

A Simple Method to Estimate Power Losses in Distribution Networks

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Abstract

This paper presents a simple approximated formulas to estimate active and reactive power losses in distribution networks. The developed formulas are derived from Elgerd's power loss formulas considering load profiles. A simple model is also proposed to estimate the load profile based on several parameters, such as average of load demand and load limitations. Derivation of the load model and loss formulas are given along with their application on a sample distribution system. The results show that the loss formulas depending on the proposed load profile can estimate the power losses in distribution networks with high precision.

1. Introduction

The load profiles in conventional networks can be estimated from the database of the distribution system, while in smart networks can be obtained directly on time from the smart systems. Estimated load profiles in conventional distribution networks helps the electrical companies in many fields; such as, covering the energy demands of consumers, taking the economic and management decisions, improving the performance of networks...etc. Many researches were carried out to determine the model of load profiles; analytical model for determining uncertainty in distribution loads presented in [1], depends on the calculation of mean and variance of loading profile in each distribution transformer. Daily load profiles for all load types carried out in [2] are modeled depending on the mean, standard deviation and the normal distribution of load profiles. The daily load profile in different seasons were also studied in [3]. Previous studies on load profile need to find mean and standard deviation in constructing load profile model. The proposed model of load profile in this paper consists of a little of load data; type of demands and their average values, maximum and minimum limitations, and the probability of load distribution between the given limitations.

Estimation of active and reactive power losses in distribution systems is one of the main goals for any electric utility company. For example, reduction of real power losses increases the performances of distribution networks and decreases the financial costs. Also, reduction of reactive power losses reduces the voltage drops and reactive power consumption and ameliorating loadability of networks [4]. For these reasons, many equations depending on generated or consumed powers were proposed to calculate power losses in transmission and distribution systems: Such as Kron's loss equation [5], Branch power loss equation [6], and Elgerd's loss equation [7]. The equations were used in the calculation of power losses in the electrical system, economic dispatch analysis [5, 8] and determining the optimal size and location of distributed generators or capacitors in primary distribution networks [4, 6, 9-12]. These equations have related with single values of power (load, generation or injections power)

but they do not have load profile functions with time (i.e. daily, monthly or yearly load profiles) to calculate the average power losses in electrical networks. This paper presents simple approximation power loss formulas to estimate the active and reactive power losses, taking load profiles into account. The full derivation of the proposed power loss formulas is discussed in this manuscript.

The contents of this paper are: simple model of load profiles, active power loss formulas based on load demands, modified power flow equations based on power profiles, an algorithm to find active and reactive power loss profiles, application on a 13 bus distribution test system and its simulation results; finally, summarizes the contributions and conclusions of this work.

2. Simple Approximated Model of Load Profiles

The profiles of residential, commercial and industrial load demands were modeled through field measurements and statistical study [2]. The daily load profiles of $P(t)$ or $Q(t)$ at specific probability can be obtained by using a normal distribution, where the Eq. (1) is the general daily load profile.

$$X_G(t) = \mu(t) + \kappa_1 \sigma(t) \quad (1)$$

where, $X_G(t)$: general daily load profile. $\mu(t)$: the mean curve of certain consumer. $\sigma(t)$: the standard deviation curve of a certain consumer based on daily load curves. κ_1 : the Gaussian distribution value at specific probability $\text{Pr}(\%)$. For example, $\kappa_1=1.3$ at $\text{Pr}=90\%$ and $\kappa_1 = 1.65$, $\text{Pr} = 95\%$. The following proposed procedures present a simple and approximated model of daily load profile depends upon a small amount of load data:

- According to Eq. (1), if the $\sigma(t)=0$, the daily load curve equal to the mean curve. Under this condition, the daily load profile can be defined as $X_G(t) = \mu(t) = \mu_{pu}(t) \times \text{average}\{\mu(t)\}$. At zero standard deviation, we can assume $\mu_{pu}(t)$ is the general behavior of residential, commercial or industrial load profile in per unit system. Where,

$$X_{base} = \text{average}\{\mu(t)\} = \frac{\text{monthly consumption(kWh)}}{24 \times 30} \quad (2)$$

