Palestine Polytechnic University

College of Engineering and Technology
Mechanical Engineering Department
Mechatronics Engineering

Graduation Project

Project Title:
Design and Control of Power Assisted Jib Crane

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Palestine-Hebron

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ABSTRACT

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Generally, cranes are used to perform a function that exceeds the capabilities of the human, which is loading, moving, and unloading the heavy weights. Jib crane is used to perform such a task. The problem with such cranes; is the vibrations that are produced due to starting of rotation motion with high load. These vibrations affect inversely the system performance. However, the expected result of this project is to design, analyze, building up the Jib crane, and finally to control its rotational and translational motions, where the angular and linear positions of the load will be controlled with minimum vibration in the load during the motion.

This report includes the graphical design of the system and all of its parts. The mathematical model of the system also will be derived. This model will be used to implement the controller that will provide the desired output signal to the motor. The system with its controller will be simulated to examine the performance of the designed controller.

The system parts will be mechanically designed; also all electrical parts will be electrically designed. Finally a prototype for the system will be implemented, with all required specifications.

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Chapter One

Introduction to Cranes

1.1 Cranes in Industry

Crane is a piece of construction equipment used to hoist heavy loads and to move them short distances. [16]

Crane can be considered as one of the most important tools used in industry to transfer loads and cargo from one spot to another. Usually cranes have very strong structures in order to lift heavy payloads in factories, in building construction, on ships, and in harbors.

In factories, cranes speed up the production processes by moving heavy materials to and from the factory as well as moving the products along production or assembly lines. In building construction, cranes facilitate the transport of building materials to high and critical spots.

Similarly on ships and in harbors, cranes save time and consequently money in making the process of loading and unloading ships fast and efficient. Until recently, cranes were manually operated. But when cranes became larger and they are being moved at high speeds, their manual operation became difficult. Consequently, methods of automating their operation are being sought. [3]

Hence, designing, and controlling these equipments became one of the most important prerequisites in the manufacturing, and as the new technologies which are represented in the Flexible Manufacturing Systems (FMS) entered the fields of life, these types of equipments had a large necessity in these fields.

1.2 Literature Review:

Cranes and hoisting systems are designed and constructed made in a wide variety of shapes and designs according to their purposes. So a brief background to each of these cranes will be provided. The reasons for selecting the Jib crane to design and control will be discussed.

1.2.1 Types of Cranes

Generally the cranes can be classified according to their structure into several categories, which can be introduced as:

1.2.1.1 Overhead Crane

Generally, there are three kinds of overhead cranes which are: bridge cranes, gantry cranes, and jib cranes. They can be considered as overhead or bridge cranes because they are equipped with a horizontal girder called the bridge from which a pulley is suspended. The pulley can then usually be moved horizontally across the bridge then to grant the lift access to a large area.

Each of these types will be discussed to provide their features.

1.2.1.1.1 Bridge Crane

A bridge crane is one of the most common kinds of cranes for industrial use. It consists of a pulley which can move back and forth horizontally across a bridge (usually a steel girder). The bridge is usually held up by steel girders or the central supports inside a warehouse. The most common kind of bridge cranes is the overhead bridge crane which is used in warehouses and manufacturing facilities as shown in Figure (1.1). Overhead bridge cranes are used to hoist and transport heavy equipment to different locations within a warehouse. Bridge cranes are generally placed on top of or suspended from long steel tracks. This allows the bridge to move back and forth granting lift access to an entire warehouse or yard.[16]

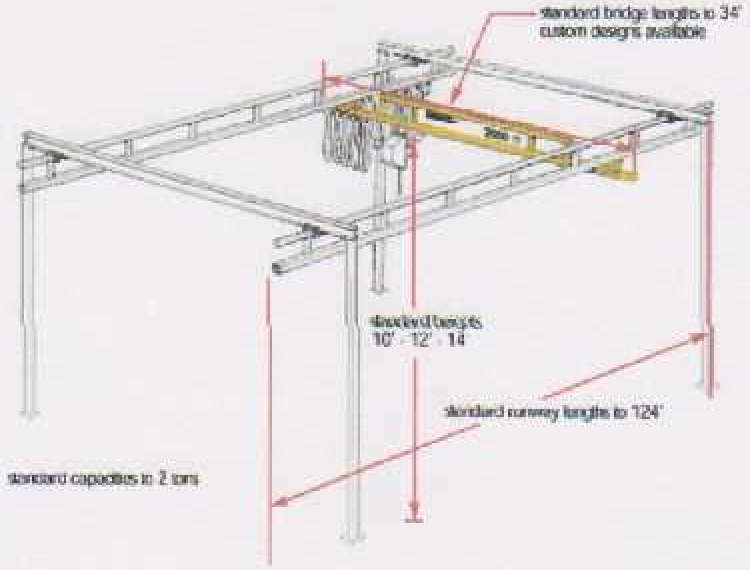

Figure (1.1): Bridge Crane.

1.2.1.1.2 Gantry Crane

A gantry crane is like a bridge crane except that it is always supported by steel girders equipped with rails or wheels on the floor. Gantry cranes are generally smaller than the bridge cranes and cover more space. Still, they are ideal for outdoor yard when there are no indoor rafters for mounting. As shown in Figure (1.2), gantry cranes feature a horizontal bridge girder equipped with a pulley that can move backward and forward and horizontally. The bridge girder is then supported by two or more vertical girders. The primary difference is that the vertical girders are situated on tracks or wheels allowing the crane to be moved backward and forward. The gantry crane system makes the easiest lifting and moving heavy equipment in shipping, loading, and industrial settings. [16]

Figure (1.2): Gantry Crane.

1.2.1.1.3 Jib Crane

Jib crane can be considered as a kind of Bridge cranes. Jib cranes feature a horizontal bridge girder equipped with a pulley that can move across the length of the bridge. The difference between jib and traditional bridge cranes is the supports. Whereas bridge cranes feature multiple supporting girders, a jib crane only has one as shown in Figure (1.3). That solitary support then pivots allowing the jib crane to service a large, circular area without taking up a lot of space. Because they only have one support, however, they can not lift as much weight as other overhead cranes.

Jib crane designs (explained later) are particularly popular servicing individual warehouse workstations. They can be small and compact, and yet still be versatile enough to reach any area within a workstation. Because they use less steel than bridge or gantry cranes, they are also more cost effective giving you the maximum yield both in space and cost. Jib-type crane models are also popular on fishing boats.[16]

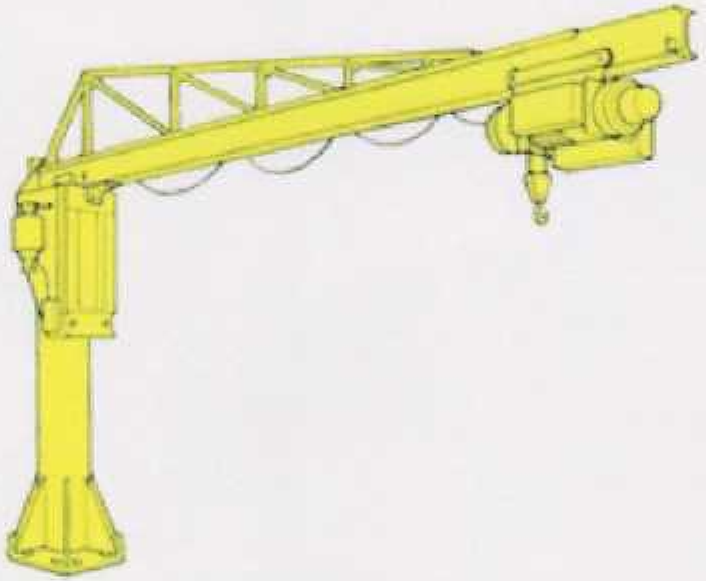

Figure (1.3): Jib Crane.

1.2.1.2 Construction Cranes

Through all of the cranes that are used in construction jobs in one way or another, the most common construction cranes are tower cranes which are shown in Figure (1.4).

These cranes are huge steel structures used to move heavy steel girders and other materials from one side of a major construction site to another.

