

# Center of Mass States and Disturbance Estimation for a Walking Biped

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**Abstract**— An on-line assessment of the balance of the robot requires information of the state variables of the robot dynamics and measurement data about the environmental interaction forces. However, modeling errors, external forces and hard to measure states pose difficulties to the control systems. This paper presents a method of using the motion information to estimate the center of mass (CoM) states and the disturbance of walking humanoid robot. The motion is acquired from the inertial measurement unit (IMU) and forward kinematics only. Kalman filter and disturbance observer are employed, Kalman filter is used for the states and disturbance estimation, and the disturbance observer is used to decompose the disturbance into modeling error and acceleration error based on the frequency band. The disturbance is modeled mathematically in terms of previous CoM and Zero moment point (ZMP) states rather than augmenting it in the system states. The ZMP is estimated using the quadratic programming method to solve the constraint dynamic equations of the humanoid robot in translational motion. A biped robot model of 12-degrees-of-freedom (DOF) is used in the full-dynamics 3-D simulations for the estimation validation. The results indicate that the presented estimation method is successful and promising.

**Index Terms**—Humanoid robot, Disturbance Observer, Kalman filter, state estimation.

## I. INTRODUCTION

The control of the Humanoid robots walking and stability is a challenging task which requires the availability of various feedback variables. The use of expensive and numerous sensors on board could be regarded as a solution. However, cost, external disturbances, unmodeled dynamics, sensor noise and sensor dynamics pose difficulties for measurement systems which are crucial for feedback control in this area.

Disturbance and Center of mass (CoM) states information play an important role in the control and stability of humanoids. The disturbance, in terms of modeling error and acceleration error, may affect the control algorithms adversely. CoM states are used in the stability criteria, directly [1-4] or through the inertial and gravity forces [1] [2].

The difficulties in the measurement systems motivate the use of intelligent techniques for the estimation of the state variables and disturbances. Systematic estimation is based on models. Biped balance is usually studied using simplified

models. A simple model is the Linear Inverted Pendulum Model LIPM [3] where the body (trunk) is modeled as a lumped mass at the Center of Mass (CoM) and moves only horizontally. This model is convenient since it can be written in discrete state space representation. Thus, the linear methods of estimation can be implemented.

Controlling the biped CoM dynamics [4] requires the zero moment point (ZMP) trajectory, the knowledge of the CoM states and the disturbance. The ZMP can be calculated by estimating the reaction forces and their location using constraint optimization [5]. The constraints are due to the leg in contact, friction and support polygon [6]. The CoM states are estimated based on process and measurement models. The models change according to the used sensors and the states. The disturbance may have significant effects of the system. It can be estimated by modeling it as an augmented step disturbance state in the system model [7] or by using a Disturbance observer [8].

In [9], the CoM position and velocity states are estimated. The augmented state principle is used to estimate the disturbance. The measurement model includes position and force measurements. The disturbance observer reported in [10] is used to estimate the external force using inertial sensors and foot force sensors. The ZMP disturbance observer in [11, 12] decomposes the ZMP error into position error and acceleration error based on the frequency band, however it doesn't consider the CoM states

In this paper, different from [9], The disturbance and the CoM position, velocity, and acceleration states of a walking biped are estimated without force or position sensors, and the disturbance is modeled mathematically as a function of the previous known ZMP, position and acceleration states. This estimation is based on the readings of the IMU and the kinematics of the robot legs through the assembled joint-encoders. The discrete model of the LIPM dynamic equations is obtained to form the process model of the linear state space representation with the ZMP as the input, and the acceleration as the output model. The input ZMP is estimated using the quadratic programming method to solve the constraint dynamic equations of the humanoid robot in translational motion. Kalman filter is used in its basic linear structure for the estimation, the acceleration from IMU serves as the

measurement for the filter. The estimated disturbance is decomposed using disturbance observer into both position (modeling) error and acceleration errors.

The rest of the paper is organized as follows: Section 2 describes the estimation of the total reaction force at each foot with its location, the CoM states and disturbance. Section 3 introduces the simulation platform and presents the simulation results. The paper conclusion is in Section 4.