- The per-unit mean profiles of residential, commercial and industrial were obtained through the results of [2], where the area under each curve is equal to one. Fig.1 shows the per unit mean profiles of residential, commercial and industrial load demands.
- Approximated load profile can be derived which are based on the values of average and minimum- maximum limits of load demands, as shown below:

$$X_G(t) = \mu_{pu}(t) X_{base} + \kappa_1 \sigma(t) \quad (3)$$

where, $\sigma(t) = \sqrt{E([X_r(t)]^2) - \mu(t)^2}$

$$\sigma(t) = \mu(t)\sqrt{\lambda - 1} = \mu_{pu}(t) X_{base}\sqrt{\lambda - 1} \quad (4)$$

where:

$$\lambda = \frac{\frac{1}{Nd} \sum_{i=1}^{Nd} [x_i(t)]^2}{\left[\frac{1}{Nd} \sum_{i=1}^{Nd} x_i(t)\right]^2} \quad (5)$$

$E(X_r) = \mu$ is the mean value of X_r , X_r is the random variable in the set $\{x_1, x_2, \dots, x_{Nd}\}$. The subset $\{x_{max}(t), \mu(t), x_{min}(t)\}$ represents the maximum deviation in the set of X_r . This subset can be used to find out the approximate value of the λ ratio. The approximate value of λ is shown in Eq. (6).

$$\lambda \approx \frac{\frac{1}{3}[x_{max}^2(t) + \mu^2(t) + x_{min}^2(t)]}{\left[\frac{1}{3}[x_{max}(t) + \mu(t) + x_{min}(t)]\right]^2} \quad (6)$$

Fig.2 clarifies the relation between maximum, minimum, and mean functions of general load demands and their average values. Replacing the average values of these functions instead of the general load functions is the second approximation that can be used to simplify the calculations of λ ratio. Where the symbol \bar{x} is the average of $x(t)$. Eq. (7) represents the approximate value of λ , based on the range of load limits.

$$\lambda \approx \frac{\frac{1}{3}[\bar{x}_{max-pu}^2 + \bar{\mu}_{pu}^2 + \bar{x}_{min-pu}^2]}{\left[\frac{1}{3}[\bar{x}_{max-pu} + \bar{\mu}_{pu} + \bar{x}_{min-pu}]\right]^2} = \frac{3[\bar{x}_{max-pu}^2 + \bar{x}_{min-pu}^2 + 1]}{[\bar{x}_{max-pu} + \bar{x}_{min-pu} + 1]^2} \quad (7)$$

Now Eq. (1) can be reformulated based on the approximate value of λ as shown in Eq. (8). Table 1 displays three numerical examples and the comparison between exact and approximated values of λ ratio, and profile values of X_G .

$$X_G(t) = \mu_{pu}(t) X_{base}(1 + \kappa_1\sqrt{\lambda - 1}) \quad (8)$$

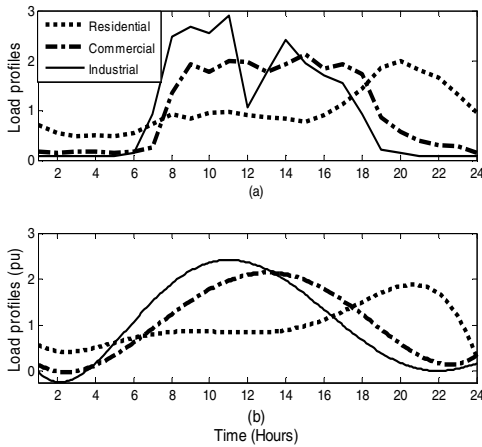


Fig. 1. The per unit mean load profiles based on a) the measurement data [2], b) 5th order polynomial function of curves in (a)

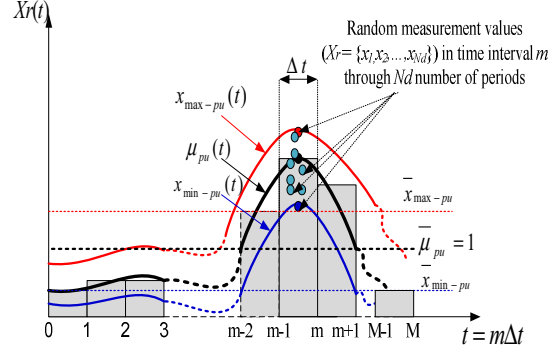


Fig. 2. Maximum, minimum, and mean functions of the general load demand based on random subset of $X_r(m)$.

Table 1: Examples compare between exact and approximate values of load profiles at specific time.