A construction crane features a large concrete base that is bolted to the ground, a tower or mast that can be up to 400 feet tall (average 150 ft), a jib crane that can slew from side to side, and a short arm with the crane's machinery and concrete counter weight system. These tower cranes can lift almost 20 tons and extend it in a 230 foot radius.[16]

1.2.1.3 Crawler Crane

A crawler crane, or crawl crane, is a mobile crane equipped with tread rather than wheels. Crawler cranes have a tread undercarriage, a cab situated on top of a pivoting (slewing) platform, a steel latticework boom hinged to the platform, and a series of cables and pulleys strung through the boom that usually end in a hook, demolition ball, or bucket which is expressed in Figure (1.5). Some modern crawl cranes have booms which are raised and lowered by hydraulics rather than cables, but both crane varieties are common. Depending on the size and weight of its platform, a crawler crane can reach well over 200 m and lift as much as 1200 metric tons. [16]

Figure (1.4): Tower Crane.

Figure (1.5): Crawler Crane.

1.2.1.4 Truck Crane

As shown in Figure (1.6), truck crane is a crane mounted on a truck. The idea behind truck cranes is to minimize the costs and inconvenience of having to load your crane onto a trailer and set it up every time you want to use it. Still, there are many different kinds of truck mounted cranes available offering different kinds of service. [16]

(a) (b)

Figure (1.6): Truck Crane: (a) Standard Truck Crane, b- Lattice boom Truck Crane.

1.2.1.5 Boom Cranes

Boom cranes (Figure 1.7) are very common on ships and in harbors. In general, a boom crane consists of a rotating base to which a boom is attached. The load hangs from the tip of the boom by a set of cables and pulleys. The rotational movement of the base along with the luff movement of the boom places the boom tip over any point in the horizontal plane that is in reach of the crane. Meanwhile, changing the elevation (luff) angle of the boom causes a change in the radial and vertical positions of the load. The structure of boom cranes supports loads in compression, whereas rotary and gantry cranes support loads in a bending fashion. This makes boom cranes more compact than rotary and gantry cranes of similar capacities. Boom cranes are mounted on ships to transfer cargo between ships or on harbor pavements to transfer cargo between ships and offshore structures. [12]

Figure (1.7): Boom Crane

This is the most famous, used, and known types of the cranes, however we will consider the overhead cranes, and exactly the jib crane, the reasons for this selection will be expressed in the next section.

1.3 Why to Consider the Jib Crane?

Today's industry demands versatile, efficient, and cost effective equipments while at the same time providing more flexibility along with significant savings through increased productivity. A jib crane can help improve materials handling efficiency and work flow. Serious consideration should be given to jib cranes for applications requiring repetitive lifting and transferring of loads within a fixed arc of rotation. [16]

As mentioned before, jib crane is one of the overhead cranes, and it is designed into several forms for different applications. So in order to explain the selection for the jib crane for building and controlling, first its designs must be studied, then comparing it with the other two types of the overhead cranes which are: Bridge, and Gantry cranes.

1.3.1 Types of Jib Crane

Jib Cranes consist of a horizontal load supporting boom, which is attached to a pivoting vertical column that is either free standing or building mounted. They enable lifting and lowering of a load within a fixed arc of rotation. Jib Cranes can be provided in a variety of capacities and configurations including motorized rotation. Below are the basic types of Jib Cranes. [14]

1.3.1.1 Free Standing Jib Cranes

Free standing jib cranes are engineered to stand by themselves on a concrete foundation without building support. They allow for 360° rotation and can be base plate mounted, foundation mounted, or sleeve insert mounted as in Figure (1.8). [14]

These types can be motorized, in their motions, which they are the rotational and the translational motions. The controlling of these motions will be a part of this project.

1.3.1.2 Mast Type Jib Cranes

Mast type jib cranes offer a low cost alternative to achieve 360° rotation without a large mounting foundation as is required by free standing models. The mast is supported top and bottom by the overhead building steel and floor. Mast type jib cranes are available in 2 designs; full cantilever and drop cantilever which are given in Figure (1.9). [14]

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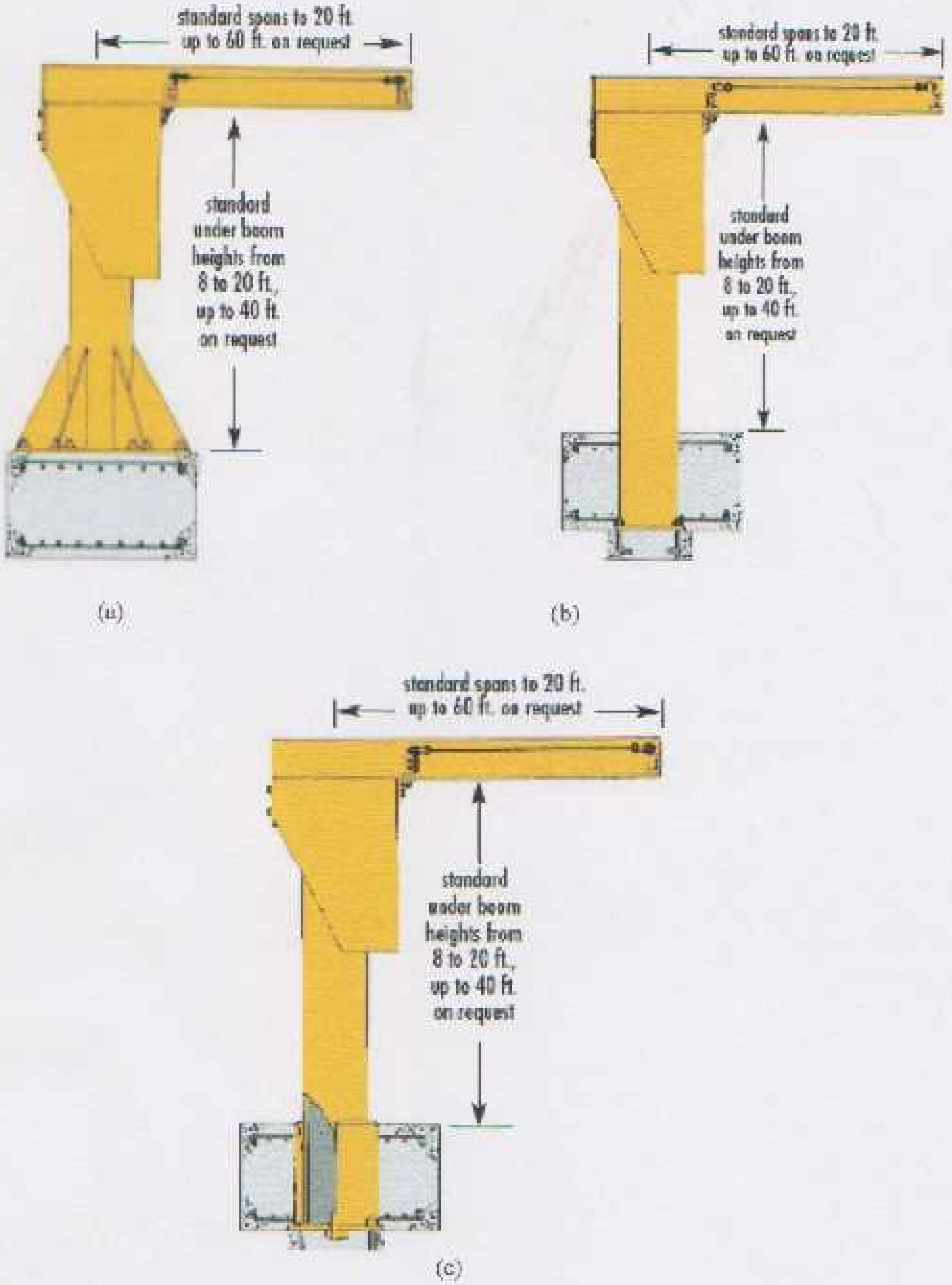


Figure (1.8): Free standing jib crane types: (a) Base plate mounted, (b) Foundation mounted, (c) Sleeve insert mounted.