## II. METHOD ARCHITECTURE AND MODEL DERIVATION

### A. ZMP estimation

The ZMP ( $p$ ) of a biped can be approximated as [13]

$$p = \frac{p_L (F_L \cdot \hat{z}_w) + p_R (F_R \cdot \hat{z}_w)}{m(\ddot{c} \cdot \hat{z}_w - g)} \quad (1)$$

Where:  $p_L$ ,  $p_R$ ,  $F_L$  and  $F_R$  are the position of the ZMP and the reaction forces at the left and right legs respectively,  $c$  is the CoM position,  $g$  is the constant gravity acceleration vector pointing to the negative  $z$ - direction in the world frame,  $m$  is the humanoid mass, and  $\hat{z}_w$  is unit vector pointing to the positive  $z$  direction in the world frame.

Equation (1) requires the calculation of the force and ZMP for each leg. The dynamic equations of a humanoid robot in translational motion in Eq (2) are used to calculate the force and its location for each leg [14] [15]

$$\begin{aligned} F_R + F_L &= m(\ddot{c} - g) \\ f_R \times F_R + f_L \times F_L + M_R + M_L &= \dot{H} \end{aligned} \quad (2)$$

where,  $H$  is the CoM angular momentum,  $M_R$  and  $M_L$  are the moments at the right and left feet respectively.  $f_R$  and  $f_L$  are the position of the right and left feet frame-origins respectively from the CoM, they are known by using the robot-leg kinematics and the joints encoders for each leg. Equation (2) can be written in matrix form as

$$CN = d \quad (3)$$

where

$$C = \begin{bmatrix} I_3 & I_3 & 0_{3 \times 3} & 0_{3 \times 3} \\ f_R \times & f_L \times & I_3 & I_3 \end{bmatrix}, N = \begin{bmatrix} F_R \\ F_L \\ M_R \\ M_L \end{bmatrix}, \quad (4)$$

and

$$d = \begin{bmatrix} m(\ddot{c} - g) \\ \dot{H} \end{bmatrix} \quad (5)$$

where  $I_m$  and  $0_{m \times l}$  are the identity and the zero matrix of dimensions  $m$  and  $m \times l$  respectively. The term  $(\ddot{c} - g)$  is directly measured from the accelerometer and then transformed to the world frame. For rotationally stable robot, the angular momentum is considered constant i.e.  $\dot{H} = 0$  [14]. The solution of Eq(2) is subjected to constraints due to the leg in contact, friction and support polygon [6]. The leg in contact constraints are just activating or deactivating the leg parameters in binary way and can be written mathematically as in

$$AN = b \quad (6)$$

where

$$A = \begin{bmatrix} A_{right} & 0_{D \times D} & A_{right} & 0_{D \times D} \\ 0_{D \times D} & A_{left} & 0_{D \times D} & A_{left} \end{bmatrix}, b = 0_{2D \times 1}, \quad (7)$$

$$A_{right} = \begin{cases} I_D & \text{left contact} \\ 0_{D \times D} & \text{right contact} \end{cases}, \quad (8)$$

and

$$A_{left} = \begin{cases} I_D & \text{right contact} \\ 0_{D \times D} & \text{left contact} \end{cases} \quad (9)$$

where  $D$  is the space dimension, for 3-dimensional space,  $D = 3$ .

The support polygon constraints are

$$-\frac{\delta_x}{2} \leq \frac{-M_{y,i}}{F_{z,i}} \leq \frac{\delta_x}{2}, \quad -\frac{\delta_y}{2} \leq \frac{M_{x,i}}{F_{z,i}} \leq \frac{\delta_y}{2}, \quad (10)$$

where the subscripts  $x, y$ , and  $z$  refer to the  $x$ -,  $y$ - and  $z$ - components respectively. The subscript  $i$  refers to right or left leg. And  $\delta$  is the foot dimension in the specified direction.

To guarantee that the feet will not slip, other restrictions do exist as friction constraints and written as

$$\begin{aligned} |F_{x,i}|, |F_{y,i}| &\leq \mu F_{z,i}, \\ F_{z,i} &\geq 0 \end{aligned} \quad (11)$$

where  $\mu$  is the coefficient of friction.

