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A time-varying load-based analytical approach for DG optimization in the distribution network

Nassim A. Iqteit | Ayşen Basa Arsoy | Bekir Çakir

Department of Electrical Engineering, Kocaeli University, İzmit, Turkey

Correspondence
Nassim A. Iqteit, Department of Electrical Engineering, Kocaeli University, İzmit, Turkey.
Email: nassimiqteit@gmail.com

Summary
This paper presents a novel analytical approach to estimate the optimal generation profiles/size and location of a distributed generation (DG) unit that can be integrated into distribution network to attain the lowest level of active loss. This approach considered the impact of significant parameters including daily load profiles, best power limits of a DG unit, and varying injected power profiles of the substation on results of minimum active loss of the distribution network and optimal calculations of DG. Additionally, the DG placement was speedily identified in this approach by using a new active loss expression along with four different scenarios. These scenarios considered a number of assumptions on the load profiles and on the state variables (voltage and angle). The obtained optimal active generation profile was also used to manage the uncertainty in the energy of the wind and photovoltaic distribution generators for achieving the highest loss reduction. Additionally, the exhaustive load flow method was used to check the validity of the analytical approach by using 13-bus and 69-bus distribution networks. The results obtained suggest the effectiveness of this approach in estimating the optimal location and profile/size of DG.

KEYWORDS
distributed generation, energy management, loss reduction, optimal location, optimal profile

1 | INTRODUCTION

One of the important aspects of research in recent years has been the incorporation of optimally sized and located distributed generation (DG) units into distribution networks. The essence is to reduce loss with the aim of improving economic and environmental conditions of electrical systems as well as enhancing the performance of distribution networks.[1] Photovoltaic (PV) and fuel cells with converter/inverter systems are the examples of renewable generators, while on the other hand, an internal combustion engine and a combustion turbine are examples of nonrenewable generators.[2] Consequently, DG units could be considered as renewable or nonrenewable generators. The main advantages of integrating optimal DG units in distribution networks are loss reduction, improved voltage profiles and stability, enhanced reliability and loadability of the system.[3,4] However, the random use of DG units could lead to redundant power losses, reverse power flow, and increased heat in feeders.[5]

Few techniques such as genetic algorithm (GA) considering various indices,[6] particle swarm optimization (PSO) and hybrid GA-PSO based on loss reductions,[7,8] and chaotic artificial bee colony (CABC) using multi objective performance index[9] are used in estimating the optimal size and location of DG units. There are also other methods and algorithms used for minimizing active loss by inserting DG units or capacitors into distribution network such as; ant lion optimization algorithm,[10] bacterial foraging optimization algorithm,[11] golden rule or 2/3 rule,[12] fuzzy logic, and full search.[13] However, these algorithms and methods require a lot of computations and are slow in convergence, while the analytical methods are computationally less demanding and are fast in convergence.[14] Many analytical methods have been proposed in determining optimal locations and size of a DG unit where the active loss is minimum. These methods include Loss Sensitivity Factor (LSF), which is used to estimate optimal bus number,[14,15] analytical expression methods,[15-17] improvement analytical (IA) method,[18] and efficient analytical (EA) method.[19] Also, other methods have been proposed for siting multiple DG units in distribution networks to minimize active loss.[20-22] A multi objective function taking the active and reactive loss into account have also been proposed to find optimal size of DG.[3] The common point between most of these methods and algorithms is the sizing and siting of DG units based on peak or average demands, with a few researches considering time varying demands.[23,24]

However, there is a dearth of information in literature with regard to the effects of the daily load profiles with their types (industrial, commercial, and residential) and energy management principle. Additionally, selection of the appropriate limits of a DG unit was absent in the methods and algorithms that have been reported in this literature. On the other hand, finding the optimal location of DG by using these methods and algorithms requires running the load flow program a large number of times. Furthermore, in these analytical methods, the changing of the substation powers was neglected when estimating the location of a DG unit. All of the abovementioned drawbacks were solved by suggesting a new analytical approach.

In this paper, a new analytical approach was proposed to estimate the DG sizing and siting in the distribution network with a minimum active loss value. This approach considered the time-varying demands and the optimal maximum limits of DG for finding the generation profiles and the size value of DG. These considerations improve the estimation results of minimum loss and optimal profiles/size of DG. Four different scenarios are also examined in this paper for estimating the optimal location of DG in the distribution network in a quick manner. These scenarios depend on the proposed active loss formula with some assumptions on phasor voltages and load demands. However, the main target of these scenarios was to reduce the number of times of running the load flow program when the optimal location of DG was identified. In addition, the proposed loss formula merged the effect of injected power profiles of the substation in the power profiles of the loads and DG, where taking this effect of substation power on active loss into account improves the estimation results of DG location. The computation procedures of the proposed approach have been detailed in this study. Moreover, the uncertainty in the output energy of the renewable energy resources and energy management principle are examined by this approach. Exhaustive load flow (ELF) method is used to check the validity of the proposed approach of each scenario by applying all of them on 13-bus and 69-bus distribution systems.

The outline of the paper is presented as follows: section 2 presents the power loss formula according to load profiles. The methods of estimating optimal profile/size and location of DG are presented in section 3. Section 4 presents numerical results including test systems and simulation results. The simulation results were divided into different sections including the base case losses of both systems, optimal location results, the optimal profiles/size of DG, and the results of integrating renewable energy DG units into the network using the energy management principle. Finally, the main contributions and conclusions are summarized in section 5.

2 | POWER LOSS FORMULAS

Elgerd's loss formula[14,18,25] was modified in the study of Iqteit, Arsoy, and Çakir[26] in order to be compatible with loss calculation by taking the variation of load demands and generation sources with time into account. The modified active power loss profile in the study of Iqteit[26] for an N-bus distribution system at each time m is given in Equation 1. This equation is an approximate formula because it was derived based on linearization principle.

$$P_{LTm} = \boldsymbol{P}_{\boldsymbol{m}}^{\boldsymbol{T}}\boldsymbol{\alpha}_m\boldsymbol{P}_m + \boldsymbol{Q}_{\boldsymbol{m}}^{\boldsymbol{T}}\boldsymbol{\alpha}_m\,\boldsymbol{Q}_m + \boldsymbol{Q}_{\boldsymbol{m}}^{\boldsymbol{T}}\boldsymbol{\beta}_{\boldsymbol{m}}\boldsymbol{P}_{\boldsymbol{m}} - \boldsymbol{P}_{\boldsymbol{m}}^{\boldsymbol{T}}\,\boldsymbol{\beta}_{\boldsymbol{m}}\,\boldsymbol{Q}_{\boldsymbol{m}}, \tag{1}$$

where, $\boldsymbol{P}_{\boldsymbol{m}}$ and $\boldsymbol{Q}_{\boldsymbol{m}}$ are the injected active and reactive power vectors in an N-bus system, respectively. The coefficient matrices $\boldsymbol{\alpha}_{\mathbf{m}}$, $\boldsymbol{\beta}_{\mathbf{m}}$, $\boldsymbol{\gamma}_{\mathbf{m}}$, and $\boldsymbol{\xi}_{\mathbf{m}}$ have the size $N \times N$. Equation 1 can be reformulated by summation form as shown in Equation 2.

$$P_{LTm} = \sum_{i=1}^{N}\sum_{j=1}^{N}\left[\begin{array}{c}\alpha_{mij}\left(P_{mi}P_{mj}+Q_{mi}Q_{mj}\right)\\ +\beta_{mij}\left(Q_{mi}P_{mj}-P_{mi}Q_{mj}\right)\end{array}\right], \tag{2}$$

where

$$\alpha_{mij} = \frac{r_{ij}}{U_{mi}U_{mj}}\cos\left(\delta_{mi}-\delta_{mj}\right);$$

$$\beta_{mij} = \frac{r_{ij}}{U_{mi}U_{mj}}\sin\left(\delta_{mi}-\delta_{mj}\right),$$

$U_{mi} \angle \delta_{mi}$ is the phasor voltage at node i and time m; $r_{ij} + j\,x_{ij}$ is the ijth entry of impedance matrix $[Z_{ij}]$; P_{mi} and P_{mj} are the active power injections at time m and at nodes i and j, respectively; Q_{mi} and Q_{mj} are the reactive power injections at time m and at nodes i and j, respectively.

The active and reactive power injected profiles at bus i and time m, with the inclusion of the DG are given by Equations 3 and 4, respectively,

$$P_{mi} = P_{DGmi} - P_{Dmi}, \tag{3}$$

$$Q_{mi} = Q_{DGmi} - Q_{Dmi}, \tag{4}$$

where, $P_{DGm1} = P_{SGm1}$ and $Q_{DGm1} = Q_{SGm1}$ for $i = 1$, $P_{mj} = -P_{Dmj}$ and $Q_{mj} = -Q_{Dmj}$ for $j \neq 1$, and $j \neq i$. P_{Dmi} and P_{Dmj} are the active load demand profiles. Q_{Dmi} and Q_{Dmj} are the reactive load demand profiles.

