2nd International Conference on Civil Engineering – Palestine

مؤتمر الهندسة المدنية الدولي الثاني – فلسطين

FINITE ELEMENT METHOD FOR PRECISE GEOID MODELING FOR GNSS POSITIONING IN PALESTINE

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Abstract

Nowadays, the use of modern and precise GNSS technologies for precise positioning is the most common tool for field surveyors. The output coordinate of GNSS are divided into geometric horizontal coordinates (latitude, longitude), or equivalently the mathematically transformed and projected coordinates (Easting, Northing), and the ellipsoidal Normal heights (h). The ellipsoidal heights (h) need to be transformed to match with properties and values of the engineering used physical/orthometric heights (H) that are typically produced using precise leveling. The transformation between both types of heights requires the availability of precise geoid model as a height reference surface (HRS). Typically, the modeling process of a Geoid requires dense networks of precise leveling, gravity and astronomical deflections of vertical. Here, the requirements of the availability of dense leveling and gravity networks for classical geoid modeling methods are overridden by the integration of the limited number of benchmarks and the freely available global geoid models (EGM2008, Eigen05c, EGM96 … etc.) is applied using finite elements method. Conceptually, the modeling area is divided into patches with dimensions (50-70km) to transform the global models' reference datum to fit to the local vertical datum. Afterward, each patch is then divided into smaller elements/meshes with the size of (5x5km) that are represented by 2nd/3rd order polynomial. To apply the least squares solution for the parameters of the polynomials, a combined system observation equations is applied using GNSS/Leveling and additionally Geoid heights and deflections of vertical by the global models for further observations and densification of the solution. To guarantee the continuity and the smoothness of the modeled surface, one least squares solution is applied for all element using zero, first and second-order continuity conditions. Finally, statistical analysis of the least squares solution and test points were used for the validation and accuracy assessment of the model. Residuals less than 3cm were obtained by the solution. Consistently, the accuracy of 1-3cm could be achieved using the test points.

Keywords: GNSS; Precise Levelling; Geoid; Finite Elements Method.

1. Introduction

The global navigation satellite systems (GNSS) has become the most used tool for precise positioning in cadastral, engineering and geodetic surveys. Generally, the GNSS systems provide solutions for point coordinates in specific global systems (WGS84/ITRF) in the form of geocentric coordinates (X,Y,Z) or ellipsoidal geographic longitude, latitude and height (λ, ϕ, h) (Torge, 2001). However, these coordinates have to be integrated in the local plane Easting/Northing coordinates and physical (orthometric heights) (H). Mathematically, the plane coordinates are computed using an official datum transformation of the 3D GNSS coordinates to the locally used geodetic datum before the application of map projection methods. On the contrary, the transformations of the ellipsoidal heights (h) to the orthometric heights (H) is related to the gravity and potential of the earth. Therefore, a global or a local model of the earth gravity/potential is required to compute the differences between both systems defined as the Geoid Undulation (N) in equations (1) (Ghilani & Wolf, 2008), see figure 1.

$$H = h - N \qquad (1)$$

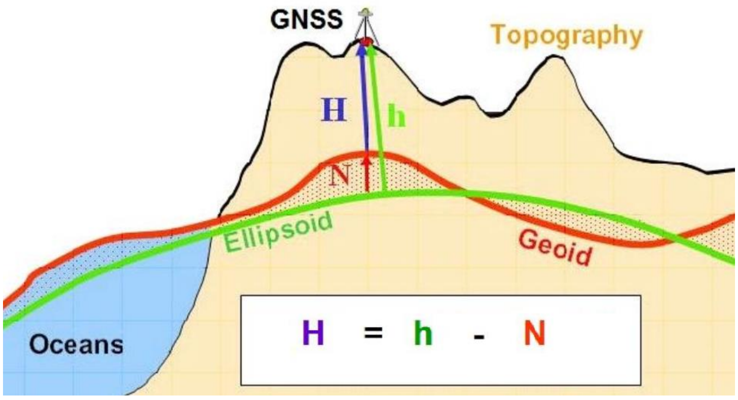


Figure 1. The relation between h, H and N. (Younis, 2013)