The support polygon and friction constraints are written in matrix inequality form for both of the legs as

$$A_{eq} N \leq b_{eq} \quad (12)$$

The matrix forms of the constraints are used directly to estimate the vector  $\hat{N}$  by solving the constraint quadratic programming problem

$$\begin{aligned} \hat{N} &= \min_N N^T (C^T C + W) N - d^T C N \\ \text{s.t.} \quad & A_{eq} N \leq b_{eq} \\ & AN = b \end{aligned} \quad (13)$$

where  $W$  is a diagonal matrix. By the knowledge of the forces and moments from Eq(13), the CoP is calculated for each foot using

$$CoP = \begin{bmatrix} -M_y / F_z & M_x / F_z & 0 \end{bmatrix}^T \quad (14)$$

### B. CoM states and disturbance estimation

The LIPM dynamics in Eq(15) are used to form the process dynamics of the estimation

$$\begin{aligned} \ddot{c}_x &= \frac{g}{z_c} (c_x - p_x) \\ \ddot{c}_y &= \frac{g}{z_c} (c_y - p_y) \end{aligned} \quad (15)$$

where  $Z_c$  is the constant height of CoM . From now on, the subscripts  $x, y$  will be ignored for convenience. The analysis shown below is applicable for both  $x$  and  $y$  dimensions.

In the ideal case, the computed  $p$  from the force “ $p^f$ ” is the same as the one computed from

$$p = c - \frac{z_c}{g} \ddot{c} \quad (16)$$

However, due to modeling error, noise, and disturbance an error between  $p^f$  and  $p$  does exist. This error “ $p^{err}$ ” is written as

$$p^{err} = p^f - p \quad (17)$$

For simplification let the error be zero, then the acceleration at the current time instance  $k$  for the discrete form of Eq(15), using the backward Euler method with sampling time  $T$ , is

$$\ddot{c}_k = T \frac{g}{z_c} (\dot{c}_{k-1} - \dot{p}_{k-1}^f) + \ddot{c}_{k-1} \quad (18)$$

where

$$\dot{p}_{k-1}^f = \frac{p_k^f - p_{k-1}^f}{T} \quad (19)$$

This idea is fruitful, since a direct relation between  $p_k^{err}$  and the previous state vector of the CoM is obtained as in Eq (20)

$$p_k^{err} = -c_{k-1} + p_{k-1}^f + \frac{z_c}{g} \ddot{c}_{k-1} \quad (20)$$

Equation(18) and Eq(19) form the process model required for the basic structure of Kalman filter. Considering the acceleration as the measurement, then the state space representation used in Kalman filter is

$$\begin{aligned} x_k &= Ax_{k-1} + Bu_k + w_k \\ Y_k &= Cx_k + v_k \\ z_k &= a_G \end{aligned} \quad (21)$$

where:

$$x = \begin{bmatrix} c & \dot{c} & \ddot{c} & p^f \end{bmatrix}^T, u = \dot{p}_{k-1}^f$$

$$A = \begin{bmatrix} 1 & T & 0.5T^2 & 0 \\ 0 & 1 & T & 0 \\ 0 & \frac{g}{z_c}T & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ -\frac{g}{z_c}T \\ T \end{bmatrix},$$

and

$$C = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix},$$

$a_G$  is the measurement acceleration vector expressed in the world frame.  $w$  and  $v$  represent the process and the measurement noises respectively. They are considered Gaussian processes with covariance's  $Q$  and  $R$  as

$$\begin{aligned} w &\sim N(0, Q) \\ v &\sim N(0, R) \end{aligned}$$

The above state space representation is valid for both  $p_x$  and  $p_y$ , by changing the input and measurement components.

### C. Error signal decomposition

The error in Eq(20) is composed of position (modeling error appears as an offset) error and acceleration error. They can be separated based on the frequency band using two low pass filters (LPF) with different frequencies. The offset error is the output of the low frequency LPF. This offset error is subtracted from the output of the higher frequency LPF to obtain the acceleration error [11, 12].