The total average active loss along period M (ie, 1 day) is computed by Equation 5.

$$P_{LTDGi} = \frac{1}{M}\sum_{m=1}^{M}P_{LTDGmi}, \tag{5}$$

From Equations 2 to 5, the average active power loss in an N-bus system can be rewritten as follows:

$$P_{LTDGi} = \frac{1}{M}\sum_{m=1}^{M}\sum_{i=1}^{N}\sum_{j=1}^{N}\left[\begin{array}{c}\alpha_{mij}\left((P_{DGmi}-P_{Dmi})P_{mj}+(Q_{DGmi}-Q_{Dmi})Q_{mj}\right)\\ +\beta_{mij}\left((Q_{DGmi}-Q_{Dmi})P_{mj}-(P_{DGmi}-P_{Dmi})Q_{mj}\right)\end{array}\right]. \tag{6}$$

3 | OPTIMAL PROFILE/SIZE AND LOCATION OF DG

According to Equation 6, the location and the profiles of DG can change the active loss in any distribution system. In order to obtain the minimum possible active loss in a distribution network, Equation 6 was applied as the main objective function to estimate the optimal profiles/size and location of DG that can be inserted into the network. In the following sections, the steps in estimating the profiles, optimal limits, and the location of DG have been discussed.

3.1 | Optimal profiles and size of DG

The optimal profiles of DG (P_{DGmi}^{opt} and Q_{DGmi}^{opt}) with respect to its location i were calculated by solving the minimization problem in Equation 7.

$$\min_{P_{DGmi},\,Q_{DGmi}}\{P_{LTDGi}\}; \tag{7}$$

The coefficients α_{mij} and β_{mij} in Equation 6 are functions with state variables (U_{mi},δ_{mi}), and they depend on the power injections, but their effects on the active power loss are very minimal compared with DG effects. For this reason, the coefficients can be assumed constants at each value of m. However, the effects of these coefficients were overcome by updating the optimization and loss calculations more than two times.

The solution of problem in Equation 7 requires the derivative of P_{LTDGi} with respect to the variables P_{DGmi} and Q_{DGmi} at both values of m and i. These derivatives are given in Equations 8 and 9, respectively.

$$\frac{\partial P_{LTDGi}}{\partial P_{DGmi}} = \frac{1}{M}\frac{\partial P_{LTDGmi}}{\partial P_{DGmi}} = 2\left(\frac{1}{M}\right)\left\{\alpha_{mii}\left(P_{DGmi} - P_{Dmi}\right) + \sum_{\substack{j=1 \\ j\neq i}}^{N}\left(\alpha_{mij}P_{mj} - \beta_{mij}Q_{mj}\right)\right\} = 0, \tag{8}$$

$$\frac{\partial P_{LTDGi}}{\partial Q_{DGmi}} = \frac{1}{M}\frac{\partial P_{LTDGmi}}{\partial Q_{DGmi}} = 2\left(\frac{1}{M}\right)\left\{\alpha_{mii}\left(Q_{DGmi} - Q_{Dmi}\right) + \sum_{\substack{j=1 \\ j\neq i}}^{N}\left(\alpha_{mij}Q_{mj} + \beta_{mij}Q_{mj}\right)\right\} = 0. \tag{9}$$

The solution of Equations 8 and 9 yields the optimal profiles of DG at each bus i. This solution is given in Equations 10 and 11, whereas the size of DG is given in Equations 12 and 13.

$$P_{DGmi}^{opt} = P_{Dmi} - \frac{1}{\alpha_{mii}}\sum_{\substack{j=1 \\ j\neq i}}^{N}\left(\alpha_{mij}P_{mj} - \beta_{mij}Q_{mj}\right), \tag{10}$$

$$Q_{DGmi}^{opt} = Q_{Dmi} - \frac{1}{\alpha_{mii}}\sum_{\substack{j=1 \\ j\neq i}}^{N}\left(\alpha_{mij}Q_{mj} + \beta_{mij}P_{mj}\right), \tag{11}$$

$$\overline{P}_{DGi}^{opt} = \frac{1}{M}\sum_{m=1}^{M}P_{DGmi}^{opt}, \tag{12}$$

$$\overline{Q}_{DGi}^{opt} = \frac{1}{M}\sum_{m=1}^{M}Q_{DGmi}^{opt}. \tag{13}$$

Equations 6 and 10-13 were used to examine the impact of the daily load profiles ($M = 24$) on the optimal calculations of DG and active loss in the N bus distribution system.

3.2 | Limitation conditions

1- Voltage profile limits

The connection point of DG should have an average voltage between the limits shown in Equation 14. In this paper, the minimum voltage was assumed as 0.95 pu, while the maximum value was 1.05 pu.

$$U_i^{min} \leq \overline{U}_{mi} \leq U_i^{max}. \tag{14}$$

2- Optimal limits of DG power profiles

It should be emphasized that in some situations, the optimal values obtained in Equations 10 and 11 fall out of the limits as provided in the general limitations of DG defined in Equation 15. As a result, selecting optimal maximum limits in such situations becomes significant in minimizing active power loss. It was assumed that the maximum limit is a fraction of total average load demands or fraction of total load profiles, which are defined in Equations 16 and 17, respectively. The value of this fraction $k\in[0,1]$ denotes the optimal limitations when the active loss is at minimum level.

$$\begin{cases} 0 \le P_{DGmi}^{opt} \le P_{DGmi}^{max} \\ 0 \le Q_{DGmi}^{opt} \le Q_{DGmi}^{max} \end{cases}, \tag{15}$$

where

Case1 Power limitations are constants and equal to fraction of total average load demands.

$$\begin{cases} \mathrm{P}_{DGmi}^{max} = k\left(\frac{1}{M}\sum_{i=1}^{N}\sum_{m=1}^{M} P_{Dmi}\right) \\ Q_{DGmi}^{max} = k\left(\frac{1}{M}\sum_{i=1}^{N}\sum_{m=1}^{M} Q_{Dmi}\right) \end{cases}, \tag{16}$$

Case2 Power limitations are profiles and equal to a fraction of total load profiles.

$$\begin{cases} P_{DGmi}^{max} = k\sum_{i=1}^{N} P_{Dmi} \\ Q_{DGmi}^{max} = k\sum_{i=1}^{N} Q_{Dmi} \end{cases}, \tag{17}$$

3.3 | Optimal location

The ELF method can be used to estimate the optimal location of DG by finding out the optimal losses at each value of both time m and location i. In other words, calculation of all entries of loss matrix $\mathbf{M_{loss}}$ in Equation 18 is obliged. Computing these entries needs a large number of numerical calculations (ie, by Newton Raphson or one of power flow programs), and this is a critical problem that was also resolved in this paper. The optimal location was discovered by assigning the minimum loss in the vector of the total active average losses at each location i, as shown in Equation 19. The Matlab function $\left[P_{LTDGi}^{opt}, i^{opt}\right] = \min(\mathrm{average}(\mathbf{M}_{\mathrm{loss}}))$ can be used to find the optimal location i^{opt}. Elgerd's loss formula at assumed base case (without DG) coefficients (α and β) and substation injected power is one way to estimate the optimal location of DG where the active loss is the minimum value.[18] However, inserting DG in different locations changes the substation injected powers, this effect has been taken into account in this paper.

$$\mathbf{M_{Loss}} = \begin{bmatrix} P_{LTDG11}\left(P_{DG11}^{opt}, Q_{DG11}^{opt}\right) & P_{LTDG12}\left(P_{DG12}^{opt}, Q_{DG12}^{opt}\right) & \cdots & P_{LTDG1M}\left(P_{DG1M}^{opt}, Q_{DG1M}^{opt}\right) \\ P_{LTDG21}\left(P_{DG21}^{opt}, Q_{DG21}^{opt}\right) & P_{LTDG22}\left(P_{DG22}^{opt}, Q_{DG22}^{opt}\right) & \cdots & P_{LTDG2M}\left(P_{DG2M}^{opt}, Q_{DG2M}^{opt}\right) \\ \vdots & \vdots & P_{LTDGim}\left(P_{DGim}^{opt}, Q_{DGim}^{opt}\right) & \vdots \\ P_{LTDGN1}\left(P_{DGN1}^{opt}, Q_{DGN1}^{opt}\right) & P_{LTDGN2}\left(P_{DGN1}^{opt}, Q_{DGN1}^{opt}\right) & \cdots & P_{LTDGNM}\left(P_{DGNM}^{opt}, Q_{DGNM}^{opt}\right) \end{bmatrix}; \tag{18}$$

$$\mathbf{M_{Loss}} \xrightarrow[\ ]{\textit{Find average by Eq.5}} \min_{i} \begin{bmatrix} P_{LTDG1} \\ P_{LTDG2} \\ \vdots \\ P_{LTDGi} \\ \vdots \\ P_{LTDGN} \end{bmatrix} \rightarrow i^{opt}. \tag{19}$$