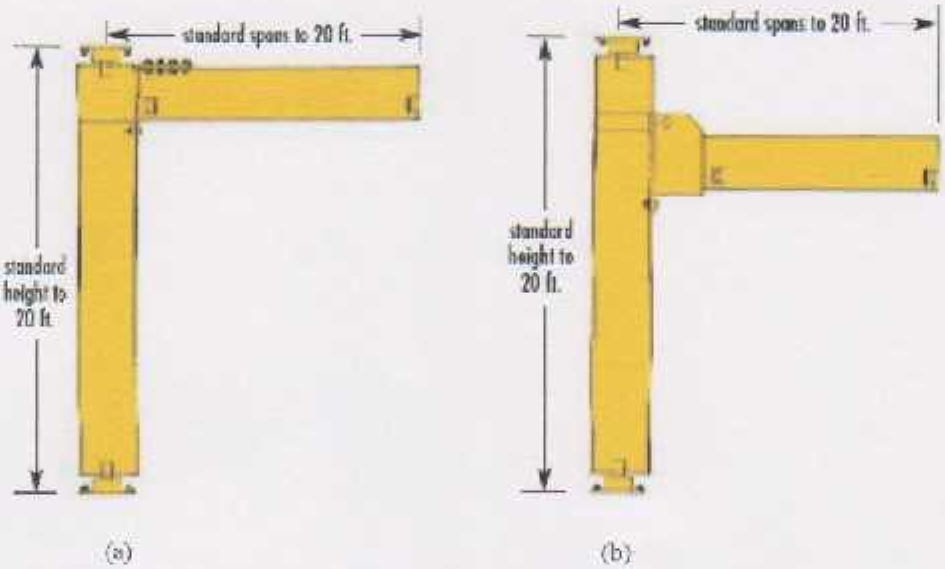


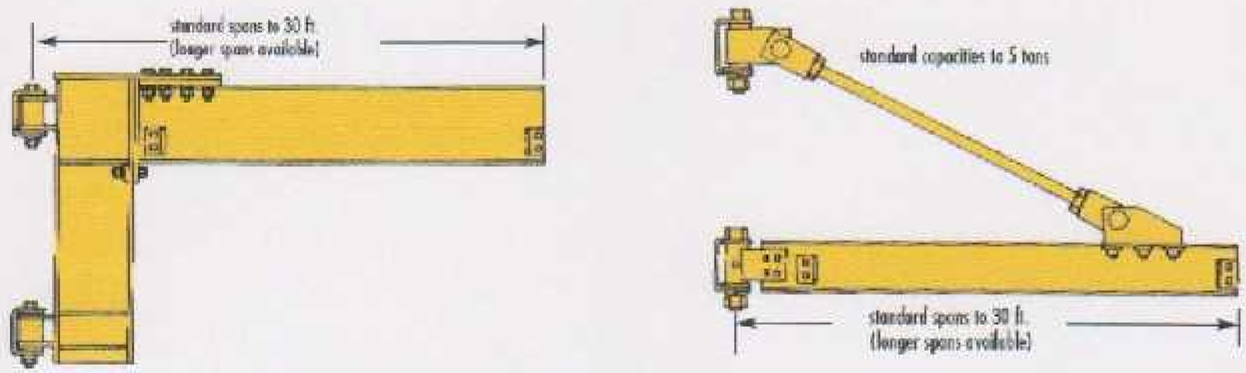
Figure (1.9): Mast jib crane types: (a) Full Cantilever, (b) Drop Cantilever.

1.3.1.3 Wall Mounted Jib Crane

Wall mounted jib cranes are capable of 200° rotation. Their design can be either cantilever or tie rod supported as in Figure (1.10). Wall Mounted Jib Cranes offer an economical alternative to Mast or Free Standing jibs. [14]

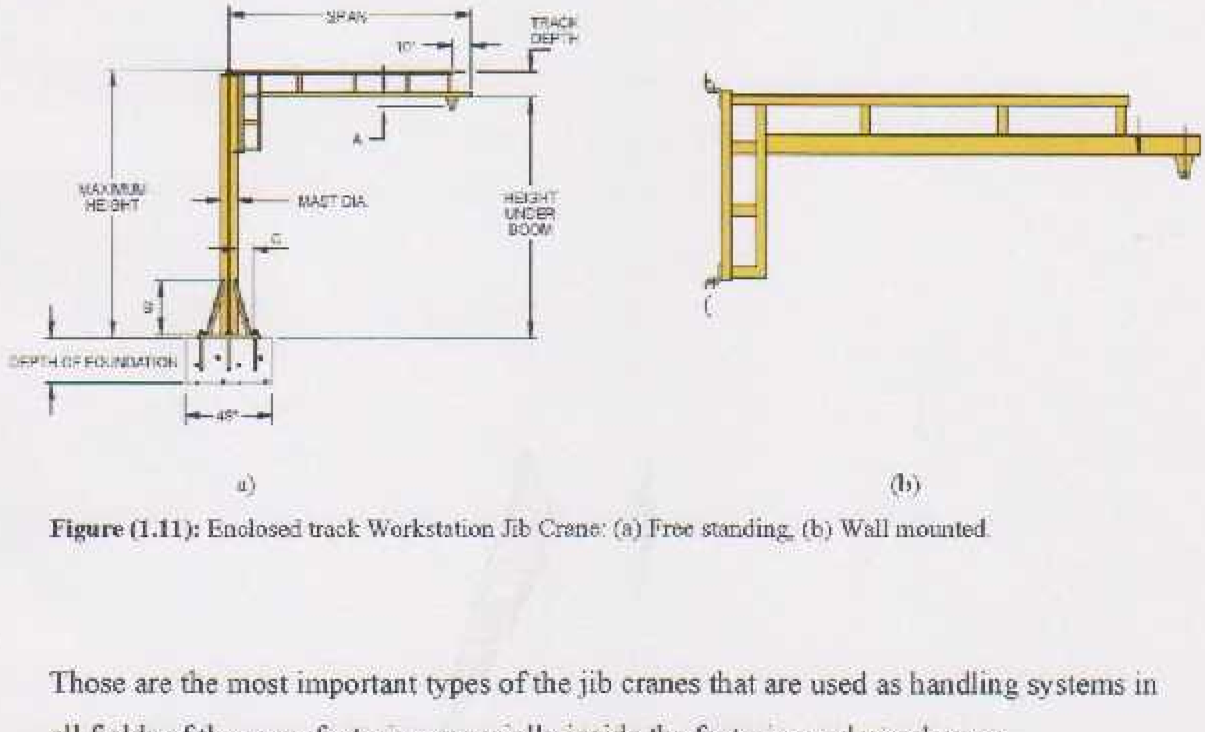
1.3.1.4 Enclosed track Workstation Jib Crane

Two types of this kind, which are: free standing and wall mounted as in Figure (1.11).



(a) (b)

Figure (1.10): Wall mounted Jib Cranes: (a) Wall Cantilever Jib, (b) Wall Bracket Tie Rod Supported Jib Cranes.



a) (b)

Figure (1.11): Enclosed track Workstation Jib Crane: (a) Free standing, (b) Wall mounted.

Those are the most important types of the jib cranes that are used as handling systems in all fields of the manufacturing especially inside the factories and warehouses.

However, reversing back to our question ***why Jib Crane to design and control?***. In order to answer this question, after the study of all types of the jib crane, we must compare it with the Gantry and Bridge Cranes.

These two other types of overhead material handling machines are shown in Figure (1.2) and Figure (1.1) respectively. Both types have a rail (identical to a boom) and trolley similar to those on the jib crane. On the gantry crane, though, the rail is rigidly attached to two supports which roll along the ground, often in tracks. On the bridge crane, one or two rails form a bridge across two fixed rails. In Figure 1.1, the one rail which forms the bridge is shown in yellow. This bridge then has two or four trolleys at each end to allow movement along the fixed rails. [13]

The three material handling machines exhibit a tradeoff between high load capacity and low space infringement. The cantilevered mount of the jib crane gives it a lower capacity than similarly sized gantry and bridge cranes, where the boom is supported at both ends. This same characteristic, however, allows the jib crane to take up significantly less floor space. The bridge crane necessitates the construction of a large frame around the perimeter and/or over the top of its rectangular work space. The gantry crane usually necessitates the mounting of track (for safety reasons) and the existence of the two mobile supporting columns along its rectangular workspace. In contrast, a jib crane merely requires the mounting of a slender mast at the center of a circular workspace or the attachment to a wall along a semicircular workspace. The large rolling mass of the gantry crane creates a safety problem and often necessitates powered motion; thus it is generally restricted to outdoor applications where the use of tracks allows for long distance movements. For this reason, jib and bridge cranes are the most prevalent overhead material handling devices *inside* assembly plants. [13]

For each handling device, the two horizontal DOF differ in feel. For the jib and gantry cranes, motion along the trolley's direction of travel requires relatively little force. Similarly, on the bridge crane, motion along the bridge is relatively easy. But for the jib crane, moving perpendicular to the boom's length is complicated by the boom's rotational inertia and the boom pivot's friction. For the gantry crane, this perpendicular motion is very difficult due to the large inertia of the crane, and as stated earlier, often requires power assist. Finally, for the bridge crane, it is the large inertia of the bridge and Resulting increased rolling resistance in its trolleys that make the perpendicular motion more difficult. [13]

Another point must be mentioned here, this project contains a wide range of the mechatronics engineering fields can be applied, where the mechanical design, mathematical modeling, electrical design, programming, and the control design, will be applied, this combination of the different fields provide a good chance to the students to get the practical applications of these fields.