## III. SIMULATION RESULTS

The simulations are carried on 12 degrees of freedom (DOF) biped model. It consists of two 6-DOF legs and a trunk connecting them. Three joint axes are positioned at the hip, two joints are at the ankle and one at the knee (Fig. 1). The numerical values of the parameters (Table ) are taken to match

our experimental humanoid robot SURALP (Sabanci University Robotics Research Laboratory Platform) [16]. The details of contact modeling and simulation algorithm are in [17]. The modeled reaction forces suffer from peaks, so that Kalman filter is used for smoothing the modeled reaction forces. The coordinate frames are shown in Fig. 2. All the measurements and calculation are in the world frame. The transformation is done using the rotational matrix obtained by the author in [18]. The body frame has an offset  $x_{offset}$ . The role of this offset parameter is to place the center of the support polygon exactly below the center of mass of the robot as shown in Fig. 3. According to the figure it has a negative value. Kalman filter parameters are listed in Table II.  $Q, R$ , and the low pass filters constants are chosen by trial and error.

The estimated ZMP, CoM states and the disturbance for the  $x$ -direction is depicted in Fig. 4. The estimated ZMP (Fig. 4.a) shows its reliability and accuracy compared to the reference ZMP and the ZMP based on modeled and filtered force. The estimated CoM position trajectories and reference desired ones are in the body frame as expressed in the world frame, the simulated trajectories are in the CoM frame as expressed in the world frame. This explains the negative offset  $x_{offset}$  in the estimated and reference CoM position trajectories in Fig. 4.b, when the offset value is included in the model, the resulted position error is  $\approx 0$  while it is  $\approx 0.02$  when it is not included (Fig. 4.d). The estimated offset is positive because  $p$  is considered negative in Eq(17). In this simulation, no external forces are considered, thus the acceleration error is always zero as in Fig. 4.d. The estimation shows its accuracy in the velocity level as shown in Fig. 4.c.

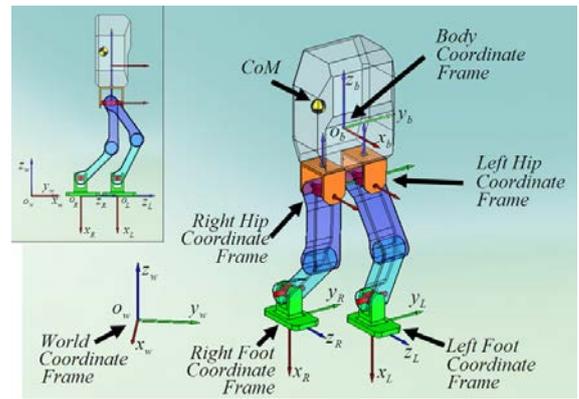


Fig. 2: Coordinate systems.  $o_w$  and  $o_b$  stand for the origins of the world and body coordinate frames, respectively. The foot coordinate frames are fixed to the foot soles.

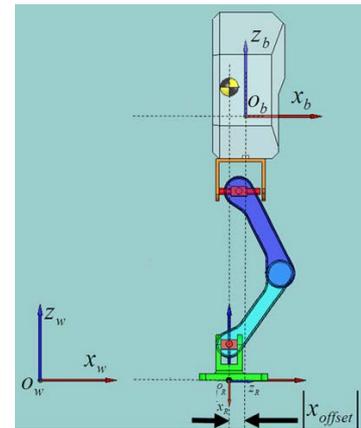


Fig. 3: body frame offset

Table I: Robot Parameters

Parameter	Value
Upper leg length	280mm
Lower leg length	270mm
Sole-ankle distance	124mm
Foot dimensions	240mm×150mm
Upper arm length	219mm
Lower arm length	255mm
Robot weight	114 kg
$x_{offset}$	25mm

