FINITE ELEMENT METHOD FOR PRECISE GEOID MODELING FOR GNSS POSITIONING IN PALESTINE

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FINITE ELEMENT METHOD FOR PRECISE GEOID MODELING FOR GNSS POSITIONING IN PALESTINE

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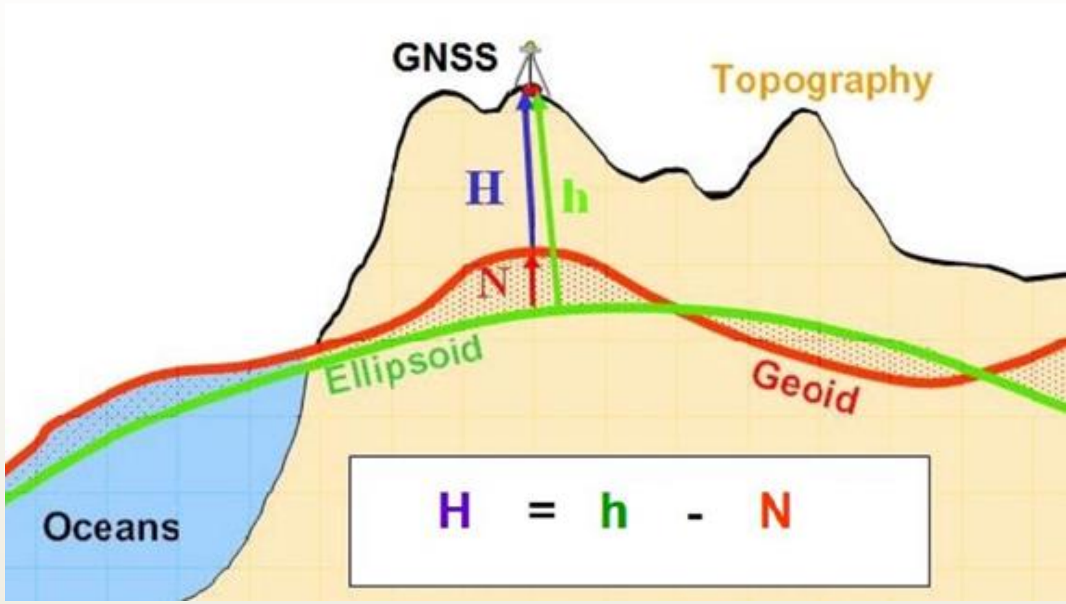
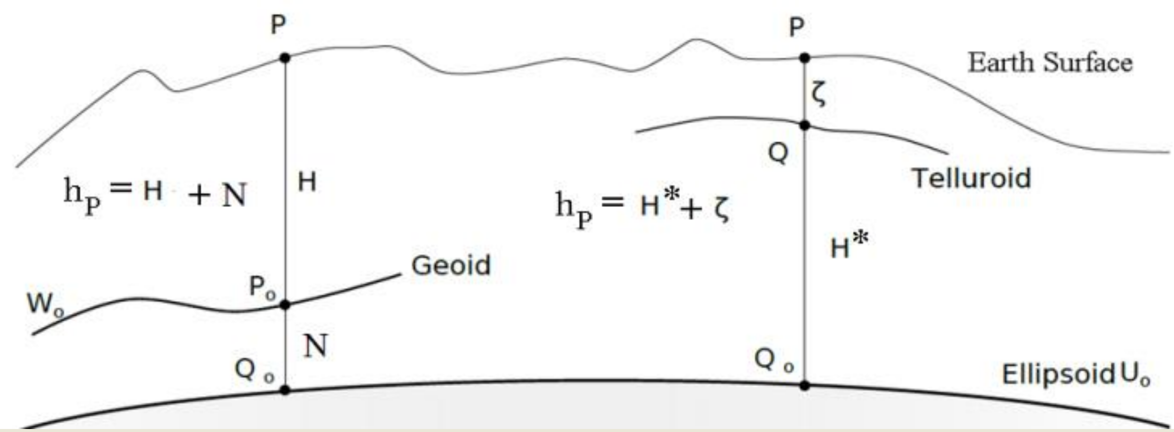
Precise Leveling Network of Palestine