Profile of set X_r at time m	Subset of X_r	λ ratio		The value of profile X_G	
		Exact	Appx	Exact	Appx
{0.9, 1.075, 1.1, 0.95, 0.9, 1.0, 0.97, 0.94, 1.0, 85, 0.96}	{1.1, 0.9, 0.988} Range $\approx \pm 10\%$	1.00512	1.00675	1.080	1.094
{0.8, 1.25, 0.7, 5, 0.7, 1.3, 1.1, 1, 1.2, 1.15, 0.85}	{1.3, 0.7, 1.01} Range $\approx \pm 30\%$	1.04304	1.05962	1.282	1.331
{1.4, 1.3, 0.55, 0.7, 1.49, 1.0, 51, 0.86, 1.39, 0.86}	{1.49, 0.5, 1, 1.006} Range $\approx \pm 50\%$	1.11992	1.15944	1.459	1.528

3. Power Losses

3.1 Exact Power Loss Formulas

The total active and reactive power losses in distribution network with N buses can be calculated by Elgerd's Eq. (9) and Eq. (10), respectively [4]. The equations show the power losses depending on the active and reactive power injections.

$$P_L = P' \alpha P + Q' \alpha Q + Q' \beta P - P' \beta Q \quad (9)$$

$$Q_L = P' \gamma P + Q' \gamma Q + Q' \xi P - P' \xi Q \quad (10)$$

where,

$$P' = [P_1 \ P_2 \ \dots \ P_i \ \dots \ P_N], \quad Q' = [Q_1 \ Q_2 \ \dots \ Q_i \ \dots \ Q_N],$$

$$\alpha = [\alpha_{ij}], \quad \beta = [\beta_{ij}], \quad \gamma = [\gamma_{ij}] \quad \text{and} \quad \xi = [\xi_{ij}].$$

While the size of matrices α , β , γ and ξ is $N \times N$. The coefficients α_{ij} , β_{ij} , γ_{ij} and ξ_{ij} are functions with magnitude and angle of phasor voltage $V_i \angle \delta_i$. Also, α_{ij} and β_{ij} depend on resistive entries r_{ij} of impedance matrix $[Z_{ij}]$, while γ_{ij} and ξ_{ij} depend on reactance entries x_{ij} . P_i and P_j are the active power injections at buses i and j , respectively; Q_i and Q_j are the reactive power injections at buses i and j , respectively. Where the power injection is the difference between the power generations and load demands on the same bus.

3.2 Power Loss Formulas Based on Load Profiles

Fig. 3 displays the change of the power injections on the nodes of a distribution network at each moment of time. The distribution system was assumed perfectly balanced in the three-phase system. The load profiles and the line segments were supposed to be perfectly balanced. The line impedance of each segment was constant with time, but the state variables ($V_i(t)$ and $\delta_i(t)$) were not constant because they depend on the power demands.

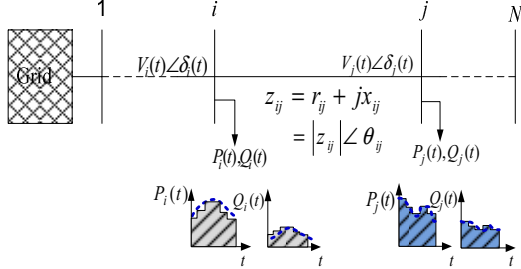


Fig. 3. Load profiles on distribution network

3.2.1 Linearization Principle and Average Integral of Power Profiles

Linearization technique and additive property of integral were used to derive a simple power loss equation based on power profiles. Profiles of active and reactive load demands, power injections and power losses are nonlinear functions with time variable. Also, active and reactive power loss relations are nonlinear with profiles of active and reactive power injections. Fig. 4 shows a general nonlinear function $X(t)$, where $X(t)$ could represent the power profiles of load demands ($P_{Di}(t)$, $Q_{Di}(t)$), power generations ($P_{Gi}(t)$, $Q_{Gi}(t)$), power injections ($P_i(t)$, $Q_i(t)$) at node i , or power losses ($P_{LT}(t)$, $Q_{LT}(t)$).

Through one period T the function $X(t)$ is divided by M regular intervals with length of Δt for each one. The length of the regular interval ($\Delta t = \frac{T}{M}$) depends on the power measurement devices (smart, digital or conventional meters) or accuracy of analysis. $X(t)$ was considered approximate linear function through each interval m . Where $X_m(t)$ is the linear approximation of $X(t)$ over interval m . X_{m_avi} is the average integral value of $X_m(t)$ through time Δt . \bar{X}_m is the weight of the average integral value of $X(t)$ in interval m .