A new active power loss formula considering the variations in substation injected power was derived by solving the equations of active and reactive power loss profiles together with the equations of power conservation. The reactive loss profile equation[26] and power conservation equations are given in Equations 20-22, respectively.

$$Q_{LTDGmi} = \sum_{i=1}^{N}\sum_{j=1}^{N}\begin{bmatrix}\gamma_{mij}(P_{mi}P_{mj}+Q_{mi}Q_{mj})\\+\xi_{mij}(Q_{mi}P_{mj}-P_{mi}Q_{mj})\end{bmatrix}, \tag{20}$$

where

$$\gamma_{mij} = \frac{x_{ij}}{U_{mi}U_{mj}}\cos(\delta_{mi}-\delta_{mj}),$$

$$\xi_{mij} = \frac{x_{ij}}{U_{mi}U_{mj}}\sin(\delta_{mi}-\delta_{mj}),$$

$$P_{SGm1} = P_{LTDGmi} + \sum_{j=1}^{N} P_{Dmj} - P_{DGmi}, \tag{21}$$

$$Q_{SGm1} = Q_{LTDGmi} + \sum_{j=1}^{N} Q_{Dmj} - Q_{DGmi}, \tag{22}$$

The derivation of the proposed formula is discussed in the following steps.

- The load profiles (P_{Dmi}, Q_{Dmi}) and the optimal profile of DG (P_{DGmi}^{opt}, Q_{DGmi}^{opt}) are known variables in the system Equations 2 and 20-22. Also, the coefficient values (α_{mij}, β_{mij}, γ_{mij} *and* ξ_{mij}) are known variables in these equations.
- The generated substation power profiles P_{SGm1} and Q_{SGm1} and power loss profiles P_{LTDGmi} and Q_{LTDGmi} are four unknown variables in the system Equations 2 and 20-22.
- After solving the system of equations, one of the results was a quadratic equation of active power loss profile as shown in Equations 23, where the factors a_{Pmi}, b_{Pmi}, and c_{Pmi} depend only on the known values that are coefficient values, load profiles, and optimal profile of DG. The exact solution of this quadratic equation is displayed in Equation 24. According to the numerical results, the approximation solution of the active loss was defined in Equation 25.

$$a_{Pmi}P_{LTDGmi}^2 + b_{Pmi}P_{LTDGmi} + c_{Pmi} = 0, \tag{23}$$

$$P_{LTDGmi} = \frac{-b_{Pmi} \mp \sqrt{b_{Pmi}^2 - 4a_{Pmi}c_{Pmi}}}{2a_{Pmi}}, \tag{24}$$

$$P_{LTDGmi} \approx \frac{-b_{Pmi}}{2a_{Pmi}}, \tag{25}$$

where

$$\mathrm{a_{Pmi}} = 1 + \left(\frac{\mathrm{F_{Ami}}}{\mathrm{F_{Bmi}}}\right)^2,$$

$$\mathrm{b_{Pmi}} = \frac{2\mathrm{F_{Ami}F_{Cmi}}}{\mathrm{F_{Bmi}^2}} + \left(2\mathrm{E_{EPmi}} + \frac{\mathrm{A}_{\alpha\,\beta\mathrm{mi}} - 1}{\alpha_{\mathrm{m11}}}\right) - \frac{\mathrm{F_{Ami}}}{\mathrm{F_{Bmi}}}\left(2\mathrm{E_{EQmi}} + \frac{\mathrm{B}_{\alpha\,\beta\mathrm{mi}}}{\alpha_{\mathrm{m11}}}\right),$$

$$\mathrm{c_{Pmi}} = \left(\frac{\mathrm{F_{Cmi}}}{\mathrm{F_{Bmi}}}\right)^2 - \frac{\mathrm{F_{Cmi}}}{\mathrm{F_{Bmi}}}\left(2\mathrm{E_{EQmi}} + \frac{\mathrm{B}_{\alpha\,\beta\mathrm{mi}}}{\alpha_{\mathrm{m11}}}\right) + \left(\mathrm{E_{EPmi}^2} + \mathrm{E_{EQmi}^2} + \frac{\mathrm{A}_{\alpha\,\beta\mathrm{mi}}\mathrm{E_{EPmi}} + \mathrm{B}_{\alpha\,\beta\mathrm{mi}}\mathrm{E_{EQmi}} + \mathrm{C}_{\alpha\,\beta\mathrm{mi}}}{\alpha_{\mathrm{m11}}}\right),$$

where

$$\mathrm{F_{Ami}} = \frac{\mathrm{A}_{\alpha\,\beta\mathrm{mi}} - 1}{\alpha_{\mathrm{m11}}} - \frac{\mathrm{A}_{\gamma\,\xi\mathrm{mi}}}{\gamma_{\mathrm{m11}}},$$

$$F_{Bmi} = \frac{B_{\alpha\ \beta mi}}{\alpha_{m11}} - \frac{B_{\gamma\ \xi mi} - 1}{\gamma_{m11}},$$

$$F_{Cmi} = \frac{A_{\alpha\ \beta mi} E_{EPmi} + B_{\alpha\ \beta mi} E_{EQmi} + C_{\alpha\ \beta mi}}{\alpha_{m11}} - \frac{A_{\gamma\ \xi mi} E_{EPmi} + B_{\gamma\ \xi mi} E_{EQmi} + C_{\gamma\ \xi mi}}{\gamma_{m11}},$$

$$E_{EPmi} = \sum_{j=1}^{N} P_{Dmj} - P_{DGmi},$$

$$E_{EQmi} = \sum_{j=1}^{N} Q_{Dmj} - Q_{DGmi},$$

$$A_{\alpha\ \beta mi} = 2\alpha_{mi1} P_{DGmi} + 2\beta_{mi1} Q_{DGmi} - 2\sum_{j=1}^{N} \alpha_{m1j} P_{Dmj} + 2\sum_{j=1}^{N} \beta_{m1j} Q_{Dmj},$$

$$B_{\alpha\ \beta mi} = 2\alpha_{mi1} Q_{DGmi} - 2\beta_{mi1} P_{DGmi} - 2\sum_{j=1}^{N} \alpha_{m1j} Q_{Dmj} - 2\sum_{j=1}^{N} \beta_{m1j} P_{Dmj},$$

$$C_{\alpha\ \beta mi} = \alpha_{mii}\left(P_{DGmi}^{2} + Q_{DGmi}^{2}\right) + P_{DGmi}\left(-2\sum_{j=1}^{N} \alpha_{mij} P_{Dmj} + 2\sum_{j=1}^{N} \beta_{mij} Q_{Dmj}\right) + Q_{DGmi}\left(-2\sum_{j=1}^{N} \alpha_{mij} Q_{Dmj} - 2\sum_{j=1}^{N} \beta_{mij} P_{Dmj}\right) + D_{EPLLmi},$$

$$D_{EPLLmi} = \sum_{\mathbf{i=1}}^{\mathbf{N}} \sum_{\mathbf{j=1}}^{\mathbf{N}} \begin{bmatrix} \alpha_{mij}\left(P_{Dmi} P_{Dmj} + Q_{Dmi} Q_{Dmj}\right) \\ +\beta_{mij}\left(Q_{Dmi} P_{Dmj} - P_{Dmi} Q_{Dmj}\right) \end{bmatrix},$$

$$A_{\gamma\ \xi mi} = 2\gamma_{mi1} P_{DGmi} + 2\xi_{mi1} Q_{DGmi} - 2\sum_{j=1}^{N} \gamma_{m1j} P_{Dmj} + 2\sum_{j=1}^{N} \xi_{m1j} Q_{Dmj},$$

$$B_{\gamma\ \xi mi} = 2\gamma_{mi1} Q_{DGmi} - 2\xi_{mi1} P_{DGmi} - 2\sum_{j=1}^{N} \gamma_{m1j} Q_{Dmj} - 2\sum_{j=1}^{N} \xi_{m1j} P_{Dmj},$$

$$C_{\gamma\ \xi mi} = \gamma_{mii}\left(P_{DGmi}^{2} + Q_{DGmi}^{2}\right) + P_{DGmi}\left(-2\sum_{j=1}^{N} \gamma_{mij} P_{Dmj} + 2\sum_{j=1}^{N} \xi_{mij} Q_{Dmj}\right) + Q_{DGmi}\left(-2\sum_{j=1}^{N} \gamma_{mij} Q_{Dmj} - 2\sum_{j=1}^{N} \xi_{mij} P_{Dmj}\right) + D_{EQLLmi},$$

$$D_{EQLLmi} = \sum_{i=1}^{N} \sum_{j=1}^{N} \begin{bmatrix} \gamma_{mij}\left(P_{Dmi} P_{Dmj} + Q_{Dmi} Q_{Dmj}\right) \\ +\xi_{mij}\left(Q_{Dmi} P_{Dmj} - P_{Dmi} Q_{Dmj}\right) \end{bmatrix}.$$