- The precise leveling network was built from the end of 1920s to the middle of 1930s.
- The network was built by means of precise leveling with a start gauge point at Gaza to define the zero level/mean sea level (MSL).
- The loops were used to connect main cities with main interest of water sources leveling.
- level instruments without the use of gravity data.
- The position of these points was not documented by coordinates
- Last densification 1970s-1980s

Introduction : Ellipsoidal vs Orthometric

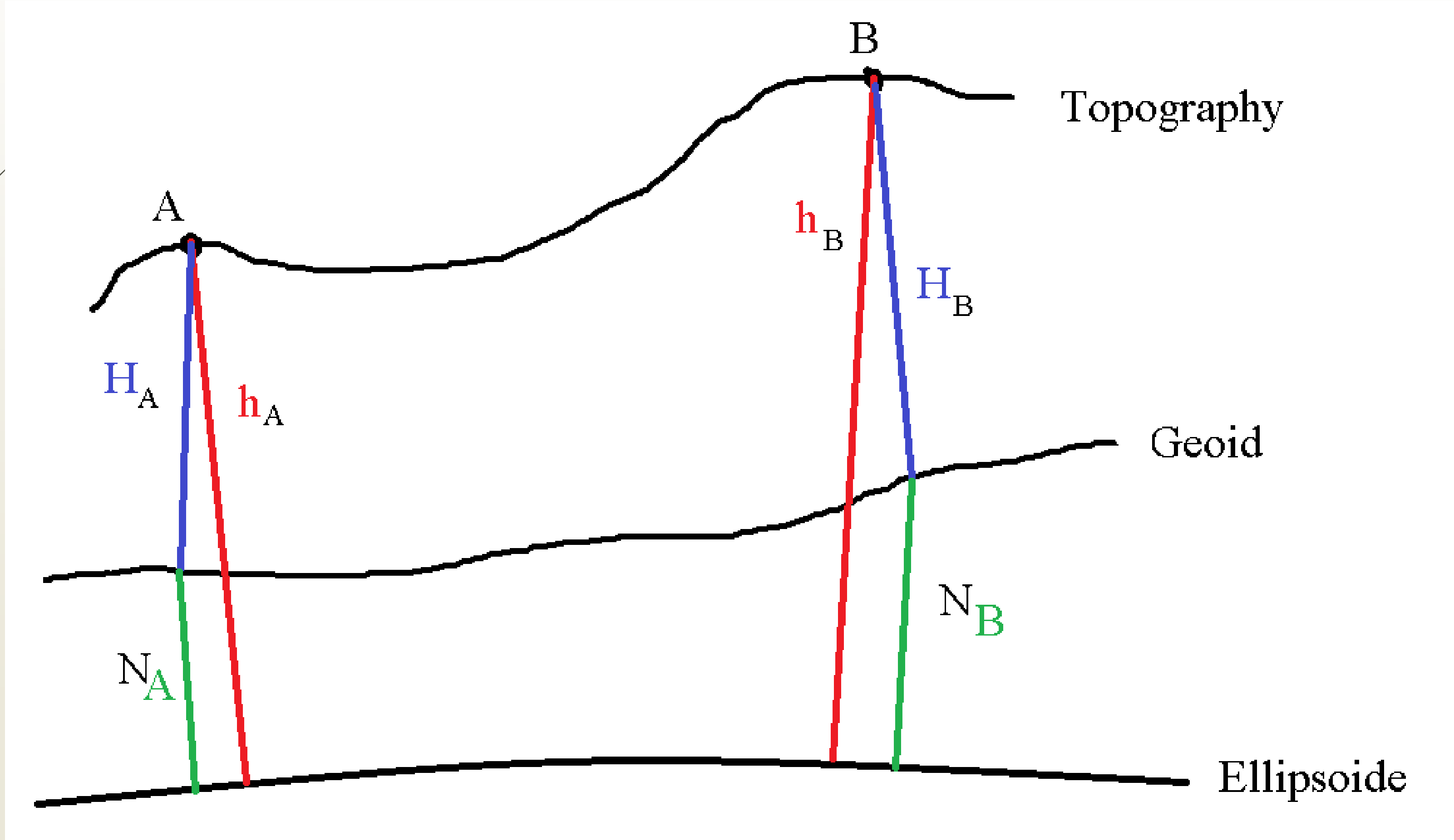
- Ellipsoidal height (h): geometric normal to the ellipsoid surface
- - h is normally measured by GNSS
- Orthometric height (H): define the direction of the plumb line referenced to reference equipotential surface (geoid)≈MSL
- - H is classically measured by precise Leveling
- Engineering heights are measured using orthometric heights
- Integration surface (Geoid) is required
- Modern alternative is the Normal heights (true geometry). Reference surface named QuasiGeoid.