Table II: Simulation Parameters

Parameter	Value
$Q$	$10^{-10}$
$R$	$4 \times 10^{-4}$
$T$	0.001

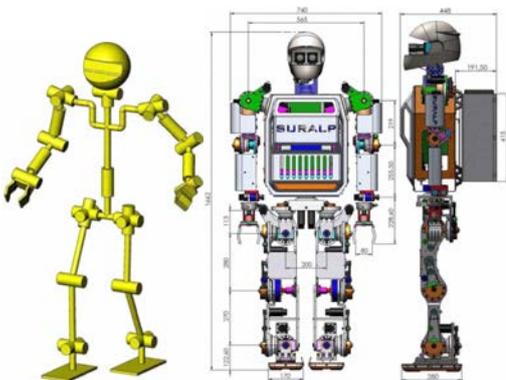


Fig. 1 The kinematic arrangement of SURALP

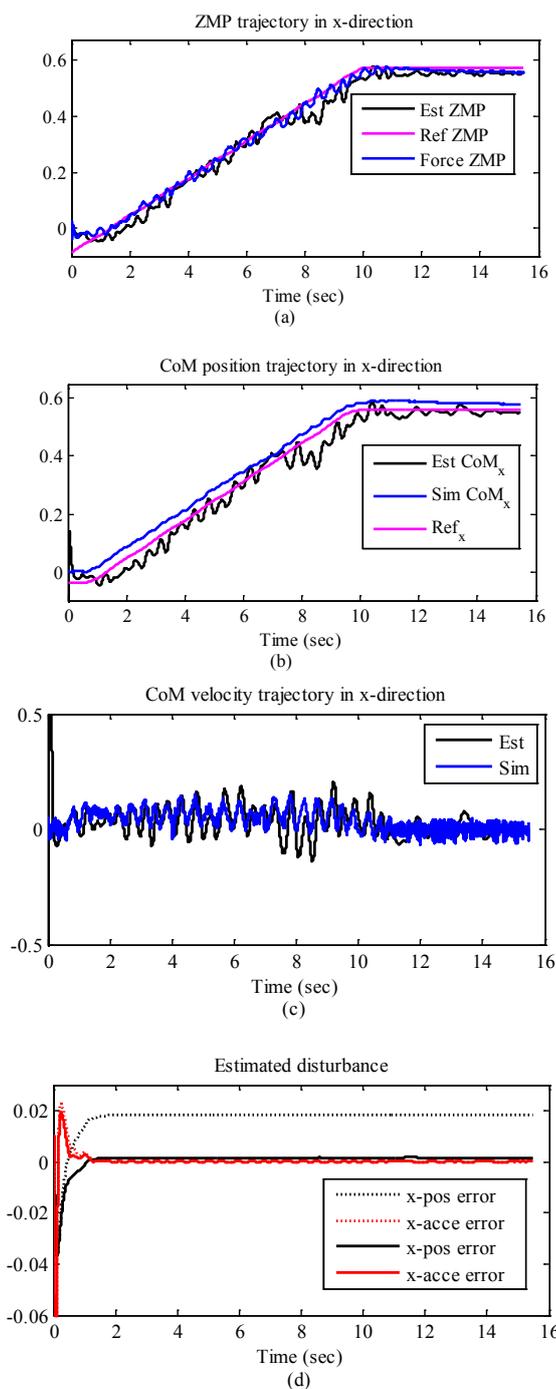


Fig. 4: ZMP, CoM states, and disturbance estimation in the  $x$  – direction, in the figure, Est refers to estimated, Ref refers to reference, Sim refers to simulation based, and Force refers to force based. (a) ZMP trajectories. (b) CoM  $x$  – direction position state. (c) CoM  $x$  – direction velocity state. (d) Estimated disturbance, in terms of position error and acceleration error, the dotted line when the offset is not included in the model and solid line when it is included.

#### IV. CONCLUSION AND FUTURE WORK

A method of estimating the CoM states and the disturbance by employing the reading of IMU and joint-encoders is shown. The disturbance is modeled mathematically in terms

of previous CoM and (ZMP) states rather than augmenting it in the system states, and it is decomposed into position and acceleration error based on frequency band using disturbance observer. Kalman filter is used for the states estimation where the estimated ZMP is used as its input. The simulations validate the proposed method and motivate us to implement it into our humanoid robot SURALP in the future.

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