In order to identify the optimal location of DG, four scenarios were examined by using the proposed Equation 25 to estimate the entries of loss matrix $\mathbf{M_{loss}}$. These scenarios depend on two assumptions of the nodes' voltage: Equation 1 before adding DG (scenarios 1 and 2) and Equation 2 nodes' voltage at $1\angle 0$ pu (scenarios 3 and 4). These assumptions considered the worst and best cases of nodes' voltage after adding DG into the network. Moreover, scenarios 1 and 3 take the effect of daily load profiles into account, while scenarios 2 and 4 consider the average effect of load profiles. The main goals of these scenarios were to reduce the number of load flow calculations and to estimate the location of DG in a quick manner. For instance, the proposed method in the study of Hung, Mithulananthan, and Bansal,[18] which is based on Elgerd's loss formula, has the same assumptions as in scenario 2 and requires two times to run the load flow program in estimating the optimal location of DG, whereas in scenarios 3 and 4, just finding the impedance matrix (requires zero times) is enough to estimate the optimal location of DG. These scenarios and their assumptions are simplified in Table 1.

The simulation results obtained by using these scenarios were compared with the results of the ELF method to prove the validity of the scenarios considered. The optimal location of DG was determined by using ELF, which requires $N \times M \times$ update data times to run load flow program, whereas the proposed scenarios required less than this value as shown in Table 1. Furthermore, in all proposed scenarios and the ELF method, the values of the "update data"

TABLE 1 Proposed scenarios of the coefficients $\alpha, \beta, \gamma, and\ \xi$

Scenario No.	Values of m	Assumptions: Coefficients α_{mij}, β_{mij}, γ_{mij}, and ξ_{mij}	Assumptions: Load demands	Aims of Assumptions: Size of $\mathbf{M}_{\text{loss}}$	Aims of Assumptions: No of running load flow program
1	1,2,...,M	Equal to the base case (without DG)	Daily load profiles	$N \times M$	$1 \times M \times$ *update data*
2	1	Equal to the base case (without DG)	Average values	$N \times 1$	$1 \times 1\times$ *update data*
3	1,2,...,M	$\alpha_{mij} = r_{ij}$, $\beta_{mij} = 0_{ij}$, $\gamma_{mij} = x_{ij}$, and $\xi_{mij} = 0_{ij}$	Daily load profiles	$N \times M$	0 (determining impedance matrix it is enough)
4	1	$\alpha_{mij} = r_{ij}$, $\beta_{mij} = 0_{ij}$, $\gamma_{mij} = x_{ij}$, and $\xi_{mij} = 0_{ij}$	Average values	$N \times 1$	0 (determining impedance matrix it is enough)

Abbreviation: DG, distributed generation.

should be run more than two times. The numerical results in this paper showed that, updating the calculations three times is enough to give accurate results. These include the update data for the base case (without DG) and two times for estimating the optimal calculation of DG and loss.

3.4 | Computational procedures

The following steps give the computational procedures for the proposed approach.

Step 1: Set the parameters of the network: line data, load profiles, and base values.
Step 2: Set the limits of the DG power profiles by assuming the initial limitation factors $k = 1$(For accuracy, the k should be updated to the optimal value).
Step 3: According to scenario 1, 2, 3, or 4 find the coefficients $\alpha, \beta, \gamma, and\ \xi$ then compute the base case losses (without DG).
Step 4: Find the optimal profiles (P_{DGmi}^{opt} and Q_{DGmi}^{opt}) and size of DG at each bus by using Equations 10 and 11, in case these optimal values are located out of the limits in Equation 15, use the maximum values {case 1 or case 2} as optimal values.
Step 5: Use the active loss formula in Equation 25 to compute the loss matrix $\mathbf{M}_{\text{loss}}$ in Equation 18 at each optimal profile of DG (*step 4*).
Step 6: Locate DG with the optimal profiles obtained in *step 4* at each node. Then update and repeat *step 5* at least more than two times.
Step 7: Compute the optimal location of DG (i^{opt}) where the active loss is minimum, by using Equation 19.
Step 8: To find the optimal k factor, locate the optimal DG at i^{opt}, then find k factor (change $k\in[0,1]$ in small step, ie, 10%) where the active loss is minimum.
Step 9: Locate the DG at i^{opt} and update k factor at the optimal value (*step 8*), then use the ELF/proposed approach to obtain the final results (profile/size and power factor of DG, voltage profile, substation power, etc).

Note: To obtain more accurate results, update the k factor in *step 2* at the optimal value that was found in *step 8*. Then repeat all steps without using *step 8*.

3.5 | The power profile model for PV and wind DG

The wind speed and the solar irradiance are random parameters and depend on the geographical region and the weather conditions. In addition, the output power of a wind turbine is proportional to the cube of wind speed, whereas the output power of the PV panel is linearly proportional to the solar irradiance. However, the wind speed can be approximated by the Normal or Weibull distribution function,[27] and the solar irradiance can be estimated by the

Gaussian or Beta distribution function.[28,29] Consequently, in this section, the uncertainty in the produced power by PV or wind DG (P_{DG}) will be approximated according to the Gaussian probability distribution function.

If the PV or wind power for DG $P_{DG} \sim N(\mu,\sigma)$, the random variable defined by $\varepsilon = \frac{P_{DG} - \mu}{\sigma}$ is a standard normal random variable $\varepsilon \sim N(0,1)$. Therefore, the cumulative distribution function $F\ (P_{DG})$ is given in Equation 26.

$$F(P_{DG}) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\varepsilon} \exp\left(-\frac{w^2}{2}\right) dw = \Phi(\varepsilon), \tag{26}$$

where$\Phi(\varepsilon)$ is the standard normal cumulative distribution function (Gaussian function) at the random variable ε. Also, μ and σ are the mean and the standard deviation of random variable P_{DG}. Then, the uncertainty model at time m for the daily generation profile of wind or solar DG at the ith bus is given in Equation 27.

$$P_{DGmi} = \overline{P}_{DGi}\left[\mu_{pu}(m) + \varepsilon(m)\sigma_{pu}(m)\right]; \tag{27}$$

$\overline{P}_{DGi}$ is the size of PV or wind generator, $\mu_{\text{pu}}\ (m)$ and $\sigma_{pu}\ (m)$ are the mean and the standard deviation of PV/wind power profile in per unit values, respectively. Both parameters were generally estimated as shown in Figure 1 by the data in the study of Engin[30] along 12 months. $\varepsilon(m)$ is the Gaussian value at time m and at specific cumulative probability $\Phi(\varepsilon) =\ \Pr\ [E < \varepsilon]$, where $\Phi(\varepsilon)$ can be estimated depending on the weather conditions and historical data.

The optimal size of any DG is calculated by Equations 12 and 13. Consequently, the optimal profile of the PV/wind DG at ith bus when the uncertainty impact is taken into account is given in Equation 28.

$$P_{DGmi} = \frac{\overline{P}_{DGi}^{opt}}{average\left[\mu_{pu}(m) + \varepsilon(m)\sigma_{pu}(m)\right]} \times \left[\mu_{pu}(m) + \varepsilon(m)\sigma_{pu}(m)\right]. \tag{28}$$

4 | NUMERICAL RESULTS

4.1 | Test systems

The proposed approach for finding the optimal profile/size and the location of DG has been tested on two distribution systems. The first system is a 13-bus distribution network having 18 MVA and 12.5 kV specifications and its single line diagram, line data, and load profiles are given in the study of Iqteit et al.[26] The base power of this network is 10 MVA at 12.5 kV. The second system is a 69-node distribution network with a total average load of 3.8021 MW and 2.6945 MVAr. The data of the lines and the loads of this system is found in.[9] The system base values are 12.66 kV and 100 MVA. The

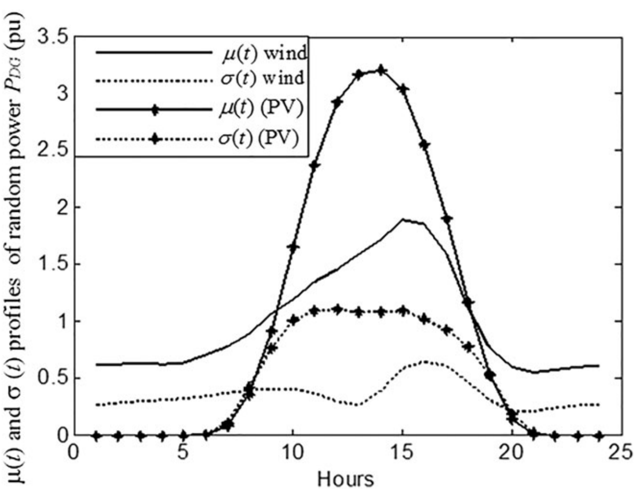

FIGURE 1 Mean and standard deviation of output power for photovoltaic (PV) and wind power distributed generation

load profiles of each node of the second system were calculated by multiplying the average demand by the suitable curve which is given in Figure 2 in article.[31] To maintain the security of both networks, the profiles/size of DG should not be greater than the total profiles/size of load demands.