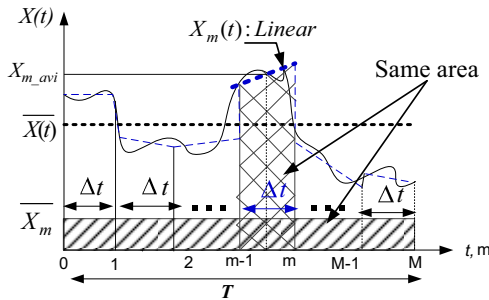


Fig. 4. Linearization principle of general nonlinear function $X(t)$

The following relations have been obtained by using fig. 4.

- The length of the regular interval m is $\Delta t = \frac{T}{M}$ and the endings of this interval are $(m-1)\frac{T}{M}$ and $\frac{T}{M}$, where $m = 1, 2, \dots, M$.
- If $X(t)$ is assumed a linear function along interval m , then the values of X_{m_avi} and \bar{X}_m can be approximated by Eq. (11).

$$\left\{ \begin{array}{l} X_{m_avi} = \frac{1}{\Delta t_m} \int_{(m-1)\frac{T}{M}}^{\frac{T}{M}} X_m(t) dt \cong X\left(\left(m - \frac{1}{2}\right)\frac{T}{M}\right) \\ \bar{X}_m = \frac{1}{T} \int_{(m-1)\frac{T}{M}}^{\frac{T}{M}} X(t) dt \cong \frac{1}{M} X\left(\left(m - \frac{1}{2}\right)\frac{T}{M}\right) \end{array} \right\} \quad (11)$$

- The average integral of $X(t)$ through the period T is approximated in Eq. (12)

$$\overline{X(t)} = \frac{1}{T} \int_0^T X(t) dt = \sum_{m=1}^M \frac{1}{T} \int_{(m-1)\frac{T}{M}}^{\frac{T}{M}} X(t) dt = \sum_{m=1}^M \bar{X}_m \Rightarrow$$

$$\overline{X(t)} \cong \frac{1}{M} \sum_{m=1}^M X_m \cong \frac{1}{M} \sum_{m=1}^M X\left(\left(m - \frac{1}{2}\right)\frac{T}{M}\right) \quad (12)$$

3.2.2 Approximated Power Loss Formulas Based on Load Profiles

According to Eq. (11), over interval m the average integral of power injection profile ($P_i(t)$, $Q_i(t)$, $P_j(t)$ or $Q_j(t)$) can be represented by single point and constant value at time $t_b = \left(m - \frac{1}{2}\right)\frac{T}{M}$. Dependent on approximated values of power injection profiles, the active power loss profile for each interval m was derived in the following equation.

$$\begin{aligned} P_{LTm} &= P_{LT}(t_b) = f\left(P_i(t_b), P_j(t_b), Q_i(t_b), Q_j(t_b)\right) \\ &= f(P_{mi}, P_{mj}, Q_{mi}, Q_{mj}) \end{aligned}$$

$$P_{LTm} = P'_m \alpha_m P_m + Q'_m \alpha_m Q_m + Q'_m \beta_m P_m - P'_m \beta_m Q_m \quad (13)$$

Similarly, the reactive power profile for each interval m can be calculated by using Eq. (13) only by replacing γ_m and ξ_m instead of α_m and β_m , respectively.

According to Eq. (12) and Eq. (13), the average approximation of active and reactive losses based on load profiles are obtained using Eq. (14) and Eq. (15), respectively.

$$P_{LT} = \frac{1}{M} \sum_{m=1}^M \left[P'_m \alpha_m P_m + Q'_m \alpha_m Q_m \right. \\ \left. + Q'_m \beta_m P_m - P'_m \beta_m Q_m \right] \quad (14)$$

$$Q_{LT} = \frac{1}{M} \sum_{m=1}^M \left[P'_m \gamma_m P_m + Q'_m \gamma_m Q_m \right. \\ \left. + Q'_m \xi_m P_m - P'_m \xi_m Q_m \right] \quad (15)$$

where,

$$P'_m = P' \left(\left(m - \frac{1}{2}\right)\frac{T}{M} \right) = [P_{m1} \ P_{m2} \ \dots \ P_{mi} \ \dots \ P_{mN}]$$

$$Q'_m = Q' \left(\left(m - \frac{1}{2}\right)\frac{T}{M} \right) = [Q_{m1} \ Q_{m2} \ \dots \ Q_{mi} \ \dots \ Q_{mN}]$$