According to these reasons, we can see that the most useful and the widest spread as a handling system in the manufacturing fields is the jib crane. So the objective of this project is to design and control the motions of the jib crane.

1.4 Output of the Project

In this project, the Jib crane (Free standing jib crane type-Base plate mounted) will be built up, with the following specifications:

- When the crane starts the rotation, the load which is hoisted starts swinging in both directions, in plane and out of plane of the jib crane, so this causes vibrations that affect the whole system, which is undesirable. So the point is to control the torque and speed of the motor which causes the rotation motion, to get the desired position with minimum vibration of the load.

- Another output for this project that is the mechanical design of the mechanical parts which are used in its design. The electrical and control designs will be implemented in the final output.

1.5 Contents of the Report

In addition to the introduction, this report will contain 6 chapters, which are: the Theoretical Background, Mathematical Model, Mechanical Design, Electrical Design, Controller Design, and the Results and Recomendations.

Theoretical Background chapter will include the theoretical design of the whole system, and the parts that the project will contain, both; the mechanical and electrical parts. Also the requirements of the system will be explained. Another thing that will be included is the design of the prototype that was built up, and all of its features.

Mathematical model chapter will include the model for the system, and the linearization for the non-linear model, also state space representation for the model.

Mechanical design chapter, as in its name, will include the mechanical design for all parts of the system.

Electrical design chapter will include the design of the system from the electrical aspect, where all the electrical parts as motors, sensors, and AC inverters, will discussed in details.

In controller design chapter, the controller that will be used to stabilize the load through out controlling the signal that applied to the AC-inverter in order to control the motor's torque and speed will be designed as a software controller.

Finally the last chapter will include some experiments, and output results. Also some of conclusions and recommendations will be included.

1.6 Estimated Cost

Table 1.1 represents the estimated costs for the parts that will be used in building up the system, then accordingly the total cost of the whole project.

This budget is prepared according to estimated prices for each part mentioned in the table, so the grand total may have plus or minus 500 NIS.

Table (1.1): Estimated Budget of the Jib Crane Project.

Item number	Item	Price (NIS)
1	Data Acquisition Card (DAC)	4000
2	AC Inverter	1500
3	1 AC motor, 2 DC motors	600
4	Boom	500
5	Mast	100
6	Machining operations	1000
7	2 Sensors	600
8	5 limit switch	200
9	2 Gear + Chain	500
10	Personal computer	1000
11	Other expanses	1000
Grand Total		**11000 NIS**

1.7 Project Schedule

For the first semester, the introduction, prototype, and model chapters are completed, and the rest chapters and system building up are distributed throughout the second semester as shown in table (1.2).

Table (1.2): Project plan for the next semester

Week number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Mechanical Design	■	■	■	■	■	■	■									
Electrical Design						■	■	■	■	■	■					
Control Design			■	■	■	■	■	■								
Building Up the project			■	■	■	■	■	■	■	■	■	■	■	■		
Writing and printing					■	■	■	■	■	■	■	■	■	■		
Testing the output													■	■	■	
Preparing the presentation															■	■

This was the time plan, and it was modulated according to the unexpected events.

Chapter Two

Theoretical Background

- Jib Crane Prototype
- Components of Jib Crane Project

Chapter Two

Theoretical Background

In order to understand the Jib crane-system principle of work, and to obtain the general idea about the motions and the mechanisms that cause these motions, also to develop the idea about reasons that causes the vibrations in the system, a prototype is built up. This prototype and its mechanical, electrical, and control design will be discussed in this chapter. Also the whole system with all of its mechanical parts are graphically designed, these parts also will be discussed according to their function. The electrical parts as motors, sensors, and control circuits that will be used in the final design also will be discussed.

2.1 Jib Crane Prototype

As a one stage of designing a mechatronics system, a prototype for the Jib crane is built up having the same idea of design for the Jib crane, but with primary control for the three basic motions, rotation, translation, and hoisting.

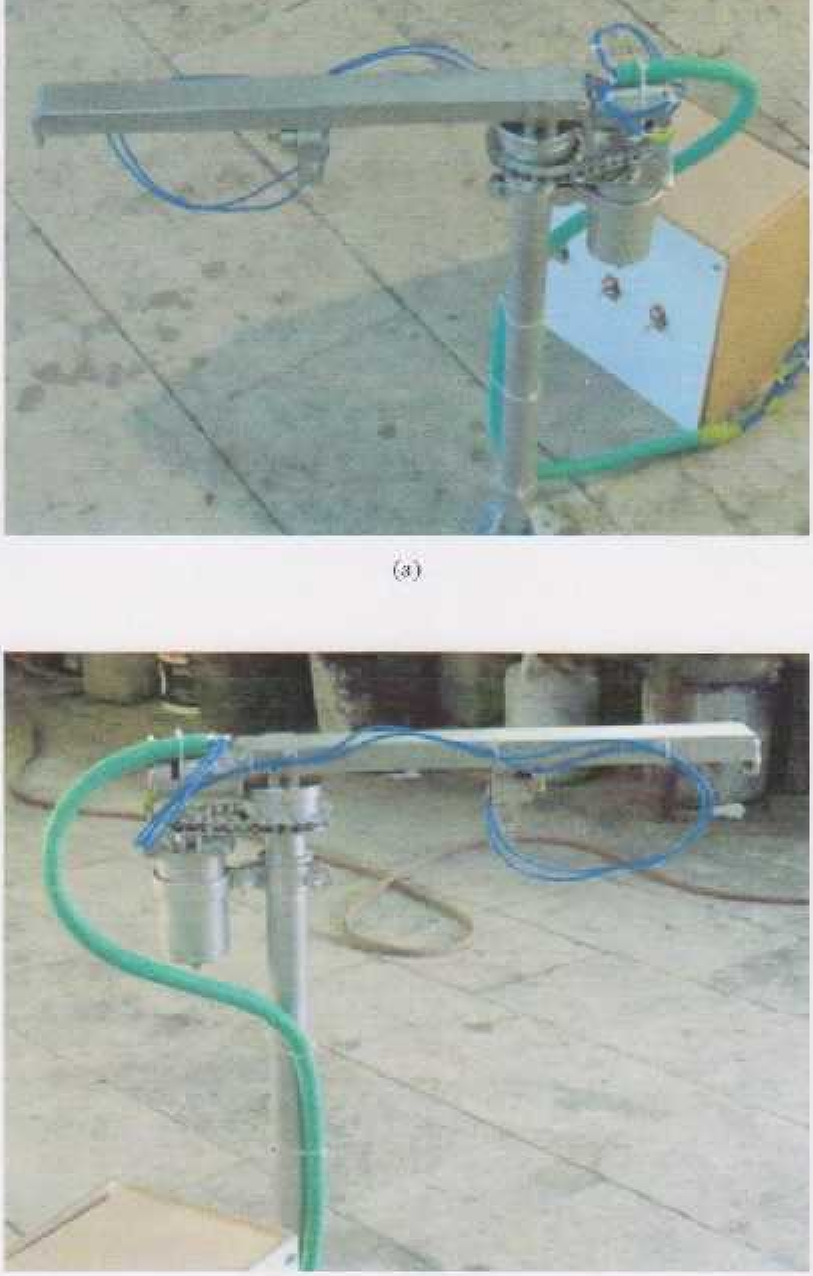
As shown in Figure (2.1), the prototype is consisted of the basic mechanical components that are similar to that in the Jib crane: mast, revolute joint, gears, motors, boom mount mechanism, boom, and the trolley.

The mechanical components that are used in the prototype will be explained later in section (2.2.1) where the parts of the Jib crane that will be designed, will be explained in details.