The global gravity/potential models are normally used to calculate the true earth gravity and potential and their related geoid undulations using the spherical harmonics model. Generally, these models are calculated using global terrestrial gravity, satellite gravity, satellite altimetry, and GNSS/Leveling measurements. The global resolution of the global models are defined by the maximum degree (m) and order (n) in equation (2). The spherical harmonic coefficients are directly used to calculate the gravitational potential (V) or its derivatives, which are caused by earth masses at a given point (Hofmann-Wellenhof & Moritz, 2006). Fortunately, most of the global models can be used freely with available online tools or the direct use of parameters (GFZ-Potsdam, 2019).

$$V(r,\bar{\phi},\lambda)=\frac{GM}{r}+\frac{GM}{a}\sum_{n=2}^{n-\max}\left(\frac{a}{r}\right)^{n+1}\sum_{m=0}^{n}(\bar{C}_{nm}\cos m\lambda+\bar{S}_{nm}\sin m\lambda)\,\bar{P}_{nm}(\sin\bar{\phi}) \tag{2}$$

Where, $(\lambda,\bar{\phi},r)$ are spherical longitude, latitude and the radial distance from the centre of the earth. $\bar{C}_{nm}$ and $\bar{S}_{nm}$ are the normalized spherical harmonics which are the defining parameters of a global model (Younis, 2015). $\bar{P}_{nm}(\sin\bar{\phi})$ is the fully normalized associated Legendre function. $\bar{P}_{nm}(\sin\bar{\phi})$ can be calculated by the recursive formulae (3a) to (3e), with the abbreviations $t=\sin\bar{\phi}$ and $u=\cos\bar{\phi}$ starting with $\bar{P}_{0,0}=1$, $\bar{P}_{1,0}=\sqrt{3}t$, $\bar{P}_{1,1}=\sqrt{3}u$ (Holmes & Featherstone, 2002), as follows:

$$\bar{P}_{n,m}=a_{nm}t\bar{P}_{n-1,m}-b_{nm}\bar{P}_{n-2,m} \quad \text{, if } (n\neq m) \tag{3a}$$

$$\bar{P}_{m,m}=u\sqrt{\frac{2m+1}{2m}}\bar{P}_{m-1,m-1} \quad \text{, if } (n=m) \tag{3b}$$

$$a_{nm}=\sqrt{\frac{(2n-1)(2n+1)}{(n-m)(n+m)}} \tag{3c}$$

$$b_{nm}=\sqrt{\frac{(2n+1)(n+m-1)(n-m-1)}{(n-m)(n+m)(2n-3)}} \tag{3d}$$

The geoid undulation (N) is then calculated as a function of the disturbing potential (T), which equals the difference between the true earth gravitational potential (V) and theoretical gravitational potential (V') on the surface of the geoid divided by the normal gravity (γ_Q), see equation (4). Here, the normal potential (V') and the normal gravity (γ_Q) on the surface of the ellipsoid are directly calculated using the ellipsoidal physical defining parameters and constants of the GRS80 or WGS84 ellipsoid (Torge, 2001).

$$N=\frac{T}{\gamma_Q} \tag{4}$$

Unfortunately, the spatial resolution and the accuracy of the global models are not sufficient to be directly used in many engineering applications or precise GNSS-Real time application where high levels of accuracy are required. For example, EGM2008 and EIGEN-06C4 have nowadays the highest spatial resolution. Their evaluated accuracy in the USA is ±8cm and approximately ±12cm in Europe and Canada. But their global accuracy is about ±24cm

(GFZ-Potsdam, 2019). Table 1 shows a group of common global geoid Models and their global accuracy as tested by GNSS/leveling control points (GFZ-Potsdam, 2019).

Table 1. A group of the common global models and their information.

Model	Max Degree	Data Used in Modeling	Global Accuracy (m)
EGM2008	2190	A, G, S(Grace)	0.24
Eigen06c4	2190	A, G, S(Goce), S(Grace), S(Lageos)	0.24
Eigen05c	360	A, G, S(Grace), S(Lageos)	0.34
EGM96	360	A, G, PGM55	0.43
S is for satellite (e.g., GRACE, GOCE, LAGEOS), A is for altimetry, and G for ground data (e.g., terrestrial, shipborne and airborne measurements).			