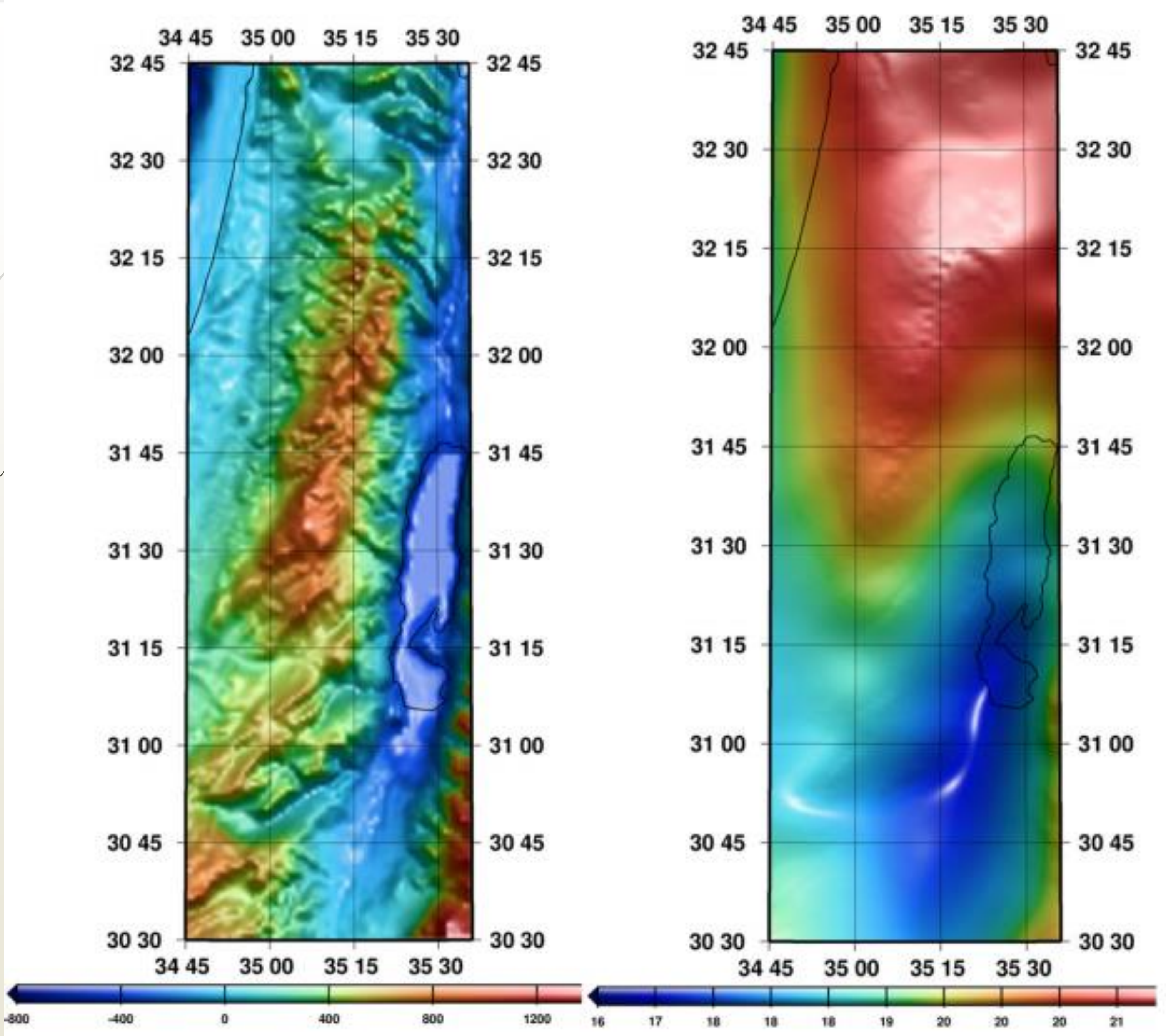




Height difference : ΔH = Δh ???