4.2 | Simulation results

A. Base case power losses of the systems

Table 2 shows the validity of the proposed loss formula in Equation 25 by comparing the loss results of the ELF method and the modified Elgerd's loss formula in Equation 6 with the proposed loss formula.

B. Optimal location results

Figures 3–6 show the active power loss of the 13 and 69 bus systems when the DG of the optimal profiles/size is integrated at each node of the network. The results of Figures 3 and 4 were obtained by using the ELF method and the proposed approach at each scenario, where the calculations were updated twice. These figures were applied at $k = 1$ of case 1 limit conditions. The figures also show the success of scenarios 1 to 3 by estimating the optimal location of DG at node 9 in the 13 bus system and at node 61 in the 69 bus system. However, scenario 4 failed to estimate the same optimal locations in both systems. It should be noted that, the failure of scenario 4 was probably due to the assumptions made, which affected the results of the optimal location. However, to avoid these failures, appropriate limits of DG (optimal k factor) should be selected and calculations should be updated more than two times. Figures 5 and 6 show the application of $k = 0.5$ at case 1 of limit conditions for both systems 13 and 69, respectively. The results of these figures show that all the scenarios' optimal locations were found at node 9 in the 13 bus system, whereas that of the 69 bus system was found at node 61. The scenarios' success in these figures ($k = 0.5$) is the optimal value in both distribution systems, and the calculations were updated three times.

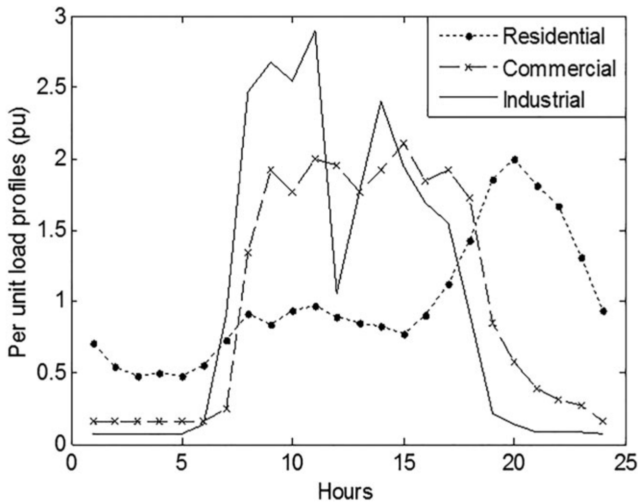

FIGURE 2 The per unit load profile of residential, commercial, and industrial

TABLE 2 Active average loss of the 13 and 69 bus system without distributed generation

	13 Bus System	69 Bus System
System Calculation Method	***P* (MW)**	***P* (MW)**
By using ELF method	0.4070	0.4464
Equation 6 (modified Elgerd's Eq.)	0.4070	0.4464
Average value of Equation 25 (proposed Equation)	0.3990	0.4422

Abbreviation: ELF, exhaustive load flow.

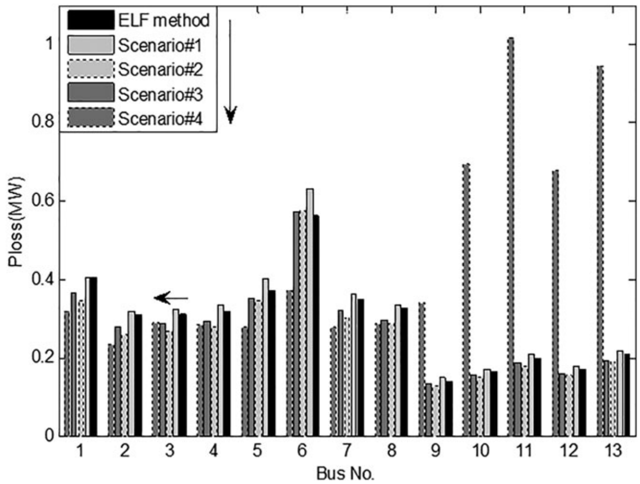

FIGURE 3 Distributed generation location with active power loss of the 13 bus system at $k = 1$ of case 1

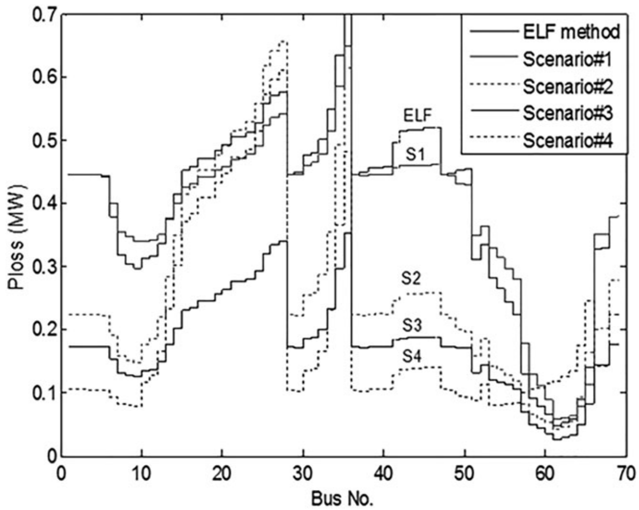

FIGURE 4 Distributed generation location with active power loss of the 69 bus system at $k = 1$ of case 1

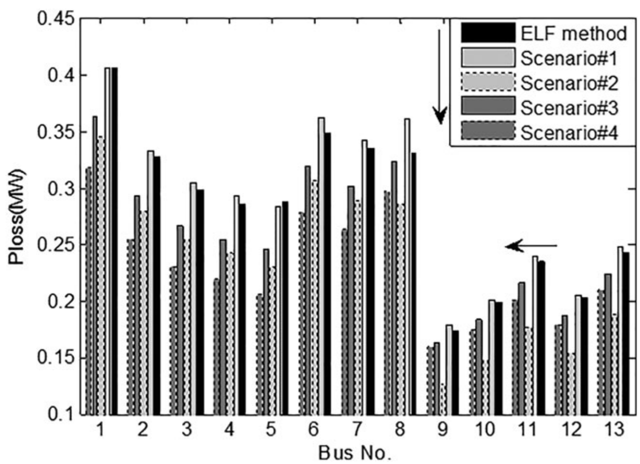

FIGURE 5 Distributed generation location with active power loss of the 13 bus system at $k = 0.5$ of case 1

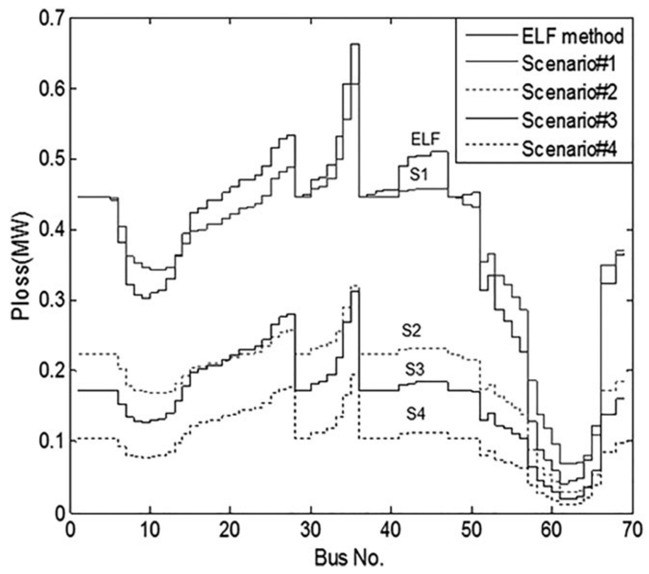

FIGURE 6 Distributed generation location with active power loss of the 69 bus system at $k = 0.5$ of case 1

The limitation conditions of DG power in case 2 are only compatible with the ELF method and scenarios 1 and 3. The results in Table 3 were obtained based on the proposed scenarios at $k = 1$, where the calculations were updated three times in order to obtain accurate results. The table also shows the systems' active losses with and without DG, and demonstrates that the ELF method needs to run the load flow program more than the proposed approach at scenarios 1 and 3. However, scenario 3 is independent of the load flow program and just knowing the impedance matrix is enough to apply Equation 25. Sometimes the results of DG size might not be accurate in scenario 3, but the losses are different in the applied scenarios because they depend on k factor. The results show that scenario 1 succeeded in estimating the optimal location of DG in both distribution systems.