The mechanism that is used to translate the trolley in the prototype is the power screw mechanism, different from the used one in the Jib crane, which is the rack and pinion mechanism. The reason for using this mechanism is that, the power screw teeth will be corroded because the bending that will be produced in the boom due to high loading, so the rack and pinion mechanism will be used.

In this small prototype a three DC motors are used to generate the needed motions in the prototype, with a rated voltage $\pm 5 - to - \pm 35\ V$ for the three motors, while in the designed Jib crane, an AC motor, is used for the rotational motion, and 2 DC motors are used for both translational and hoisting operations. But their specifications don't determined yet. The use of DC motors in the prototype is due to their small size, and the control of them is simple.

The control that is used in the prototype is simple, i.e. the method depends on change the polarity of the DC motor using the toggle switch in order to change the direction of rotation. One toggle switch is used for each motor.



(a)

(b)

Figure (2.1): Jib Crane Prototype: (a) prototype and control box. (b) prototype detailed parts.

In the final Jib crane design, the control for the AC motor that is used to generate the rotational motion will be using an AC inverter, where its input signal will be from a software controller that will be designed using MATLAB. There will be two signal inputs to the controller, the first will be the desired angular position of the boom, and the second is the feedback signal from a sensor which measures the actual angular position.

AC inverter is used in order to change the output voltage according to the input voltage throughout changing the frequency, where a constant voltage-to-frequency ratio must be maintained. The output voltage from the AC inverter will be the input of the motor of the rotational motion, and this voltage signal will affect the output of the motor, and accordingly the speed of rotation. As the needed torque increases, the rotational speed will be decreased.

2.2 Components of Jib Crane project

Jib Crane consists of a horizontal load supporting boom, which is attached to a pivoting vertical column that is either free standing or building mounted. They enable lifting and lowering of a load within a fixed arc of rotation. The jib crane that will be designed in this project will be of the same components as detailed in Figure (2.2). The whole design and all of its parts are designed using the mechanical design tool box of the CATIA drawing design program.

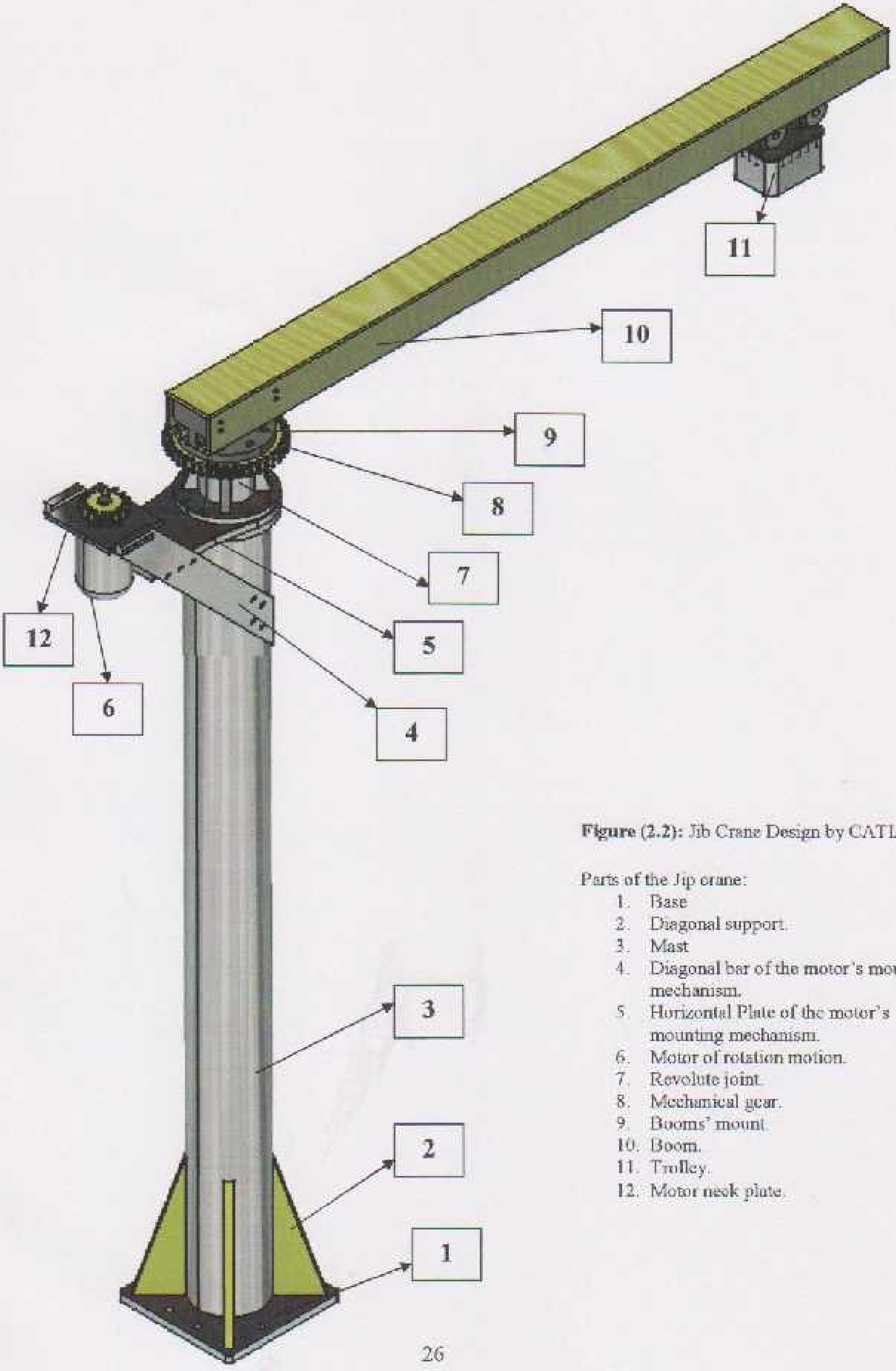


Figure (2.2): Jib Crane Design by CATIA:

Parts of the Jip crane:

1. Base
2. Diagonal support.
3. Mast
4. Diagonal bar of the motor's mounting mechanism.
5. Horizontal Plate of the motor's mounting mechanism.
6. Motor of rotation motion.
7. Revolute joint.
8. Mechanical gear.
9. Booms' mount.
10. Boom.
11. Trolley.
12. Motor neck plate.

So the following list represents the components of the system design:

2.2.1 Mechanical Parts:

- *Base*: it will be a square plate of metal welded to the mast and also connected to a supported concrete in order to provide rigid system without vibrations which produced by the rotation of the jib.

- *Mast*: it is a tube vertical cylinder of a designed thickness and material, with a height acknowledges to the desired function. It is shown clearly in Figure (2.2).

- *Lower flange*: shown in Figure (2.3.a). It represents the base of the revolute joint where the crane will be rotated through it.

- *Upper flange*: shown in Figure (2.3.b). It represents the other part of the revolute joint which is described previously.

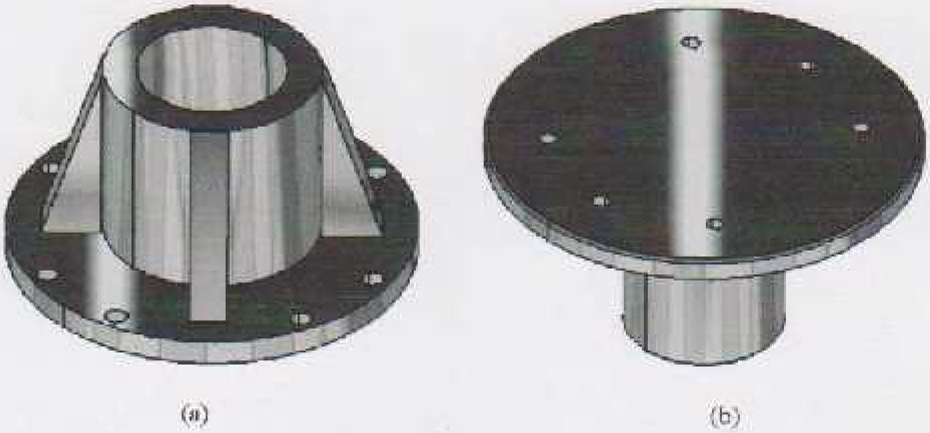
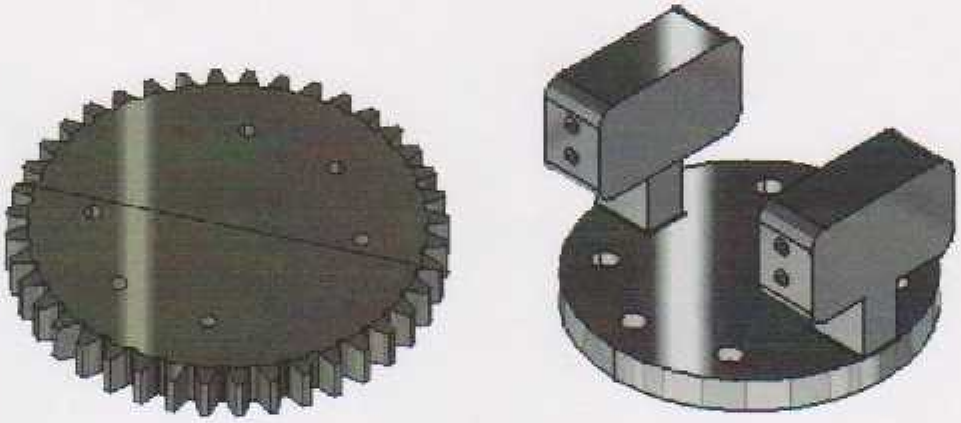