$$P_{mi} = P_i \left(\left(m - \frac{1}{2}\right)\frac{T}{M} \right) \text{ and } Q_{mi} = Q_i \left(\left(m - \frac{1}{2}\right)\frac{T}{M} \right)$$

$$\alpha_m = \alpha \left(\left(m - \frac{1}{2}\right)\frac{T}{M} \right) = [\alpha_{mij}], \beta_m = \beta \left(\left(m - \frac{1}{2}\right)\frac{T}{M} \right) = [\beta_{mij}],$$

$$\gamma_m = \gamma \left(\left(m - \frac{1}{2}\right)\frac{T}{M} \right) = [\gamma_{mij}], \xi_m = \xi \left(\left(m - \frac{1}{2}\right)\frac{T}{M} \right) = [\xi_{mij}].$$

While the size of matrices α_m , β_m , γ_m and ξ_m is $N \times N$. The

coefficients α_{mij} , β_{mij} , γ_{mij} and ζ_{mij} are calculated for each interval m .

where,

$$\alpha_{mij} = \alpha_{ij} \left(\left(m - \frac{1}{2} \right) \frac{T}{M} \right) = \frac{r_{mij}}{V_{mi}V_{mj}} \cos(\delta_{mi} - \delta_{mj}) \quad (16a)$$

$$\beta_{mij} = \beta_{ij} \left(\left(m - \frac{1}{2} \right) \frac{T}{M} \right) = \frac{r_{mij}}{V_{mi}V_{mj}} \sin(\delta_{mi} - \delta_{mj}) \quad (16b)$$

$$\gamma_{mij} = \gamma_{ij} \left(\left(m - \frac{1}{2} \right) \frac{T}{M} \right) = \frac{x_{mij}}{V_{mi}V_{mj}} \cos(\delta_{mi} - \delta_{mj}) \quad (16c)$$

$$\zeta_{mij} = \zeta_{ij} \left(\left(m - \frac{1}{2} \right) \frac{T}{M} \right) = \frac{x_{mij}}{V_{mi}V_{mj}} \sin(\delta_{mi} - \delta_{mj}) \quad (16d)$$

where,

$$r_{mij} = r_{ij}, \quad x_{mij} = x_{ij}, \quad \alpha_{mij} = \alpha_{mji}, \quad \beta_{mij} = -\beta_{mji}, \quad \gamma_{mij} = \gamma_{mji}, \quad \zeta_{mij} = -\zeta_{mji}.$$

3.3 Algorithm of Power Losses Estimation

The coefficients α_{mij} , β_{mij} , γ_{mij} and ζ_{mij} are calculated for each interval m . Power flow equations in reference [5] were reformed to include the state variables and power injections at each value of m . where the state variables V_{mi} and δ_{mi} can be calculated by solving the modified power flow equations (Eq. (17) and Eq. (18)). Numerical methods such as Newton-Raphson method is used to solve Eqs. (17) and (18). The power injections P_{mi} and Q_{mi} are calculated by Eq. (11).

$$P_{mi} = \sum_{j=1}^N V_{mi}V_{mj}Y_{mij} \cos(\theta_{mij} - \delta_{mi} + \delta_{mj}) \quad (17)$$

$$Q_{mi} = -\sum_{j=1}^N V_{mi}V_{mj}Y_{mij} \sin(\theta_{mij} - \delta_{mi} + \delta_{mj}) \quad (18)$$

where, Y_{mij} is the entry of admittance matrix at values of m , i and j , Fig. 5 represents an algorithm to find the profiles and the total average of active and reactive power losses.

4. Applied Example

The following applied example was formed through reconfiguring the distribution test system given in [13] and its load profiles were estimated depending on the proposed simple approximation model of load profiles.

18MVA, 12.5kV, 13 bus distribution network, is shown in fig.6. Nodes 1 and 2 are unloaded. Nodes 3, 4, 8, 10, 12, and 13 have only residential loads. Nodes 6 and 9 have only commercial loads. Node 7 is connected only with industrial load. Buses 5 and 11 have a mixed loads. The line and the load data are shown in table 2 and 3, respectively. The base values of this network are 10 MVA and 12.5kV.

Table 2: Line data of a 13 bus radial distribution network.