- $(h_B - N_B) - (h_A - N_A) = H_B - H_A$
- A geoid surface/model is required

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21

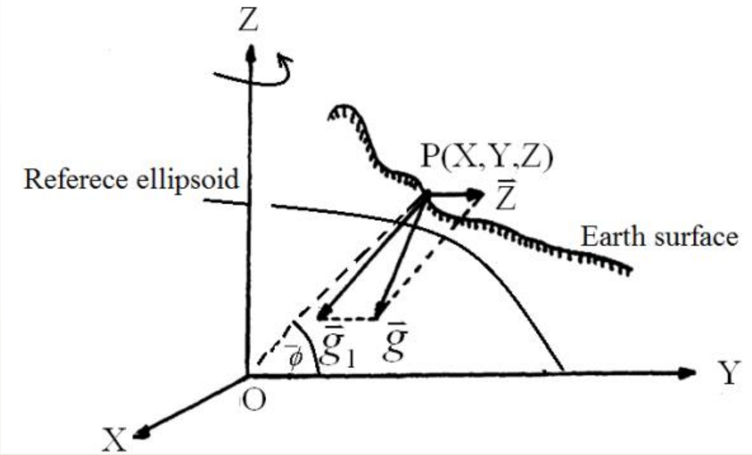
Basic Relations

Global Geoid/Potential Model

- True Earth potential W by earth gravity (g) : gravitational (V) + centrifugal (Ω)

$$\Omega = 0.5\omega^2 r^2 \cos\bar{\phi} = \frac{1}{2}\omega^2(X^2+Y^2)$$

$$V(r,\bar{\phi},\lambda) = \frac{GM}{r} + \frac{GM}{a}\sum_{n=2}^{n-\max}\left(\frac{a}{r}\right)^{n+1}\sum_{m=0}^{n}(\bar{C}_{nm}\cos m\lambda + \bar{S}_{nm}\sin m\lambda)\bar{P}_{nm}(\sin\bar{\phi})$$



- Normal Earth Potential U caused by normal gravity (γ)

$$U(u,\beta) = \frac{GM}{E}\tan^{-1}\frac{E}{u} + \frac{1}{2}w^2a^2\frac{q}{q_0}\left(\sin^2\beta - \frac{1}{3}\right) + \frac{1}{2}w^2(u^2+E^2)\cos^2\beta$$

- Disturbing potential T = W-U

$$N = \frac{T}{\gamma_Q}$$

Model	Max Degree	Data Used in Modeling	Global Accuracy (m)
EGM2008	2190	A, G, S(Grace)	0.24
Eigen06c4	2190	A, G, S(Goce), S(Grace), S(Lageos)	0.24
Eigen05c	360	A, G, S(Grace), S(Lageos)	0.34
EGM96	360	A, G, PGM55	0.43

S is for satellite (e.g., GRACE, GOCE, LAGEOS), A is for altimetry, and G for ground data (e.g., terrestrial, shipborne and airborne measurements).