The results in Table 3 illustrate that scenario 1 with power limits at case 2 is the preferred scenario to estimate the optimal location of DG. Table 4 explains the effects of the k factor on the calculation results of optimal size and location of DG. The results in this table were taken after updating the calculations of the ELF method and scenario 1 three times. The losses and size of DG were computed when the k factor is equal to 1, 0.75, 0.5, and 0.25. Figure 7 and Table 4 indicate the optimal value of k, which is equal to 0.5 in both systems; at this value, the results of the ELF method and scenario 1 are close to the minimum active loss and the optimal size value of DG. In the 69 node system, the k factor dominates on the power loss and DG size values, because the optimal size is independent of Equations 10 and 11 in most values of m. Correspondingly, in the 13 bus system, the k factor does not have an effect on the active loss at $k > 0.3$, because in most values of m, the values of Equations 10 and 11 were less than the maximum limits at the given k.

Figures 8 and 9 compare the average voltages at each node before and after adding DG in both the 13 and 69 bus systems, respectively. These figures were obtained in each node of both systems by using the ELF method at optimal size values and optimal limitation conditions ($k = 0.5$ of case 2). Node 9 was found as the optimal location of DG in the 13 bus system, while node 61 was the optimal location in the 69 bus system. At the optimal location nodes of

TABLE 3 Optimal location of distributed generation at case 2 of limit conditions when $k = 1$

System	Scenario No	Base loss P_L (MW)	Location (Bus)	DG Size P_{DG} (MW)	DG Size Q_{DG} (Mvar)	Loss with DG P_L (MW)	No of Running Load Flow
13 bus	ELF	0.407	9	3.95	2.38	0.179	936
	1	0.407	9	3.71	2.29	0.2835	72
	3	0.364	9	3.33	2.66	0.2587	0
69 bus	ELF	0.4464	61	3.54	2.40	0.1644	4968
	1	0.4463	61	3.14	2.38	0.0371	72
	3	0.1728	9 or 53	3.65	2.69	0.1293	0

Abbreviations: DG, distributed generation; ELF, exhaustive load flow.

DG, the voltage increased from 0.9399 to 0.9802 in the 13 bus system, whereas in the 69 bus system, it increased from 0.9048 to 1.002. Furthermore, the voltage values in both systems were found within the acceptable limits at the DG optimal location.

TABLE 4 The effects of limit conditions (*k* factor in case 2) on the results of optimal size and location of distributed generation

System	*k* Factor in Case 2	Base Loss P_L (MW)	Location (Opt Bus)	ELF Method P_{DG} (MW)	ELF Method Q_{DG} (MVAr)	ELF Method Loss with DG P_L (MW)	Scenario#1 P_{DG} (MW)	Scenario#1 Q_{DG} (Mvar)	Scenario#1 Loss with DG P_L (MW)
13 bus	1	0.407	9	3.95	2.38	0.1792	3.71	2.29	0.2835
	0.75	0.407	9	3.90	2.36	0.1783	2.63	1.63	0.2179
	0.50	**0.407**	**9**	**3.55**	**2.14**	**0.1736**	**3.31**	**2.04**	**0.1789**
	0.25	0.407	9	2.60	1.61	0.2006	2.84	1.83	0.1976
69 bus	1	0.4464	61	3.54	2.40	0.1644	3.14	2.38	0.0371
	0.75	0.4464	61	2.78	1.95	0.1123	2.35	1.78	0.0097
	0.50	**0.4464**	**61**	**1.88**	**1.32**	**0.0404**	**1.57**	**1.19**	**0.0688**
	0.25	0.4464	61	0.94	0.67	0.1144	0.784	0.594	0.2144

The bold numbers were computed when the *k* factor is an optimal value.

Abbreviations: DG, distributed generation.

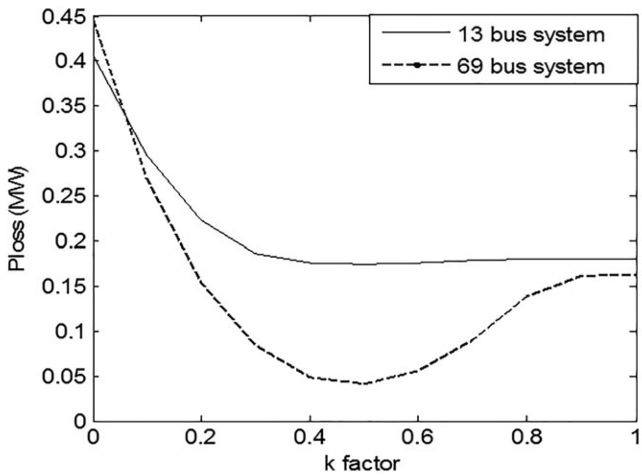

FIGURE 7 The relation between *k* factor and active loss

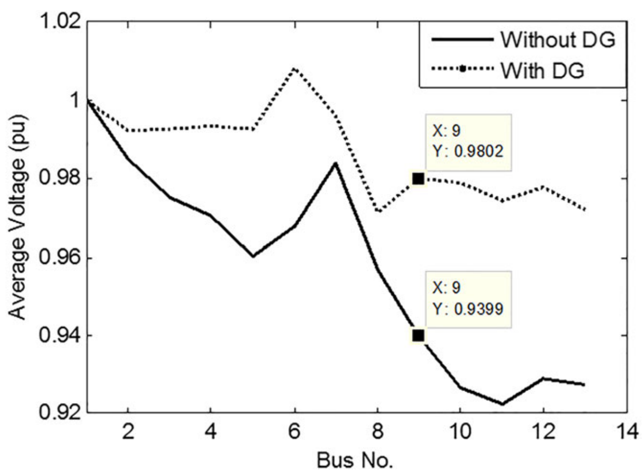

FIGURE 8 Average voltages with locations of distributed generation in the 13 bus system at $k = 0.5$ of case 2

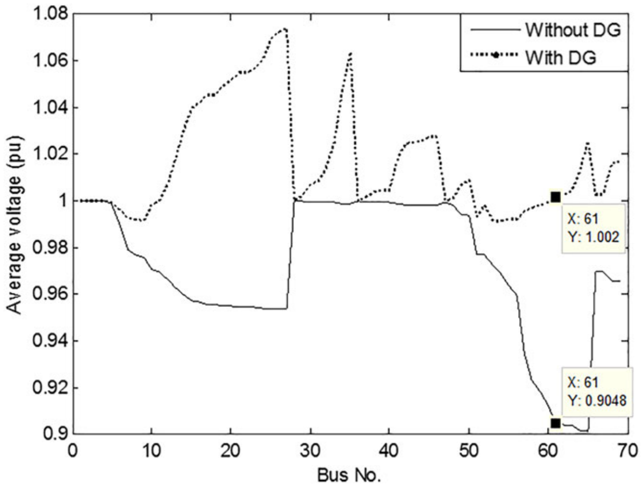

FIGURE 9 Average voltages with locations of distributed generation in the 69 bus system at $k = 0.5$ of case 2

C. Optimal profile and size results of DG

Determining the optimal profiles/size of DG at a specific node in the network (ie, i^{opt}) by the ELF method gives more accurate results compared with the proposed scenarios. For this reason, all the results shown in Figures 10–13 were obtained by using the ELF method at the optimal location node of both the 13 and 69 bus systems and at the optimal power limitation conditions ($k = 0.5$ of case 2). The calculations of this method were updated three times, and this was sufficient to acquire the convergence solutions. Figures 10 and 12 show how the active loss in the 13 and 69 bus systems decreases when using DG, respectively. Figures 11 and 13 show the optimal active and reactive power profiles of DG in both the 13 and 69 bus systems, respectively. These figures show that adding DG of size 4.146 MVA and power factor 0.86 at node 9 of the 13 bus system yields loss reduction of 57.25%. Correspondingly, adding DG of size 2.3 MVA and power factor of 0.82 at node 61 of the 69 bus system produces loss reduction up to 91.03%.

Table 5 presents the change in the substation powers of the 13 and 69 bus systems with and without using DG. All the results indicate the advantages of adding optimal DG at the optimal location for the reduction in active power loss, as well as improving the voltage profile and supplied energy.