BUS(n _s)	BUS(n _r)	R (pu)	X(pu)	½ B (pu)
1	2	0.0043	0.0121	0.000035
2	3	0.0060	0.0170	0.000050
3	4	0.0031	0.0090	0.000026
4	5	0.0090	0.0250	0.000073
2	7	0.0030	0.0085	0.000025
4	6	0.0170	0.0221	0.000043
2	8	0.0290	0.0380	0.000074
8	9	0.0230	0.0290	0.000056
9	10	0.0400	0.0500	0.000010
10	11	0.0290	0.0380	0.000074
9	12	0.0370	0.0460	0.000010
12	13	0.0220	0.0270	0.000060

Table 3: Load data of a 13 bus radial distribution network

BUS	P (pu)	Q (pu)	Range (± R%)	Probability (Pr%)	Type of load
1	0	0	-	-	-
2	0	0	-	-	-
3	0.3045	0.1828	5	90	R
4	0.4059	0.2435	30	90	R
5	3.0351	2.2519	15	90	80%R+20%I
6	0.7060	0.4353	10	90	C
7	1.0509	0.7006	20	90	I
8	1.0631	0.6571	10	90	R
9	0.4578	0.2861	13	90	C
10	0.8299	0.5532	15	90	R
11	0.4450	0.2966	50	90	75%R+25%C
12	1.0580	0.6172	20	90	R
13	0.3046	0.1521	5	90	R

R: residential C : Commercial I: Industrial

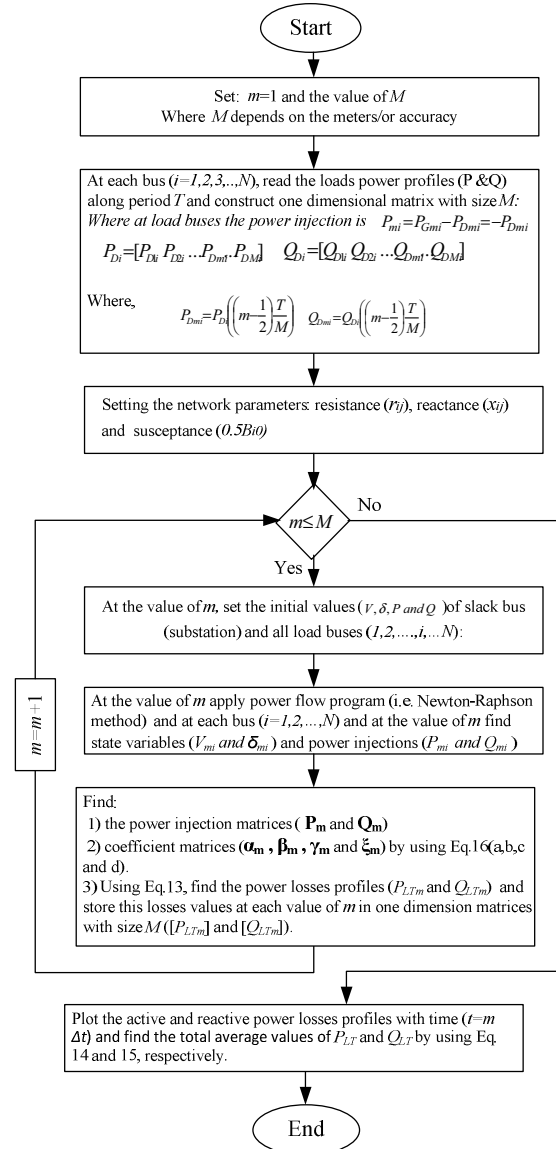


Fig. 5. Algorithm to find active and reactive power losses by using simple formulas.

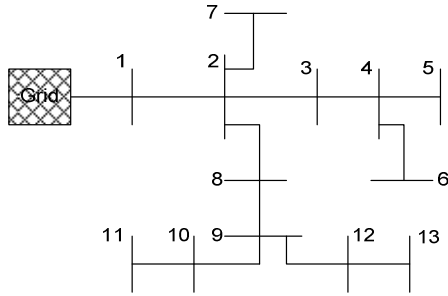


Fig. 6. 13 bus radial distribution network.

5. Simulation Results and Discussion

The proposed model of load profile has been used to estimate the load profiles at each node of 13 bus system. Type of load, maximum and minimum limitations and the probability percent of load distribution were considered besides the load mean value to estimate each load profiles at each node of the system. The consideration details of our case study given in table 3, where the probability percent of all distributed loads are assumed 90%. Fig.7 shows the estimated load profiles of 13 bus system at each node of the network. Table 4 shows the average values of active and reactive load profiles that shown in fig.7. The effects of limitations on load profiles can be noted by comparing the values of P and Q in table 3 and 4. The estimated load profiles increasing by increasing mean of load demand, a range of load limitations, and probability factor κ_1 .