Figure (2.3): Revolute joint: (a) lower flange, (b) upper flange

- *Gears*: there are 3 gears of spur type, the first connected to the upper flange, the second is connected to the motor of rotation, and the third is connected to the motor of the translational motion. The gear that is designed by CATIA is shown in Figure (2.4.a), these gears can be modeled as needed specifications, as number of teeth, pressure angle, and module.

- *Boom mount*: shown in Figure (2.4.b). It is a mechanical part, which represents the connection mechanism between the boom and the revolute joint, and it is connected to the gear that is connected to the upper flange.

- *Boom*: it is the horizontal beam where the trolley translates. As shown in Figure (2.5), the profile of this beam will provide tracks for the rollers of the trolley. The thickness of the profile will be designed according to the maximum load and other reaction forces as the centrifugal force, also its length will be considered according to the desired request.



(a) (b)

Figure (2.4): Mechanical parts: (a) spur gear, (b) boom mount part.

Figure (2.5): Designed boom of the Jib Crane.

- *Rotational motor mount mechanism:* it represents a mechanical mechanism used to mount the motor of rotational motion. As shown in Figure (2.6); it is consists of a two diagonal bars that are connected to a horizontal plat which is fixed over the mast. Upon the diagonal bars, the motor is positioned throughout a horizontal plate (motor neck plate) which it is connected to the diagonal bars.

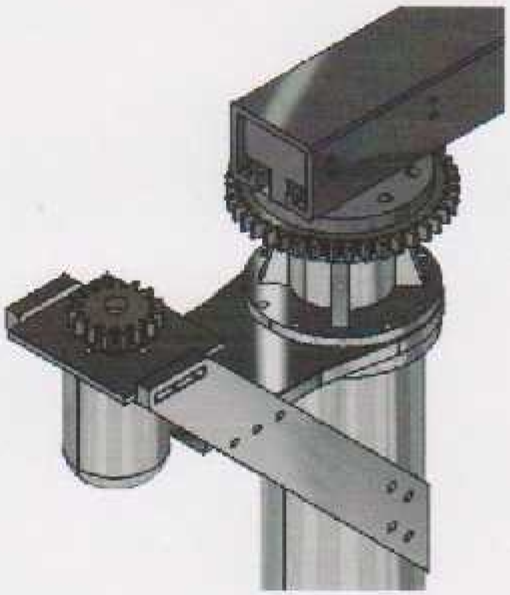


Figure (2.6): Motor mount mechanism.

- *Trolley*: it is a subsystem combines from electrical and mechanical parts, where its function is to translate along the boom of the crane, and to hoist the load up and down. The whole trolley as a closed box is shown in Figure (2.7). As shown in the Figure, it is combined from the following mechanical parts:

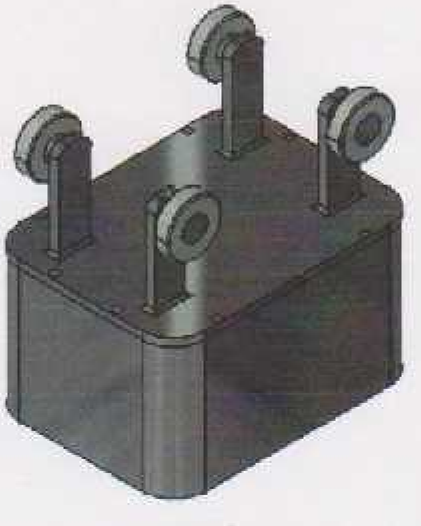


Figure (2.7): Designed Trolley of the Jib crane.

-Box: where the hoisting motor (shown in Figure (2.8.a)) is mounted inside it, and it has a square pocket at its bottom as a passport for the hoisting cable.

-Cover: it provides the connecting mechanism between the trolley's box, and the boom throughout the rollers. That is clear in Figure (2.7).

-Pulley: it will be connected to the hoisting motor, and will be selected according to the hoisting cable, length and diameter. Its design is shown in Figure (2.8.b).

-Rollers: they will be metal rings with internally fitted bearings, connected to the trolley in order to roll over the boom, as they are shown in Figure (2.7).

(a) (b)

Figure (2.8): Trolley parts: (a) Hoisting motor, (b) Pulley.

-Rack and pinion: it represents the translational mechanism for the trolley along the boom, where the pinion is connected to the motor of the translation motion, and the rack is fixed on the boom, so as the motor rotates, the trolley then will be translated.

2.2.2 Electrical Parts

As a mechatronics system, and as provided in the title of this project, this system will be power assisted, and according to that, there will be electrical parts; either provides input signals or receives output signals from the control system; these components are listed and expressed as follows:

- *Motors*: there will be 1 AC, and 2 DC motors which are used to provide the motion in the different axes, and these motors will be controlled by contactors, and an AC controller (inverter) will be used to control the output torque, and angular velocity from the rotational motion's motor as previously mentioned.

- *Sensors*: in order to control the rotational speed and output torque from the motor of the rotational motion; a feedback signals are needed, the sensors will provide these signals. There are 2 sensors, the first is to determine the position of the trolley with respect to the mast, and the second one will be used to specify the angular position of the boom.

- *Micro switches*: for the rotational and translational motions, the motors must be stopped at some position, to prevent the parts from collides each other. Five limit switches will be used, 2 for translational motion, 2 for rotational motions, and one for hoisting.

-*Data Acquisition Card (DAC)*: the output signals from the sensors are analog. These signals are needed to be conditioned in order to be input to the controller, which is software, and it is built in a non-embedded computer using MATLAB, so these signals are needed to be converted into digital. The output signal from the controller is digital, so this signal must be converted into analog to be input to the AC inverter. DAC is needed to do all the previous operations.

-Non-embedded Computer (PC): this processor is required to build up the software controller as previously described, where the controller that is expected to be used is state feedback controller and DAC will be attached to this PC.

-Control panel: it will contain contactors, AC controller (Inverter), and other electrical components, also the operating switches by which the discrete motions will be controlled.

All of these parts will be mechanically and electrically designed, and all of their specifications will be provided later according to the needed design.

Chapter Three

Mathematical Modeling of the Jib Crane

- Derivation of the Model
- State-Space Model of the Crane

Chapter Three

Mathematical Modeling of the Jib Crane

This chapter contains the mathematical model of the Jib crane. This model will be derived using the Lagrange approach. As known the Jib crane is a physical system, as a result, the equations of this model are nonlinear. The non-linear output model will be then linearized using the Taylor series method, a brief explanation to this principle also will be included. In order to do the linearization, the operating points are needed. The method that is used to determine these points which is Optimization method also briefly expressed. The state space model of the system will be shown, and finally, from this representation, the transfer functions that relate the outputs and inputs in the system will be determined.