D. Renewable energy DG and energy management

In this paper, the proposed approach presents a technique to estimate the optimal daily generation profiles of DG by taking into account the effects of the daily load profiles and the minimum active loss, as well as the uncertainty effect of output power in a PV and wind DG. Nonetheless, in realistic terms, it is impossible to find the active and reactive

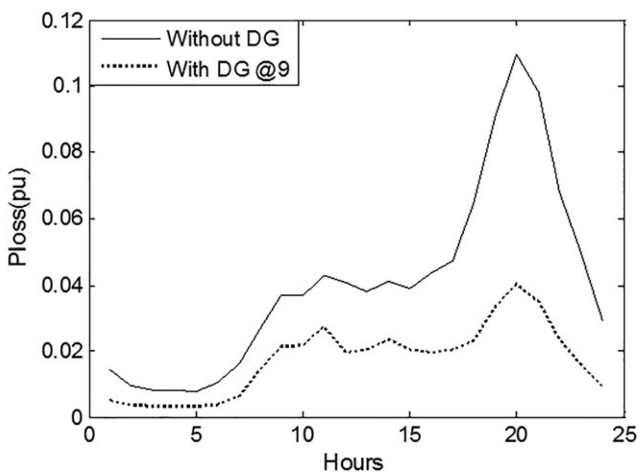

FIGURE 10 Active power loss profiles of the 13 bus system with and without distributed generation

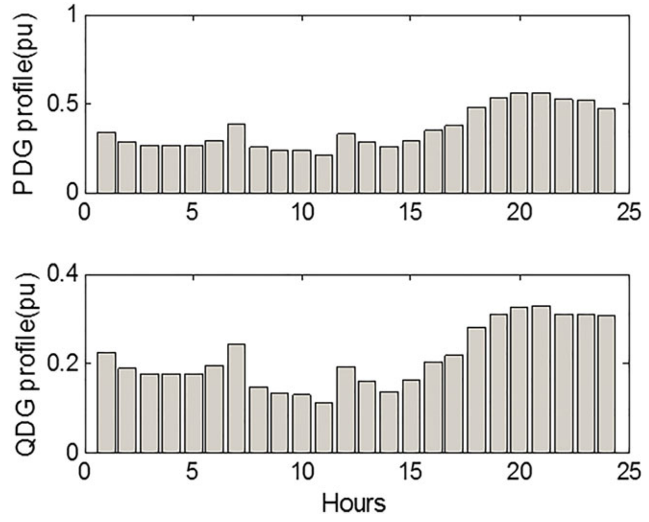

FIGURE 11 Distributed generation power profiles of the 13 bus system at bus 9

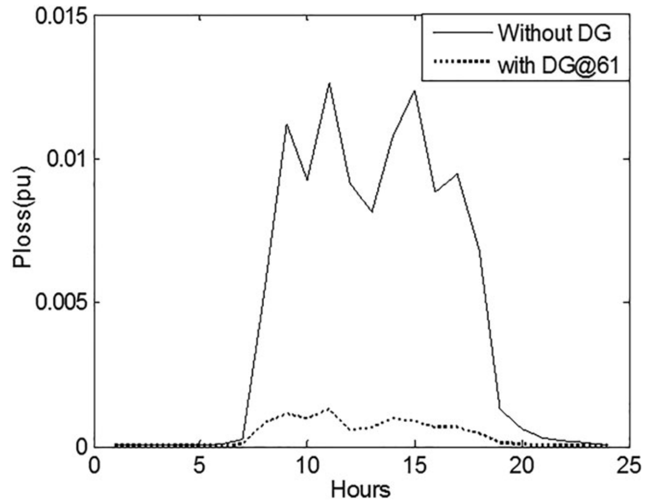

FIGURE 12 Active power loss profiles of the 69 bus system with and without distributed generation

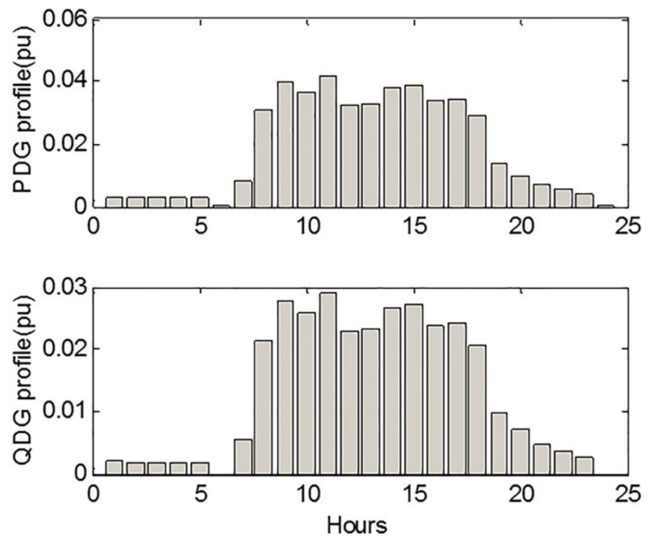

FIGURE 13 Distributed generation power profiles of the 69 bus system at bus 61

TABLE 5 Substation powers with and without using distributed generation

System		P_{SG1} (MW)	Q_{SG1} (MVAr)
13 bus	Without DG	11.783	8.275
	With DG @9	8.00	5.760
69 bus	Without DG	4.27	2.90
	With DG @61	1.99	1.395

Abbreviation: DG, distributed generation.

profiles of the renewable energy DG identical with the optimal profiles. For example, the PV DG does not exhibit reactive profile and its maximum active energy is obtained during the middle of the day. Energy management (EM) is the appropriate method to match the profiles of renewable energy DG units and the optimal profiles.

Table 6 shows the benefit of using the optimal profiles over the renewable energy profiles (PV and wind) when the same size of DG is applied. For example, in the 69 bus system, siting DG of size 2.3 MVA and power factor 0.82 at node 61 yields reduction loss 91.03% depending on the optimal profiles and 81.85% based on wind profile. However, according to the proposed approach, the EM can be done on renewable DG (PV or wind) by using optimal profiles as references. Table 6 also compares the results of the proposed approach and the approach A1 discussed in the study of Hung et al.[23] It can be noted that the base case loss (without DG) is different in both approaches because of the fact that the loss in approach A1 was computed based on the general daily load profile. It should be further emphasized that the load types (industrial, commercial, and residential) at each node of the system were considered in the loss calculation.

The present study shows that the energy of renewable DG (PV solar or wind) was only managed according to the obtained optimal active profile, as shown in Figures 14 and 15, where the output powers of wind and PV were estimated by Equation 28 at $\Phi(\varepsilon) = 0.9$. In this case, the PV/wind DG was located at optimal node 61 of the 69 bus system. The

TABLE 6 Active loss reduction and the optimal size of different types of distributed generation for the 69 bus systems

System	Optimal Bus	P_{loss} (*MW*)	S_{DG} (*MVA*)	PF_{DG}	Active Loss Reduction %	EM Decision	Approach
Without DG	...	0.4463	0	...	0	...	Proposed
DG based on opt Prof	61	0.0403	2.30	0.82	91.03	Ref of EM	
Solar DG (PV)	61	0.2027	2.30	1	54.48	Yes	
Wind DG	61	0.1065	2.30	0.82	76.01	Yes	
Without DG	...	0.1577	0	...	0		Approach A1 in the study of Hung et al[23]
Solar DG (PV)	61	0.1017	2.94	1	35.50	No ref profile	
Wind DG	61	0.0351	3.68	0.82	77.74		

Abbreviation; DG, distributed generation; EM, energy management, ELF, exhaustive load flow; PV, Photovoltaic.

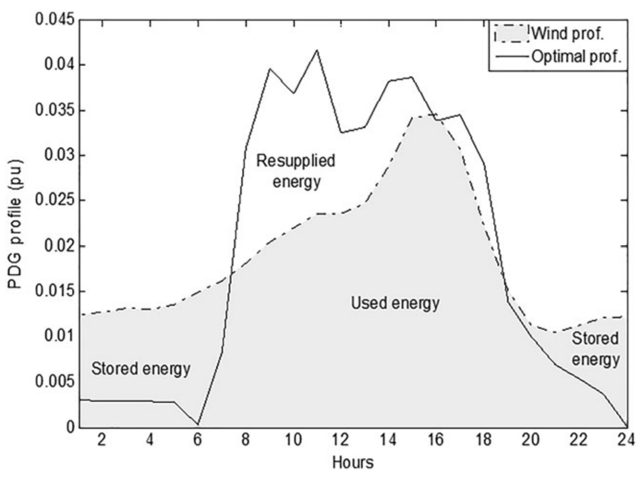

FIGURE 14 Active energy management of wind distributed generation at node 61

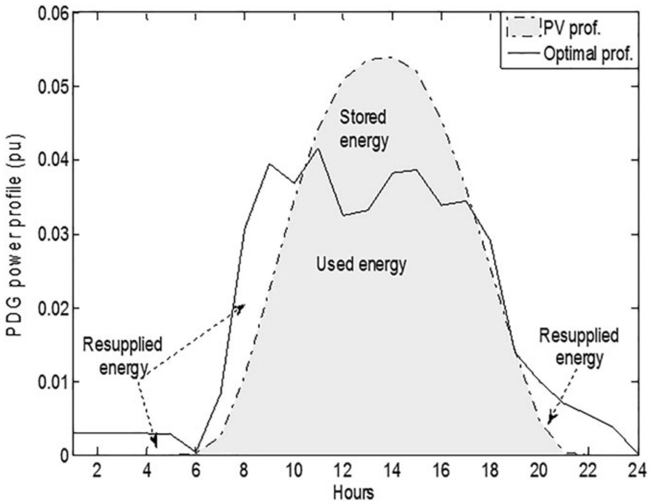

FIGURE 15 Active energy management of photovoltaic solar distributed generation at node 61