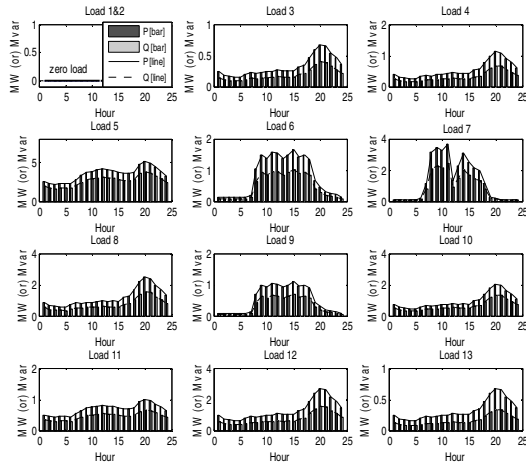


Fig. 7. load profiles of a 13 bus radial distribution network

Table 4: Average values of load profile based proposed model

bus	P proposed model (pu)	Q proposed model (pu)
1	0	0
2	0	0
3	0.3207	0.1925
4	0.5346	0.3207
5	3.5194	2.6112
6	0.7815	0.4818
7	1.2750	0.8500
8	1.1761	0.7270
9	0.5210	0.3256
10	0.9623	0.6415
11	0.6812	0.4541
12	1.2830	0.7484
13	0.3207	0.1601

Algorithm for power loss calculations is achieved using MATLAB. Table 5 includes the results of average power losses of 13 node radial distribution network. The results were estimated by using the load profile and their average and maximum values. Also, fig. 8 shows the active and reactive power loss profiles using the proposed equations of power losses. Table 5 confirms that the results of power loss equations are very close to the results of power losses when they are directly calculated by power flow programs. The power loss values are different when using single values average/maximum demand and power profiles because the power losses of the network are a nonlinear function of load profiles. For example, the active power loss based on the load profiles equals to 0.0407pu, but based on the average values of these load profiles equals to 0.03453pu. The active power loss based on maximum values of the load profiles is equal to 0.1466pu. The calculation of active power loss by using Eq. (14) is more accurate and effective than using Eq. (9) and the same way calculating reactive power loss by using Eq. (15) is more accurate and effective than using Eq. (10).

Fig. 8 shows that the maximum active power loss is equal to 0.11pu, while the maximum reactive loss is equal to 0.19pu. Fig.8 also shows that the maximum active and reactive power losses happened at time 20:00.

Table 5: Power losses in 13 bus radial distribution network

Calculations based on	Active/Reactive	losses formula	Power flow calculation
Maximum load demands	$P(pu)$	0.14660	0.14660
	$Q(pu)$	0.26761	0.26762
Average load demands	$P(pu)$	0.03452	0.03453
	$Q(pu)$	0.06502	0.06561
Load power profiles	$P(pu)$	0.04070	0.04070
	$Q(pu)$	0.07622	0.07615

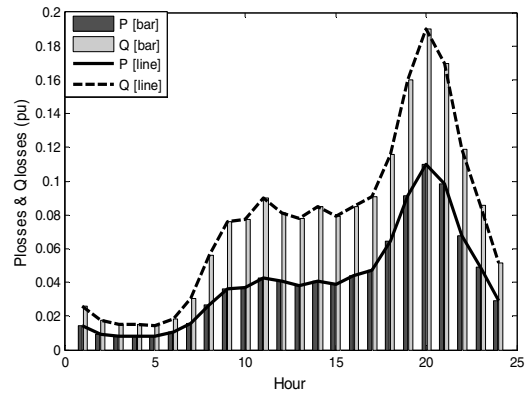


Fig. 8. Active and reactive power losses profiles of 13 bus radial distribution network

Fig.9 shows the real power loss profiles at two type of load models: 1) discrete load model as shown in fig.7. 2) 5th order polynomial functions load model. The average active power loss in discrete mode is 0.04070pu, while in the polynomial model is 0.040707pu.

The proposed load and loss models can be used in the estimation of power losses in networks, economic analysis and to determine the optimal distribution generators that can be inserted in distribution networks.