As a result of the poor accuracy of the global geoid models direct use, local geoid models are required for higher levels of accuracy to be suitable for the height accuracy levels of GNSS and precise leveling (Torge & Müller, 2012). Generally, the computations of the local geoid models require dense networks of gravity (g)/ gravity anomaly (Δg) or gravity disturbance (δg) points, deflections of vertical (η, ξ) by astronomical observations or zenith camera in addition to the dense network of precise leveling points accurately measured by means of precise GNSS techniques (Farahani, et al., 2017). Some of these methods use global models as basic input and introduce the local variations by using local reference leveling and gravimetric points in a specific way related to the selected method, as in the case of Stokes formula /Remove restore method. Furthermore, the spherical cap harmonics (SCH) methods and their modifications can support local solutions employing global models as initial computations of the modeling parameters (Younis, et al., 2013). Differently, the least squares collocation methods and radial basis functions methods use integrated solution using different types of observation using specific mapping function and its derivations for the observed values with respect to the geoid undulations (N) (Sumaruk, et al., 2019). In table 2, some of the common methods of the local geoid models are listed.

Table 2. List of common classical methods of Geoid modeling with their required input.

Method	Basic function/method	Required observations
Stokes formula /Remove restore method	$N = \frac{a}{4\pi\gamma_m}\iint_\sigma S(\psi)\Delta g \; d\sigma$ $S(\psi) = \sum_{n=2}^{\infty}\frac{2n+1}{n-1}P_n(\psi)$	•Terrestrial gravity anomalies •Global models gravity anomalies •Digital Terrain Model
Least Square Collocation	$T(P) = f(P) = \sum_{k=1}^{q} b_k\varphi_k$ $B_{ik} = L_i(\varphi_k)$ $\varphi_k = L_k^Q K(P,Q) = C_{Pk}$ $K(P,Q) = \sum_{n=2}^{\infty}\sigma_n^2\left(\frac{R^2}{r_P r_Q}\right)^{n+1} P_n(\psi_{PQ})$	•Terrestrial gravity anomalies •Terrestrial gravity disturbances •Deflections of vertical •Height fitting points
Spherical Radial basis functions (SRBF)	$\Psi_j(i,j) = \sum_{l=0}^{\infty}\psi_l\left(\frac{R}{r_i}\right)^{l+1} P_l(\theta_{ij})$	•Terrestrial gravity anomalies •Terrestrial gravity disturbances •Deflections of vertical •Height fitting points
Spherical Cap Harmonics (SCH) (combined data)	$V = \frac{GM}{r}\sum_{k=0}^{k\max}\left(\frac{R}{r}\right)^{n(k)}\sum_{m=0}^{k}\left(C'_{nm}\cos m\alpha + S'_{nm}\sin m\alpha\right)\overline{P}_{n(k),m}(\cos\theta)$	
Adjusted Spherical Cap Harmonics (ASCH) (combined data)	$V = \frac{GM}{r}\sum_{k=0}^{k\max}\left(\frac{R}{r}\right)^{n(k)+1}\sum_{m=0}^{k}\left(C'_{nm}\cos m\alpha + S'_{nm}\sin m\alpha\right)\overline{P}_{n,m}(\cos\vartheta)$	
Kriging GIS Interpolation	$\hat{Z}(s_0) = \sum_{1}^{n}\lambda_i Z(s_i)$	•Height fitting points

In many places, like the situation in Palestine, there are no precise leveling, gravimetric, and deflections of vertical networks available to be used for geoid modeling using the classical geoid modeling approaches. Furthermore, the precise leveling network was subjected to dramatic damage in the past decades without continuous repair and expansions. Nowadays, only 10s of the points are still available to be measured by GNSS methods. Also, these point points were subjected to earth kinematics caused local crustal deformations and tectonic plates movements (Younis, 2019). For GNSS height measurements, global geoid models like EGM2008 or EGM96 are used in the post-processing of the observations, which can be practically used to only set a single arbitrary reference height point for a small area project (Younis, 2018). Indeed, the land surveyors in Palestine use ILLUM12 geoid model for real-time kinematic (RTK) methods, which is almost installed on all GNSS geodetic and surveying receivers/Data Collectors. Principally, ILLUM12 was built using GIS kriging interpolation using dense precise leveling points in the occupied Palestinian areas in the year 1948. But there was not a sufficient number of reference points in the West Bank areas leading to poor accuracy up to 20cm in the large extent projects (Steiberg & Tuchin, 2009). This means that an alternative method is required to create a geoid model as a height reference system.