TABLE 7 The loss reduction after management of the energy of photovoltaic and wind distributed generation at node 61

System with DG@61	Size of DG P_{DG} *(MW)*	Size of DG Q_{DG} *(MVAr)*	Active Loss Reduction % (without EM)	Active Loss Reduction % (after EM by only Using Opt P_{DG})
Wind	1.88	1.32	81.85	86.35
PV	1.88	0	61.77	67.89

Abbreviation; DG, distributed generation; EM, energy management, ELF, exhaustive load flow; PV, Photovoltaic.

average active power of the renewable DG and the optimal active profile is 1.88 MW. This implies that the area under the optimal curve and the PV/wind curve is the same. Figures 14 and 15 show three types of energy: used energy, stored energy and resupplied energy. Table 7 illustrates that the active loss reduction rate will increase when the stored energy and resupplied energy of the renewable DG are managed together.

5 | CONCLUSIONS

This paper presents a new analytical approach to find the optimal profile of DG and its location in distribution network where the active power loss is minimum. The effects of daily load profiles, optimal limit of the DG powers, and energy balance conservation were considered in calculating the DG profile/size and location. The proposed approach also examined different scenarios in estimating the DG location in the network in a quick manner by reducing the load flow calculations. The principles of these scenarios depend on a new power loss formula and some assumptions on the load demand profiles and phasor voltages. Comparing the results of the scenarios with the ELF method showed that scenario 1 was more efficient than the other scenarios, while the quickest was scenario 4; however, the obtained result was not accurate. Additionally, applying the proposed approach on the 13-bus and 69-bus distribution systems resulted in the following conclusions: (a) the proposed approach at all suggested scenarios gave an appropriate estimation of DG location, but scenario 1 and in some instances scenario 3 gave a good estimation of optimal size and power losses; (b) the maximum limits of DG powers had a big effect on the results of the optimal DG profiles/size with the best maximum limits of DG found to be about half of the total load demand profiles in both systems in this study; (c) the results (active loss, profile/size, and location of DG) of the ELF method and the proposed approach at scenario#1 were closely related; (d) the best loss reduction was yielded by integrating DG with optimal profiles at the optimal location; nonetheless, the application of energy management principle to the renewable energy DG units according to the optimal profiles improved the percentage of loss reduction better than without the application of energy management. Finally, reducing active and reactive losses, improving voltage profile, and supplying energy are the general benefits of integrating the optimal DG at the optimal location into the distribution network.

LIST OF SYMBOLS AND ABBREVIATIONS

$A_{\alpha\beta mi}$, $B_{\alpha\beta mi}$, and $C_{\alpha\beta mi}$	Mathematical expressions that depend on the load demands and DG power besides the coefficients α and β at time m and bus i
$A_{\gamma\xi mi}$, $B_{\gamma\xi mi}$, and $C_{\gamma\xi mi}$	Mathematical expressions that depend on the load demands and DG power besides the coefficients γ and ξ at time m and bus i
D_{EPLLmi} and D_{EQLLmi}	Mathematical expressions that depend on the load demands besides the coefficients {α, β} and {γ, ξ} at time m and bus i, respectively
E_{EPmi}, E_{EQmi}	Mathematical expressions that depend on the load demands and DG power (active and reactive powers, respectively)
F_{Ami}, F_{Bmi}, F_{Cmi}	Mathematical expressions that depend on $A_{\alpha\beta mi}$, $B_{\alpha\beta mi}$, $C_{\alpha\beta mi}$, $A_{\gamma\xi mi}$, $B_{\gamma\xi mi}$, $C_{\gamma\xi mi}$, E_{Epmi} and E_{Eqmi}
M	The period of time (ie, 1 day is 24 hours $M = 24$)
$\mathbf{M_{Loss}}$	The active loss matrix at the optimal size of DG
N	The number of busses in the distribution network
P_{DG}	The random power of the photovoltaic or the wind DG
P_{DGmi}	The active power of DG at time m and bus i
P_{Dmi}	The active load demand at time m and bus i
P_{LTDGi}	The average of active loss profile when the DG is located at the bus i
P_{LTDGN}	The average of active loss profile when the DG is located at the bus N
P_{LTDGmi}	The active loss profile when the DG is located at the bus i
$P_{LTDGim}\left(P_{DGim}^{opt}, Q_{DGim}^{opt}\right)$	The active loss at time m and bus i when the optimal DG is located at time m and bus i
$P_{LTDGNM}\left(P_{DGNM}^{opt}, Q_{DGNM}^{opt}\right)$	The active loss at time M and bus N when optimal DG is located at time M and bus N
P_{LTm}	The active loss profile
$\boldsymbol{P}_m$	The active loss profile vector for all nodes in the distribution system
P_{mi}, P_{mj}	The active injected power at time m for the nodes i and j, respectively
P_{SGm1}	The active power profile associated with the substation located at bus 1
P_{DGmi}^{opt}	The optimal active power profile of DG
$\overline{P}_{DGi}^{opt}$	The average value of the optimal active power profile of DG
P_{DGmi}^{max}	The max limit of the DG active power profile
Q_{DGmi}	The reactive power of DG at time m and bus i
Q_{Dmi}	The reactive load demand at time m and bus i
Q_{LTDGmi}	The reactive loss profile when DG is located at the ith bus
$\boldsymbol{Q}_m$	The reactive loss profile vector for all nodes of the distribution system
Q_{mi}, Q_{mj}	The reactive injected power at time m for the node i and j, respectively
Q_{SGm1}	The reactive power profile associated with the substation located at bus 1
Q_{DGmi}^{opt}	The optimal reactive power profile of DG
$\overline{Q}_{DGi}^{opt}$	The average value of the optimal reactive power profile of DG
Q_{DGmi}^{max}	The max limit of the DG reactive power profile
$\boldsymbol{T}$	Transpose
$V_{mi}V_{mj}$	The voltage profile at node i and j, respectively
V_i^{min}, V_i^{max}	Min. and Max. of the average voltage at node i
$\overline{V}_{mi}$	The average value of the voltage profile at node i
$[Z_{ij}]$	Impedance matrix
a_{Pmi}	Mathematical expressions that depend on values of F_{Ami} and F_{Bmi}
b_{Pmi}	Mathematical expressions that depend on values of F_{Ami}, F_{Bmi}, F_{Cmi}, $A_{\alpha\beta mi}$, $B_{\alpha\beta mi}$, E_{Epmi} and E_{EQmi}
c_{Pmi}	Mathematical expressions that depend on values of F_{Ami}, F_{Bmi}, F_{Cmi}, $A_{\alpha\beta mi}$, $B_{\alpha\beta mi}$, E_{Epmi}, E_{Eqmi} and $C_{\alpha\beta mi}$
i, j	ith and jth busses
i^{opt}	The optimal location of DG
k	The fraction of average total load demand/total load profile

m	Time $m = 1,2,3..M$
r_{ij}	The ijth resistance entry in the impedance matrix
x_{mij}	The ijth reactance entry in the impedance matrix
$\Phi(\varepsilon)$	The cumulative normal probability at Gaussian value ε
$\boldsymbol{\alpha}_m$	The matrix of the coefficients α_{ij} at time m and in the $ijth$ entry
α_{mij}	The coefficient that depends on the voltages, angles and resistance r_{ij} at busses i and j in the time m
$\boldsymbol{\beta_m}$	The matrix of the coefficients β_{ij} at time m and in the $ijth$ entry
β_{mij}	The coefficient which depends on the voltages, angles and resistance r_{ij} at busses i and j in the time m
γ_{mij}	The coefficient that depends on the voltages, angles and reactance x_{ij} at busses i and j in the time m
$\delta_{mi}\delta_{mj}$	The profile of the angle at node i and j, respectively
ε	The standard normal random variable
ξ_{mij}	The coefficient that depends on the voltages, angles and reactance x_{ij} at busses i and j in the time m
μ	The mean value of random variables
μ_{pu}	The mean value in per unit (μ/size of DG)
σ	The standard deviation of random variables
σ_{pu}	The standard deviation in per unit (σ/size of DG)
update data	The number of the updating data in the proposed approach
DG	Distributed generation
ELF	Exhaustive load flow
EM	Energy management
PV	Photovoltaic

ORCID

Nassim A. Iqteit https://orcid.org/0000-0002-2444-6600

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