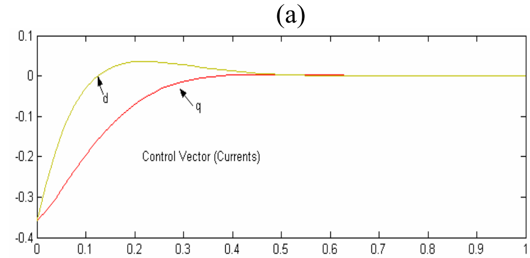


(c)

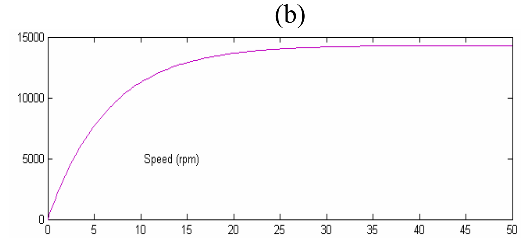
Figure 3: Case I Simulation



(a)



(b)



(c)

Figure 4: Case II Simulation

From the previous simulation, we noticed that changing the weighting matrices do not effect the optimal control or the optimal states.

3. Conclusion

In this paper, we obtained the optimal trajectories of states (IM fluxes) and controls (IM currents). These trajectories are obtained by minimizing the quadrature performance measure or total energy of the induction motor.

The simulation is carefully done to obtain the controls, states trajectories using matlab and simulink programs.

Using different state and control weighting matrices (Q and R) didn't affect the response as shown in the figures (3a, 3b, 4a, 4b), so that the speed response wasn't affected too as shown in figures (3c and 4c).

Moreover, the speed responses of the different cases presented in section 4 are not different as shown in figures (3c and 4c).

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