Local Geoid Modeling

Method	Basic function/method	Required observations
Stokes formula /Remove restore method	$N = \frac{a}{4\pi\gamma_m}\iint_\sigma S(\psi)\Delta g \, d\sigma$ $S(\psi) = \sum_{n=2}^{\infty} \frac{2n+1}{n-1} P_n(\psi)$	•Terrestrial gravity anomalies •Global models gravity anomalies •Digital Terrain Model
Least Square Collocation	$T(P) = f(P) = \sum_{k=1}^{q} b_k \varphi_k$ $B_{ik} = L_i(\varphi_k)$ $\varphi_k = L_k^Q K(P,Q) = C_{Pk}$ $K(P,Q) = \sum_{n=2}^{\infty} \sigma_n^2 \left(\frac{R^2}{r_P r_Q}\right)^{n+1} P_n(\psi_{PQ})$	•Terrestrial gravity anomalies •Terrestrial gravity disturbances •Deflections of vertical •Height fitting points
Spherical Radial basis functions (SRBF)	$\Psi_j(i,j) = \sum_{l=0}^{\infty} \psi_l \left(\frac{R}{r_i}\right)^{l+1} P_l(\theta_{ij})$	•Terrestrial gravity anomalies •Terrestrial gravity disturbances •Deflections of vertical •Height fitting points
Spherical Cap Harmonics (SCH) (combined data)	$V = \frac{GM}{r} \sum_{k=0}^{k\max} \left(\frac{R}{r}\right)^{n(k)} \sum_{m=0}^{k} \left(C'_{nm} \cos m\alpha + S'_{nm} \sin m\alpha\right) P_{n(k),m}(\cos\theta)$	
Adjusted Spherical Cap Harmonics (ASCH) (combined data)	$V = \frac{GM}{r} \sum_{k=0}^{k\max} \left(\frac{R}{r}\right)^{n(k)+1} \sum_{m=0}^{k} \left(C'_{nm} \cos m\alpha + S'_{nm} \sin m\alpha\right) P_{n,m}(\cos\vartheta)$	
Kriging GIS Interpolation	$\hat{Z}(s_0) = \sum_{1}^{n} \lambda_i Z(s_i)$	•Height fitting points

Geoid Models used in Palestine

- In GIS software global EGM2008 and EGM96 are used by default
- In GNSS data collectors ILLUM12 is used (GIS kriging low dense reference points in Westbank)

Model	$N - N_G$ (cm)		
	RMSE	Min	Max
Eigen06c4	10.0	−2.8	40.3
EGM2008	9.5	−5.2	40.2
Eigen05c	50.4	−75.6	118.8
EGm96	55.1	−113.5	78.2
EGG97	48.8	−97.0	106.9

id	h	H	N=h-H	ΔN_{ILUM12}	$\Delta N_{EGM2008}$	$\Delta N_{EIGEN05c}$
2	-0.253	-18.421	18.168	0.205	-0.378	0.461
3	-206.304	-225.055	18.751	0.549	-0.079	0.422
4	-253.984	-273.293	19.309	0.547	0.093	0.359
5	-230.864	-250.653	19.789	-0.819	-1.295	-1.430
6	-181.11	-201.162	20.052	-0.619	-1.168	-1.193
7	771.443	752.234	19.209	-0.012	-0.292	0.709
8	966.502	946.8	19.702	-0.002	-0.265	0.720
12	131.16	110.258	20.902	0.231	-0.688	-0.508
19	-223.522	-243.062	19.54	0.219	-0.629	-1.368
101	920.753	901.108	19.645	0.097	-0.049	0.946
103	899.048	879.507	19.541	0.129	-0.082	0.930
104	914.223	894.535	19.688	0.187	0.053	1.036
106	409.25	391.575	17.675	-0.004	-0.433	0.612
107	37.703	19.249	18.454	-0.005	-0.625	-0.701
108	22.787	4.233	18.554	-0.022	-0.667	-0.687
109	747.558	727.487	20.071	0.000	-0.367	0.165
110	658.778	638.821	19.957	0.015	-0.422	0.079
111	67.19	47.6	19.59	-0.100	-0.785	-0.715
112	131.16	110.258	20.902	0.231	-0.688	-0.508
		Min	17.675	-0.819	-1.295	-1.430
		Max	20.902	0.549	0.093	1.036
		RMSE		0.324	0.381	0.819

Current status in Palestine

- Limited number of precise leveling benchmarks
- No gravimetric networks
- No astronomical networks
- No official Geoid Model to applied in all GNSS data collectors and GIS software
- Global models are not suitable for engineering levels of accuracies
- ILLUM12 has dense reference points in the occupied Palestine1948 and settlements' areas
- ILLUM12 is designed for IGD05/12 Datum in which GNSS networks are designed in Palestine
- ILLUM12 will not be compatible with the new geodetic datums in Palestine
- Is it possible that the PLA can adopt an Israeli Geoid ?