Jäger (1999) introduced the use of Finite Element Method (FEM) to define a Height Reference Surface (HRS) representing the Geoid/Quasi-Geoid undulations in a project called (Digital Finite Elements Height Reference Surface – DFHRS). The advantage of this concept is that it is possible to be applied in the areas with minimal number of leveling or GNSS field observations by the densification of global geoid models with high degree and order observations (Geoid Undulations and deflections of vertical) to define the surface and its change in directions (DFHRS, 2000). Conceptually, the area to be modeled is divided into sub-areas (patches) to fit the global model with the local datum by applying datum transformation. Also, each patch is divided into smaller elements (meshes) to represent the geoid undulations using polynomials. The accuracy is usually obtained in the range of (0.01-0.1) m depending on the model design and accuracy and density of field observations. To guarantee the continuity and smoothness of the finite elements surface, different continuity conditions and levels are introduced in the least squares solutions (Janpaule, et al., 2013).

2. Finite Elements Method for Geoid Modeling

The basic principle of FEM to represent the local geoid of an area of interest is to divide the area into subareas (patches) (Jäger, 1999). The patches are used to fit the global geoid models (GGM) observations to the local datum. Multiple patches are normally used to override the limited accuracy of the GGMs and local errors and variations in local datum of the precise leveling networks (Jäger, et al., 2012). The fitting of GGM to the local datum can be easily applied employing Molodensky datum transformation approach. The reason to use the Molodensky approach is that it applies 3/7 datum transformation parameters directly for the geographic coordinates and ellipsoidal height (λ,ϕ,h) without the need to calculate the geocentric coordinates (X,Y,Z) that are the basic input in the similarity/Helmert transformation methods. Another advantage of Molodensky transformations is the possibility to separate the height components from the horizontal components during the solution to find the transformation parameters, which means that only heights can be used to calculate the 3D transformation parameters (Lu, et al., 2014). The formula of datum fitting using Molodensky for the height components is:

$$dN = (N_e(1 - e^2 sin\phi))m - (N_e e^2 sin\phi cos\phi sin\lambda)r_x + (N_e e^2 sin\phi cos\phi cos\lambda)r_y$$
$$-(cos\phi cos\lambda)T_X + (cos\phi sin\lambda)T_Y + (sin\phi)T_Y \quad (5)$$

$$N_{GGM} = N_{Local} + dN \quad (6)$$

Where, dN is the correction to be added to geoid undulation calculated from GGM as a fitting value of the global model value to the local datum as explained in equation (6). Also e^2 and N_e are the ellipsoidal 2nd eccentricity squared and the radii of curvature of the reference ellipsoid at given position, m is the scale factor, while the angles r_x and r_Y are the angular rotations around the ellipsoidal X and Y-axis in the geocentric coordinate system. TX, TY and TZ are the translations of origin. N_{Local} is the local geoid undulation to be integrated in the GNSS system as explained in equation (1) and figure 1, While, N_{GGM} is the geoid undulation calculated using GGM, like EGM2008, EIGEN06-C4 … etc., using equations (2) and (4). In the next step, the patches are divided into smaller square areas (meshes), which are used to represent the local HRS of geoid undulations (N_{FEM}) using two dimensional 2nd or 3rd order polynomials in equation (7). In figure 2, the principle of patching and meshing is explained. While the patches are drawn with thick lines, the meshes are drawn with thin lines.

$$N_{FEM} = a_0 + a_1 x + a_2 y + a_3 x^2 + a_4 y^2 + a_5 xy + a_6 x^3 + a_7 y^3 + a_8 yx^2 + a_9 xy^2 \quad (7)$$