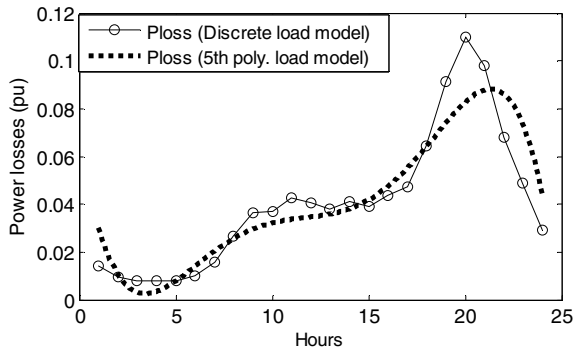


Fig. 9. Comparison between the power losses profiles at discrete load model and 5th order polynomial model of load profiles.

6. Conclusions

This paper presents a simple approximated formulas to estimate the active and reactive power losses in distribution networks based on load profiles. Also, it presents a simple approximated model to estimate load demand profiles. The models of loads and losses are examined on a 13 bus distribution network. The following conclusions can be drawn from this study: 1) type of load and its average value, the range of load changes, and the normal distribution factor κ_1 are the main parameters that can effect the behavior of load profile. 2) The developed loss formulas give to more precise results when compared to the power loss formula based on average/maximum load demand. 3) the proposed formulas of active and reactive power losses is easy to apply, is capable of using discrete load profiles, and has a small error when using an appropriate number of time intervals (M). Because of these features, the simple approximated formulas can be used not only in power loss analysis but also economic dispatch and estimate the optimal setting of distributed generators that can be inserted into distribution networks.

7. References

- [1] D. Nguyen, "Modeling Load Uncertainty in Distribution Network Monitoring", *IEEE Transactions on Power Systems*, vol. 30, no. 5, pp. 2321-2328, 2015.
- [2] J. Jardini, C. Tahan, M. Gouvea, S. Ahn and F. Figueiredo, "Daily load profiles for residential, commercial and industrial low voltage consumers", *IEEE Transactions on Power Delivery*, vol. 15, no. 1, pp. 375-380, 2000.
- [3] E. BOBRIC, G. CARTINA and G. GRIGORAS, "Clustering Techniques in Load Profile Analysis for Distribution Stations", *Advances in Electrical and Computer Engineering*, vol. 9, no. 1, pp. 63-66, 2009.
- [4] D. Hung and N. Mithulananthan, "Loss reduction and loadability enhancement with DG: A dual-index analytical approach", *Applied Energy*, vol. 115, pp. 233-241, 2014.
- [5] Saadat, "Power Systems Analysis", McGraw-Hill, 2000.
- [6] C. Chang, "Reconfiguration and Capacitor Placement for Loss Reduction of Distribution Systems by Ant Colony Search Algorithm", *IEEE Transactions on Power Systems*, vol. 23, no. 4, pp. 1747-1755, 2008.
- [7] O. Elgerd, "Electric Energy Systems Theory: An Introduction", New York: McGraw-Hill, 1971.
- [8] L. Wang and C. Singh, "Environmental/economic power dispatch using a fuzzified multi-objective particle swarm optimization algorithm", *Electric Power Systems Research*, vol. 77, no. 12, pp. 1654-1664, 2007.
- [9] D. Hung, N. Mithulananthan and R. Bansal, "Analytical strategies for renewable distributed generation integration considering energy loss minimization", *Applied Energy*, vol. 105, pp. 75-85, 2013.
- [10] D. Hung, N. Mithulananthan and R. Bansal, "Analytical Expressions for DG Allocation in Primary Distribution Networks", *IEEE Transactions on Energy Conversion*, vol. 25, no. 3, pp. 814-820, 2010.
- [11] D. Hung and N. Mithulananthan, "Multiple Distributed Generator Placement in Primary Distribution Networks for Loss Reduction", *IEEE Transactions on Industrial Electronics*, vol. 60, no. 4, pp. 1700-1708, 2013.
- [12] M. Shahzad, I. Ahmad, W. Gawlik and P. Palensky, "Load Concentration Factor Based Analytical Method for Optimal Placement of Multiple Distribution Generators for Loss Minimization and Voltage Profile Improvement", *Energies*, vol. 9, no. 4, p. 287, 2016.
- [13] W. Grady, M. Samotyj and A. Noyola, "Minimizing network harmonic voltage distortion with an active power line conditioner", *IEEE Transactions on Power Delivery*, vol. 6, no. 4, pp. 1690-1697, 1991.