The Idea of solution

- Due to difference o the reference frame of local and global model datums, a fitting approach can be applied
- 7 parameters Moldensky transformation is applied due to the possible separation between vertical and horizontal control points

$$dN = (N_e(1 - e^2 sin\phi))m - (N_e e^2 sin\phi cos\phi sin\lambda)r_x + (N_e e^2 sin\phi cos\phi cos\lambda)r_y$$

$$-(cos\phi cos\lambda)T_X + (cos\phi sin\lambda)T_Y + (sin\phi)T_Y$$

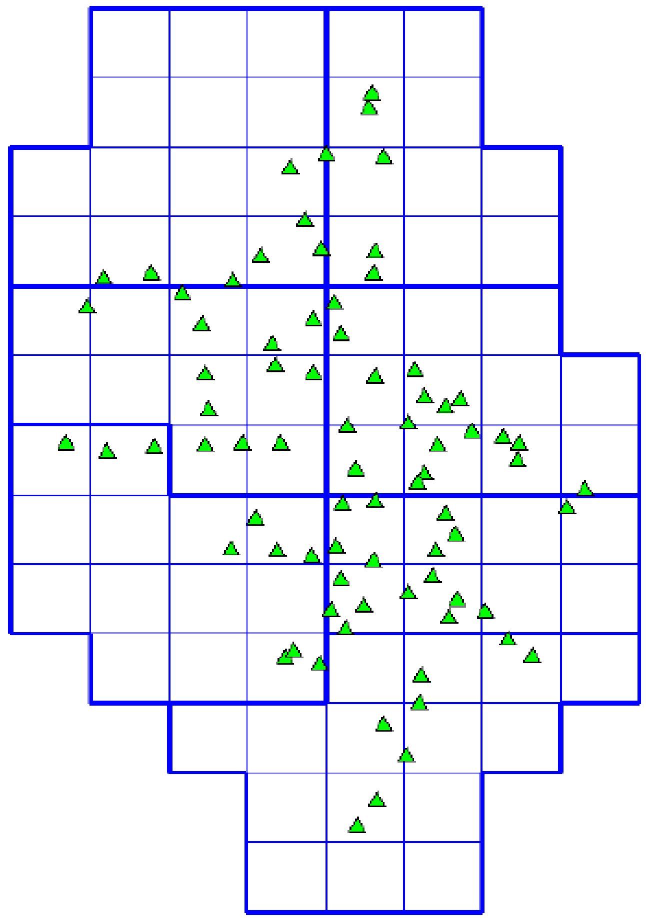
$$N_{GGM} = N_{Local} + dN$$

Finite Elements Method

- The study area is divided to sub areas (Patch) to fit the global model
- Each Patch is divided to smaller elements (meshes)
- A mesh has geoid undulation by a polynomial 2nd or 3rd order

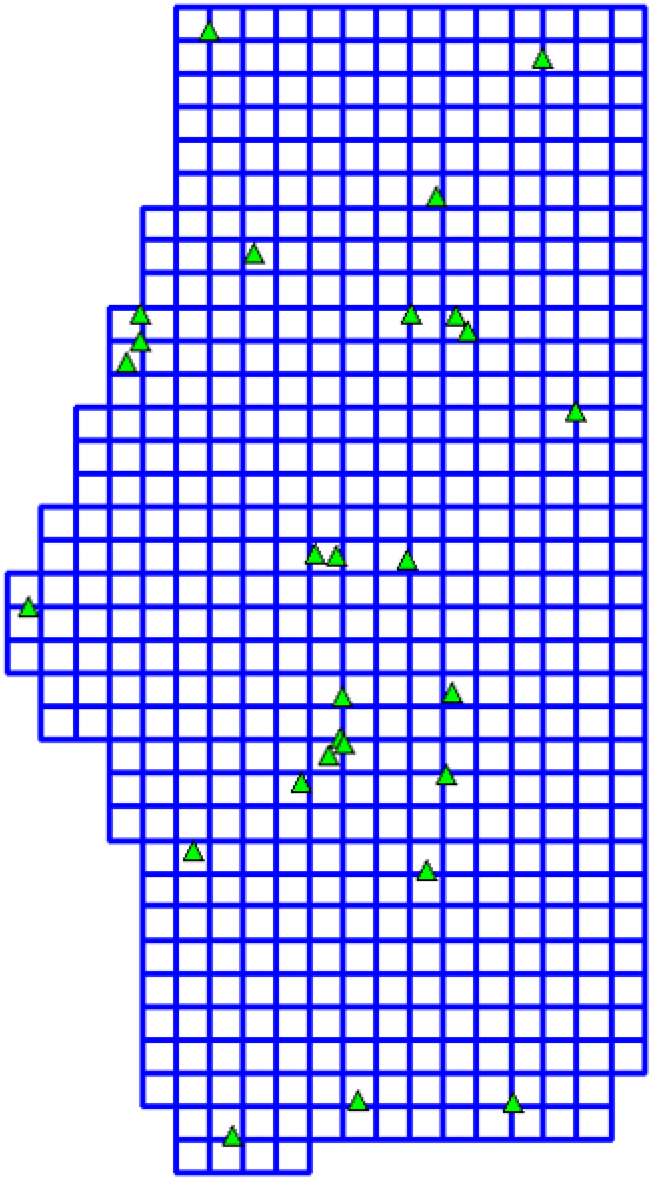
$$N_{FEM} = a_0 + a_1 x + a_2 y + a_3 x^2 + a_4 y^2 + a_5 xy + a_6 x^3 + a_7 y^3 + a_8 yx^2 + a_9 xy^2$$