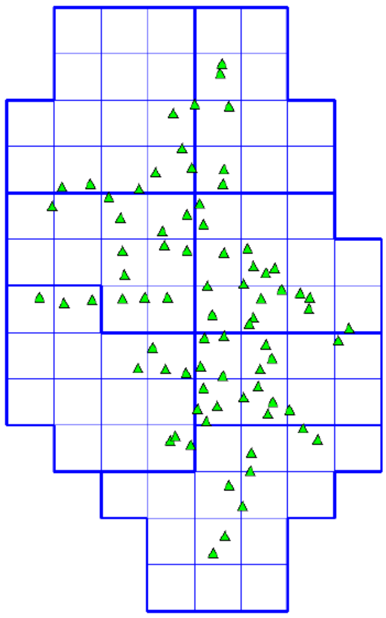


Figure 2. The concept of patches and meshes. (DFHRS, 2000)

To guarantee the continuity and smoothness of the modeled surface, three levels of continuity conditions are applied in the least squares solutions. The first level is c_0 condition; a point at the common border between two meshes should have the same value of N, if it is calculated from both meshes. The second level is the c_1 condition represented by the first derivative. Here, the slope of the surface at the border should be the same for both neighboring meshes. Finally, the third level is c_2 that keeps the deflection of the surface of both neighboring meshes the same at the common border, which can be represented using the second derivatives of the polynomial (Schneid, 2006). The continuity conditions c0, c1, and c2 at the common border between two neighboring meshes i and j are mathematically introduced in equations (8a – 8c), respectively.

$$N_{FEM_i} = N_{FEM_j} \tag{8a}$$

$$\frac{\partial N_{FEM_i}}{\partial x} = \frac{\partial N_{FEM_j}}{\partial x} \quad and \quad \frac{\partial N_{FEM_i}}{\partial y} = \frac{\partial N_{FEM_j}}{\partial y} \tag{8b}$$

$$\frac{\partial^2 N_{FEM_i}}{\partial x^2} = \frac{\partial^2 N_{FEM_j}}{\partial x^2} \quad and \quad \frac{\partial^2 N_{FEM_i}}{\partial y^2} = \frac{\partial N_{FEM_j}}{\partial y^2} \tag{8c}$$

To create a height reference surface (HRS) using the finite element method (FEM), it is required to calculate the polynomial coefficients in equation (7) for all meshes. As a result, ten unknowns $(a_0,\ a_1,\ a_2, \ldots,\ a_9)$ per mesh will be available. Additionally, six extra unknowns per patch $(m,\ r_X,\ r_Y,\ T_X,\ T_Y,\ T_Z)$ need to be calculated during the least squares adjustment to enable the use of fitted GGM observations in the adjustments. As a rule of thumb, a patch size of 50x50km and mesh size of 5x5 km are identical to calculate a cm-accuracy geoid model with consideration of accurate and will distributed height fitting points. For example, a rectangular area with the extent of 100x200km can be divided into 8 patches and 800 meshes. Whereas, the total number of unknowns to be calculated by the least squares solutions is about 8048 parameters (Janpaule, et al., 2013).

The basic type of observations in the least squares solutions to calculate the coefficients of the meshes is the height fitting points, where the orthometric heights (H) is measured by precise leveling and ellipsoidal height (h) is measured by GNSS. Referring to equation (7). The height fitting point observation equation located at the mesh (i) with an expected residual v reads (Jäger, 1999):

$$h + v = H + N_{FEM_i} \tag{9}$$

In many countries around the world, networks of points with observed astronomic longitude and latitude (Λ, Φ) by classical astronomic measurements or by using modern star observations by zenith cameras are available to be used in local modeling (Kühtreiber, 1999). The measurement of ellipsoidal longitude and latitude (λ, ϕ) using

GNSS techniques for the astronomical points enables the calculation of the deflections of vertical in east-west direction (η) and north-south direction(ζ). In a plane coordinate system, the observation equations of the deflections of vertical components of a point in mesh (i) can be introduced in the form of system of equations (10a) and (10b) (Jäger, 2006):