3.1 Derivation of the Model

Lagrange approach will be used to derive the equation of motion of the jib crane. This approach depends on the conservation of energy principle. The Lagrange's Equation is given in two forms (a) and (b), as shown in Equation (3.1):

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} + \frac{\partial V}{\partial q_i} = F_i \text{ ....}(a)$$

$$\frac{d}{dt}\left(\frac{\partial \ell}{\partial \dot{q}_i}\right) - \frac{\partial \ell}{\partial q_i} = F_i \text{ ....}(b)\text{ ,} \qquad i = 1,2,3,\ldots,n \qquad (3.1)$$

$$\ell = T - V \text{ ....}(c)$$

Where:

- T: Kinetic energy of the system
- V: Potential energy of the system
- F_i: Non-conservative generalized forces (external applied forces in the direction of generalized coordinates q_i).
- q_i: Generalized coordinates, or the linear and angular displacements that are used to specify the location of the masses and inertias.

Form (b) of the Lagrange Equation can be derived from form (a), by the partial derivation of Equation (3.1.c), then substitution in Equation (3.1.a). [2]

The terms of Equation (3.1.b) can be expressed as:

$\frac{d}{dt}\left(\frac{\partial \ell}{\partial \dot{q}}\right)$: represents the acceleration term.

$\frac{\partial \ell}{\partial q}$: represents the gravity term.

As shown in Figure (3.1), the crane can be modeled as a vertical cylinder, horizontal boom, and the trolley, this model provides a clear view which helps in developing the equation of motion. [12]

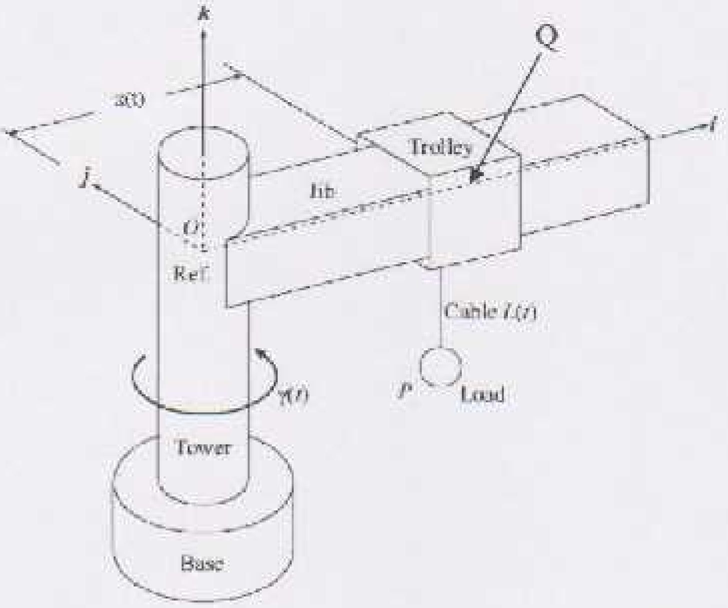


Figure 3.1: A 3D model of a rotary crane

Now the position vectors of the load and the trolley will be derived, assuming that the trolley is positioned at point Q, which is located at displacement x(t) from the tower center as in Figure (3.1). The coordinate system that is considered to develop the position vectors is shown in Figure (3.2), so these vectors can be written, as:

$$\overrightarrow{r_L} = \{[x - L\cos(\theta)\sin(\phi)]i + [L\sin(\theta)]j - [L\cos(\theta)\cos(\phi)]k\} \quad (3.2)$$

$$\overrightarrow{r_T} = x_i$$

Where $\overrightarrow{r_L}$ is the position vector for the load represented in the three coordinates i, j, k, $\overrightarrow{r_T}$ is the position vector of the trolley, and it is represented in the i coordinate, $\theta(t)$ is the out-of-plane oscillation angle, $\phi(t)$ is the in-of-plane oscillation angle, and $L(t)$ is load line length.

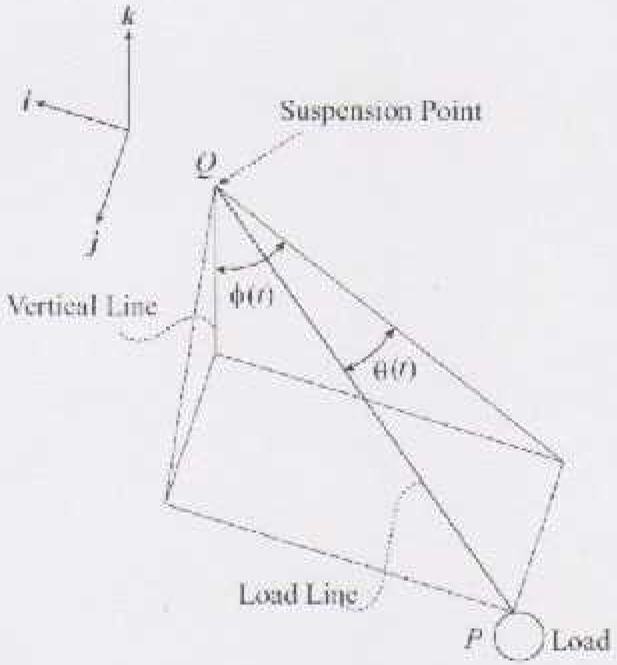


Figure 3.2: Oscillation angles of the load: $\phi(t)$ and $\theta(t)$.

The terms in Equation (3.2) are the position components in x, y, and z coordinates, where the load line is projected on the three coordinates.

The velocity of the trolley and the load can be found from the substitution in:

$$\overrightarrow{v(t)} = \frac{dr}{dt} + \vec{\omega} \times \vec{r} \tag{3.3}$$

$$\vec{\omega} = \dot{\gamma} k \tag{3.4}$$

The velocity vector $\overrightarrow{v(t)}$ consists of two terms, the first is the derivative of the position vector, and the second is the tanjential velocity due to aguler velcoty $\omega(t)$ which is the angular velocity of the crane.

The kinetic and potential energies are given as:

$$T = \frac{1}{2} m v^2 \tag{3.5}$$

$$T = \frac{1}{2} J \omega^2 \tag{3.6}$$

$$V = mgh. \tag{3.7}$$

Where Equations (3.5) and (3.6) provide the kinetic energy for linear velocity, and angular velocity respectively, and Equation (3.7) provides the potential energy.

Where:

- m: The mass of moving body.
- J: The inertia of the moving body.
- h: The vertical height of the body from a reference frame, and in this system is in the Z direction.

Since the velocities of the load and trolley are vectors, so their kinetic and potential energies become:

$$T = \frac{1}{2} m \dot{r_L} . \dot{r_L} + \frac{1}{2} M \dot{r_T} . \dot{r_T} + \frac{1}{2} J_o (\dot{\gamma})^2 \qquad (3.8)$$

$$V = -mgL \cos(\theta)\cos(\phi) \qquad (3.9)$$

Where J_o: The moment of inertia of the boom about Z-axis.

m: The load mass (6Kg).

M: The trolley mass (4Kg).

So the kinetic energy of the system can be given as:

$$
\begin{aligned}
T =& \frac{1}{2} m\left(\dot{L}\sin(\theta) + x\dot{\gamma} + L\cos(\theta)(\dot{\theta} - \dot{\gamma}\sin(\theta))\right)^2 \\
&+ \frac{1}{2} m\left(\dot{x} - \dot{L}\sin(\phi)\cos(\theta) - L(\dot{\gamma}\sin(\theta) - \dot{\theta}\sin(\theta)\sin(\phi) + \dot{\phi}\cos(\theta)\cos(\phi))\right)^2 \\
&+ \frac{1}{2} m\left(-\dot{L}\cos(\phi)\cos(\theta) + L(\dot{\theta}\sin(\theta)\cos(\phi) + \dot{\phi}\cos(\theta)\sin(\phi))\right)^2 \\
&+ \frac{1}{2} M((\dot{x})^2 + x^2(\dot{\gamma})^2) + \frac{1}{2} J_0(\dot{\gamma})^2
\end{aligned}
\tag{3.10}
$$

And the potential energy is given in Equation (3.9).