- Continuity conditions between neighboring meshes are applied
- C0: equality , C1: Slope , and C2: deflection
- Fitting points : definitions of the datum for the patches
- Fitted Global Models geoid undulations : define the mesh parameters
- Deflections of vertical : define the variations in change the directions



Input	Observation equation
The height fitting point	$h + v = H + N_{FEM_i}$
Deflections of vertical	$\eta + v = -\frac{\partial N_{FEM_i}}{\partial x}$ $\zeta + v = -\frac{\partial N_{FEM_i}}{\partial x}$
Global Geoid Models	$N_{GGM} + v = N_{FEM_i} + dN_j$ $\eta_{GGM} + v = -\frac{\partial N_{FEM_i}}{\partial x} + \frac{\partial (dN)_j}{\partial x}$ $\zeta_{GGM} + v = -\frac{\partial N_{FEM_i}}{\partial y} + \frac{\partial (dN)_j}{\partial y}$
Continuity conditions	$N_{FEM_i} = N_{FEM_j}$ $\frac{\partial N_{FEM_i}}{\partial x} = \frac{\partial N_{FEM_j}}{\partial x}$ and $\frac{\partial N_{FEM_i}}{\partial y} = \frac{\partial N_{FEM_j}}{\partial y}$ $\frac{\partial^2 N_{FEM_i}}{\partial x^2} = \frac{\partial^2 N_{FEM_j}}{\partial x^2}$ and $\frac{\partial^2 N_{FEM_i}}{\partial y^2} = \frac{\partial N_{FEM_j}}{\partial y^2}$

Design in the Westbank



Number of patches	6
Number of meshes	541
Control points	28
GGM Geoid undulations	13525
GGM deflections of vertical	27050
Continuity conditions	13241
Polynomial parameters	5410

Results

- The use of EGM2008 (equivalently EIGEN06c4) for model densification could provide the best results
- Approximate patch size 50x50km and mesh size 5x5 km could provide the required accuracy
- 1-3 cm geoid model could be achieved in the Westbank of Palestine
- The model can easily transformed to predefined grid file to be uploaded to GIS software and GNSS data collectors in different company defined formats : Leica, Stonex , Trimble, Carlson … etc.

Densification model	Value (m)	Control points	test points
EGM2008	Min residual	-0.019	-0.034
	Max residual	0.026	0.043
	RMSE	0.013	0.016
EIGEN05c	Min residual	-0.083	-0.105
	Max residual	0.123	0.145
	RMSE	0.092	0.117

Conclusions

- The global geoid model are invalid for the direct use in precise GNSS positioning techniques like static observation and RTK positioning
- The use of ILLUM12 is invalid in the areas A and B, where no reference points were used in the modeling (good results near the settlements and Highways e.g.: road 60 and 90. normally the points are renamed by U-points)
- The height system should be unified in all software and GNSS instrument by official adoption of a precise geoid.
- The precise leveling network must be updated and densified
- A geoid model must be compatible with any newly defined reference frame in Palestine
- A securing and usage policies must introduced by new laws to guarantee the sustainability of the defined frame
- Specifications and guidelines must be available to define the allowed field strategies by GNSS in different types of leveling works.


Thank you