$$\eta + v = -\frac{\partial N_{FEM_i}}{\partial x} \quad (10a)$$

$$\zeta + v = -\frac{\partial N_{FEM_i}}{\partial x} \quad (10b)$$

The number of observations needed to calculate a large number of unknowns is high compared to the number of available field observations. In many countries, like the situation in Palestine, there are not enough available dense networks of reference precise leveling points to satisfy the requirements to solve for the unknowns. On the contrary, a limited number of precise leveling points are still existing in the field with the ability to be measured by GNSS. Thus, GGMs geoid undulations (N) and deflections of vertical(η, ζ), which can be interpolated directly from the model, are and implemented in FEM principle as extra observations to override this problem. Here, 25 points are to be used per mesh leading to 75 observations. Additionally, 5 common points will be shared at the border between neighboring meshes to apply the continuity conditions in equations (8a – 8c). Using the functional model in equations (5), (6) and (10a-10b), the direct observation equations of the GGM geoid undulations (N_{GGM}) and their deflections of vertical$((\eta, \zeta)_{GGM})$ in mesh (i), with consideration of datum component dN related to the patch (j), are given in equations (11a – 11c) (Schneid, 2006).

$$N_{GGM} + v = N_{FEM_i} + dN_j \quad (11a)$$

$$\eta_{GGM} + v = -\frac{\partial N_{FEM_i}}{\partial x} + \frac{\partial (dN)_j}{\partial x} \quad (11b)$$

$$\zeta_{GGM} + v = -\frac{\partial N_{FEM_i}}{\partial y} + \frac{\partial (dN)_j}{\partial y} \quad (11c)$$

In this FEM principle, the height fitting points define the datum for the computed geoid at the containing patch (j). On the other hand, the deflections of vertical introduce the change directions up/down of the geoid surface at the given position. The GGM fitted geoid undulations introduce a dense number of observations to define the geoid reference surface at each mesh. Besides, the computed deflections of vertical of the 25 GGM points guarantee extra observation to control the directions of change of the geoid surface in both x and y directions. The common GGM geoid undulation points at the borders between the neighboring meshes with their deflections guarantee the smoothness of the surface by applying the extra continuity conditions, as explained in equations (8a -8c) (Schneid, 2006).

3. Data and Results

Due to the limited possibility of field measurements, the study area of this research is limited to the West Bank of Palestine. Historically, the precise leveling network of Palestine was established during the British mandate parallel to the fieldwork of the horizontal triangulation network during the period (1923-1947) (The Palestine Exploration Fund, 2019). Although the network had a limited number of points concentrated in the water sources areas (Gavish, 2010), the last densification/rehabilitation of the network in the West Bank was done at the end of 1970s to the middle of 1980s. Afterward, the network was subjected to dramatic destruction intentionally or due to urban expansions and unavailability of densification and rehabilitation plans by the Palestinian official departments. As a result, only a few number of precise leveling Bench Marks are still existing.As GNSS is becoming more frequently used of land and engineering surveying, a Geoid model became an important requirement to transform the measure ellipsoidal heights (h) to local orthometric heights (H) rather than the need for reference Bench Marks during the real-time 3D positioning techniques. The Geoid model in GNSS devices still has to be compatible with these Bench Marks to allow the integration with the differential/engineering leveling techniques. Therefore, the Bench Marks have to be the basic input data for the modeling process of a local Geoid.

Field observations by GNSS techniques of precise Bench Marks were achieved for approximately 30 points distributed all around the West Bank. To achieve the requirements of control accuracy, the points were observed using the static mode for 3 hours or RTK-Network control method for 2 minutes as control averaging method

(RICS, 2010). Additionally, 9 ITRF points were used from the surrounding area inside the Palestinian occupied lands (1948). For these points, information about precise orthometric height and the GNSS raw observations data were available online by the IGS/ITRF services (SOPAC, 2019). The differences between the measured geoid undulations (N) according to equation (1) were compared to the common geoid models, which are often applied in RTK-GNSS receivers, GNSS-post processing software, and GIS software in Palestine. In table 3, the differences (ΔN) between the observed undulation and the computed undulations from Ilum12, EGM2008 and Eigen05c are given for a group of reference GNSS/Leveling points for an initial evaluation of these models in Palestine.

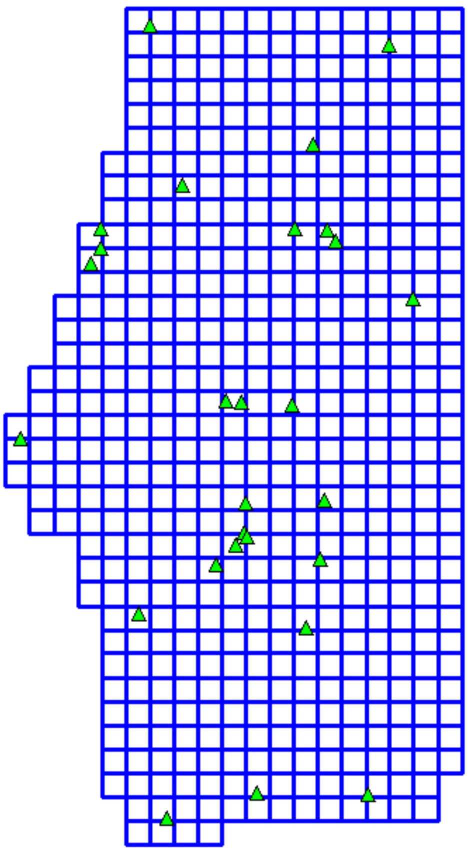
Table 3. The evaluation of the geoid models in Palestine.

id	h	H	N=h-H	ΔN_{ILUM12}	$\Delta N_{EGM2008}$	$\Delta N_{EIGEN05c}$
2	-0.253	-18.421	18.168	0.205	-0.378	0.461
3	-206.304	-225.055	18.751	0.549	-0.079	0.422
4	-253.984	-273.293	19.309	0.547	0.093	0.359
5	-230.864	-250.653	19.789	-0.819	-1.295	-1.430
6	-181.11	-201.162	20.052	-0.619	-1.168	-1.193
7	771.443	752.234	19.209	-0.012	-0.292	0.709
8	966.502	946.8	19.702	-0.002	-0.265	0.720
12	131.16	110.258	20.902	0.231	-0.688	-0.508
19	-223.522	-243.062	19.54	0.219	-0.629	-1.368
101	920.753	901.108	19.645	0.097	-0.049	0.946
103	899.048	879.507	19.541	0.129	-0.082	0.930
104	914.223	894.535	19.688	0.187	0.053	1.036
106	409.25	391.575	17.675	-0.004	-0.433	0.612
107	37.703	19.249	18.454	-0.005	-0.625	-0.701
108	22.787	4.233	18.554	-0.022	-0.667	-0.687
109	747.558	727.487	20.071	0.000	-0.367	0.165
110	658.778	638.821	19.957	0.015	-0.422	0.079
111	67.19	47.6	19.59	-0.100	-0.785	-0.715
112	131.16	110.258	20.902	0.231	-0.688	-0.508
		Min	17.675	-0.819	-1.295	-1.430
		Max	20.902	0.549	0.093	1.036
		RMSE		0.324	0.381	0.819

In table 3, the anomalies in the geoid undulations vary in magnitude due to different reasons. In the case of Ilum12, only few number of the points had smaller difference less than 1cm, because these points were themselves or near points used in the GIS interpolation during the modeling process of the Ilum12, while higher anomalies are related to reference points that are located in areas under the Palestinian administration far away from the Ilum12 reference points . On the other hand, the global models had anomalies depending on the density of data in this area used during their modeling and their accuracies in addition to spatial resolution of the model represented by the maximum degree and order of the model. Also, the global models have different coordinate systems and datum compared to the local datum of Palestine. Therefore, the introduction of datum transformation parameters can reduce the error and increase the accuracy assumed for the model. For example, EGM2008 with maximum degree of 2190 can achieve the accuracy of 10cm by introducing the datum parameters after the application of the local datum fitting parameters, while Eigen05c has results worse than 15cm (Younis, 2018).

The control points were used as the primary input of the finite element method for modeling for the Geoid in the West Bank of Palestine. According to the distribution of points, the study area was divided into 6 patches varying in dimensions in the range of 40-65km. In a second step, the patches were divided into meshes with dimensions of 5x5km represented with 3rd order polynomial, as explained in equation (7). For extra observations, geoid undulations and deflections of vertical were obtained from the global geoid models to get more reliable modeling for changes in values and directions. Finally, continuity conditions were used between meshes and patches using the levels of continuity in equations (8a-8c).

The distribution of the control points, meshing process and the summary of observations used in the least squares solution are given in figure 3. To enable the solution of this large number of observations and unknowns, the normal equations matrix is directly calculated without the use of the complete design matrix. Additionally, block matrix Cholesky decomposition solution algorithm is applied to avoid the process of normal equations matrix inversion, which gives the ability to deal with the problems of huge memory usage and time consumptions during least squares solution of the modeling process (Younis, 2015).



Number of patches	6
Number of meshes	541
Control points	28
GGM Geoid undulations	13525
GGM deflections of vertical	27050
Continuity conditions	13241
Polynomial parameters	5410

Figure 3. The distribution of the control points and 5x5km meshing of the study area.

The calculations of the geoid model using the finite element method has been applied using densified observations from EGM2008 and EIGEN05c. The final output for the model is a database file that stores the 10 parameters of the polynomial for all meshes. To validate the results of the least squares solution, a statistical evaluation of the residuals of the height fitting points was applied. additionally, a comparison for external height points was done to introduce the reliability and usability of the model through the differences between observed and computed geoid undulation values. The results using EGM2008 densification (maximum degree and order of 2190) have shown accurate values much better than the use of EIGEN05c(maximum degree and order of 360). This is due to the larger difference in spatial resolution and the accuracy of the models. The Summary of the results of the adjustment and validation process is shown in table 4.

Table 4. Summary of geoid modeling adjustment by FEM.

Densification model	**Value (m)**	**Control points**	**test points**
	Min residual	-0.019	-0.034
EGM2008	Max residual	0.026	0.043
	RMSE	0.013	0.016
	Min residual	-0.083	-0.105
EIGEN05c	Max residual	0.123	0.145
	RMSE	0.092	0.117

The output is finally stored as polynomial parameters for each mesh in a database, which can be accessed by a local software in personal computers or GNSS-RTK service providers (Jäger, et al., 2010). Alternatively, the geoid model is gridded utilizing GIS interpolation methods. Finally, the geoid model is restored in grid formats supported by GNSS receivers data collectors formats like Trimble, Leica, Stonex, Topcon … etc.

4. Conclusions

The differences between field-measured geoid undulation, which are defined by equation (1), and the values obtained from the different freely available global geoid models and Ilum12 were used to evaluate the reliability of using these models for the transformation between the GNSS observed ellipsoidal height (h) and the orthometric height (H) in Palestine as explained by the results in table 3. It was also clearly found that the currently used geoid models in Palestine (Ilum12, EGM2008, Eigen … etc.) do not fit the requirements and the accuracy levels of GNSS measurement techniques, precise engineering leveling, and GIS applications. Thus, The finite elements method (FEM) approach, which is introduced in section 2, was found to be more proper approach for the modeling of a height reference surface/Geoid in Palestine due to the possibility of its application in the areas with limited number of precise leveling points and without the requirement of dense gravimetric networks, which are typically needed for classical geoid modeling approaches introduced in table 2. In the design of FEM for accurate geoid modeling, the basic considerations are the number of accurate GNSS/leveling control points, their distribution, the size of the patch, the size of the mesh, and the selection of the proper geoid model for densification of observations. For a 1-3 cm level of accuracy, it was found that the patch size of 50x50km containing mesh with dimensions of 5x5km are proper design elements. Where, each patch should contain at least 3 control points for datum transformation of global geoid fitting, as explained in equation (5). As well, EGM2008 could provide the best results compared to other models due to its 10 km spatial resolution that provides good information about the global values and change in the geoid undulations. Finally, the accuracy of 1-3cm could be achieved for more than 90% of the control and test points. In the end, it must be considered that year after year the precise leveling Bench Marks are being dramatically demolished in the same way it is happening for the horizontal triangulation network. So, the official departments are highly recommended to lead projects for rehabilitation and densification of the precise leveling network of Palestine. As a result, the integration between engineering leveling will be more easily achieved over long distances in a unified system for GNSS and land surveying techniques.

Acknowledgment

For their help and efforts in the fieldwork and data collection, I would like to thank my colleague Eng. Ahmad Herbawi and my students from Palestine Polytechnic University: Mohammad Assfor, Mohanad Turman , Wafa Zamaara and Hussien Abu Saada. Finally, I would like to thank Eng. Ihab Shaheen and GROMA-Geosystems for their support by providing GNSS-receivers and fieldwork help.

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