Now we can obtain the Lagrange term if we substitute the kinetic and potential energies in Equation (3.1.c), that is:

$$\ell = T - V$$

So the Lagrange can be given as:

$$
\begin{aligned}
\ell =& \frac{1}{2} m\left(\dot{L}\sin(\theta) + x\dot{\gamma} + L\cos(\theta)(\dot{\theta} - \dot{\gamma}\sin(\theta))\right)^2 \\
&+ \frac{1}{2} m\left(\dot{x} - \dot{L}\sin(\phi)\cos(\theta) - L(\dot{\gamma}\sin(\theta) - \dot{\theta}\sin(\theta)\sin(\phi) + \dot{\phi}\cos(\theta)\cos(\phi))\right)^2 \\
&+ \frac{1}{2} m\left(-\dot{L}\cos(\phi)\cos(\theta) + L(\dot{\theta}\sin(\theta)\cos(\phi) + \dot{\phi}\cos(\theta)\sin(\phi))\right)^2 \\
&+ \frac{1}{2} M((\dot{x})^2 + x^2(\dot{\gamma})^2) + \frac{1}{2} J_0(\dot{\gamma})^2 + mgL\cos(\phi)\cos(\theta)
\end{aligned}
\tag{3.11}
$$

The generalized coordinates in this system are chosen to be the physical variables that affect the motion of the system, and accordingly causes the vibration in it, so the generalized coordinates vector is $\vec{q} = \{\theta, \phi, x, \gamma\}$,so the generalized forces corresponding to the generalized displacement vector are $\vec{F} = \{0, 0, F_x, T_\gamma\}$ respectively, where F_x is the force that is provided from the motor that translates the trolley along the boom, and T_γ is the torque provided from the motor of rotational motion. Hence, the partial derivation of the Lagrange with respect to the generalized coordinates then, using Lagrange Equation which is expressed in Equation (3.1.b), that is:

$$\frac{d}{dt}\left(\frac{\partial \ell}{\partial \dot{q}_i}\right) - \frac{\partial \ell}{\partial q_i} = F_i$$

Considering a constant cable length, such that $\frac{dL}{dt} = 0$, the following equations of motion for the system according to the generalized coordinates and generalized forces and moments, can be obtained:

$$\begin{aligned}&\ddot{\theta} + (x/L\cos(\theta) - \cos(\theta)^2\sin(\theta) - \sin(\theta)^2\sin(\phi))\ddot{\gamma} + (\sin(\theta)\sin(\phi))\ddot{x}/L\\&+(\dot{x}\dot{\gamma}/L)\cos(\theta) + (x(\dot{\gamma})^2 L)\cos(2\theta) - (1/4)(\dot{\gamma})^2 L^2\sin(4\theta) - (1/2)(\dot{\gamma})^2\sin(2\theta)\\&-\dot{\gamma}\dot{\phi}\cos(\theta)^2\cos(\phi) + (1/2)(\dot{\phi})^2\sin(2\theta) + (g/L)\sin(\theta)\cos(\phi) = 0\end{aligned} \quad (3.12.a)$$

$$L\ddot{\phi}\cos(\theta)^2-(\cos(\theta)\cos(\phi))\ddot{x}+((1/2)L\sin(2\theta)\cos(\phi))\ddot{\gamma}$$
$$+L\dot{\gamma}\dot{\theta}(\cos(\phi)\cos(\theta)^2)-L\dot{\phi}\dot{\theta}\sin(2\theta)+L(\dot{\phi})^2\cos(\phi)\sin(\theta) \qquad (3.13.a)$$
$$+g\cos(\theta)\sin(\phi)=0$$

$$(m+M)\ddot{x}-mL\sin(\theta)\ddot{\gamma}+(mL\sin(\theta)\sin(\phi))\ddot{\theta}-(mL\cos(\theta)\cos(\phi))\ddot{\phi}$$
$$-(M+m)x(\dot{\gamma})^2-m(\dot{\gamma})^2L\dot{\theta}\cos(\theta)-2m\dot{\gamma}\dot{\theta}L\cos(\theta)+2m\dot{\theta}\dot{\phi}L\sin(\theta)\cos(\phi) \qquad (3.14.a)$$
$$+m(\dot{\theta})^2L\cos(\theta)\sin(\phi)+mL(\dot{\phi})^2\cos(\theta)\sin(\phi)=F_x$$

$$((m+M)(x)^2+J_0+mL^2\sin(\theta)^2-2mxL\cos(\theta)\sin(\theta)+mL^2\cos(\theta)^2\sin(\theta)^2)\ddot{\gamma}$$
$$-mL\sin(\theta)\ddot{x}+(mL^2\sin(\theta)^2\sin(\phi)-mL^2\cos(\theta)^2\sin(\theta)+xmL\cos(\theta))\ddot{\theta}$$
$$+(1/2)mL^2\sin(2\theta)\cos(\phi)\ddot{\phi}+2mL^2(\dot{\theta})^2\cos(\theta)\sin(2\theta)-mLx(\dot{\theta})^2\sin(\theta)$$
$$-mL^2(\dot{\theta})^2\sin(2\theta)\sin(\phi)-2mLx\dot{\gamma}\dot{\theta}\sin(2\theta)+(1/2)m\dot{\gamma}L^2\dot{\theta}\sin(4\theta) \qquad (3.15.a)$$
$$+mL^2\dot{\gamma}\dot{\theta}\sin(2\theta)-2mL^2\dot{\theta}\dot{\phi}\sin(\theta)^2\cos(\phi)+mL^2\dot{\phi}\dot{\theta}\cos(\theta)^2\cos(\phi)$$
$$-(1/2)mL^2(\dot{\phi})^2\sin(2\theta)\sin(\phi)+2(M+m)x\dot{x}\dot{\gamma}-m\dot{x}\dot{\gamma}L\cos(\theta)\sin(\theta)=T_\gamma$$

If these equations are considered deeply, we can see that for Equations (3.12.a), (3.13.a) where the generalized coordinates are $\theta(t)$ and $\phi(t)$ respectively; are affected by the position, speed and acceleration of the trolley which is represented by x(t), and its derivatives, and also by the angular velocity and angular acceleration which is represented by the derivatives of $\gamma(t)$ supported from the motor of the rotational motion.

Other thing must be mentioned, which is the Corulus acceleration terms, which produced from the multiplying of two velocities in two different axes, this term is presented in all equations, which means that there is a relative motions with respect to other, that is during the motion of the load on the trolley in one axis, it also that there is motion of the load, or trolley in the other axis.

3.2 State-Space Model of the Crane

In order to make easier dealing with Equation (3.12.a) through to Equation (3.15.a), and to obtain the transfer functions of the linear model of the system, so the system will be presented in the state space model. Since the states are usually selected according to be equal to the number of degrees of freedom or number to the energy-storage elements in the system, the states in the dynamic mechanical systems are selected to be the displacements and their derivatives, and because each of the four equations is of second order, so the states will be eight, which are:

$$\begin{Bmatrix} x_1 = \theta(t) \\ x_2 = \phi(t) \\ x_3 = x(t) \\ x_4 = y(t) \\ x_5 = \dot{\theta}(t) \\ x_6 = \dot{\phi}(t) \\ x_7 = \dot{x}(t) \\ x_8 = \dot{y}(t) \end{Bmatrix} \tag{3.16}$$

According to these states, there derivatives will be as shown in Equation (3.17):

$$\begin{Bmatrix} \dot{x}_1 = x_5 \\ \dot{x}_2 = x_6 \\ \dot{x}_3 = x_7 \\ \dot{x}_4 = x_8 \\ \dot{x}_5 = \ddot{\theta}(t) \\ \dot{x}_6 = \ddot{\phi}(t) \\ \dot{x}_7 = \ddot{x}(t) \\ \dot{x}_8 = \ddot{y}(t) \end{Bmatrix} \tag{3.17}$$

Substituting these states into Equations (3.12a), (3.13a), (3.14a), (3.15a), so they will become as follows:

$$\dot{x}_5 + ((x_3/L)\cos(x_1) - \cos(x_1)^2\sin(x_1) - \sin(x_1)^2\sin(x_2))\dot{x}_8$$
$$+1/L(\sin(x_1)\sin(x_2))\dot{x}_7 + x_7x_8L\cos(x_1) + x_3(x_8)^2L\cos(2x_1)$$
$$-(1/4)(x_8)^2L^2\sin(4x_1) - (1/2)L^2(x_8)^2\sin(2x_1) - x_8x_6\cos(x_1)^2\cos(x_2)$$
$$+(1/2)(x_2)^2\sin(2x_1) + (g/L)\sin(x_1)\cos(x_2) = 0 \qquad (3.12.b)$$

$$L\dot{x}_6\cos(x_1)^2 - (\cos(x_1)\cos(x_2))\dot{x}_7 + ((1/2)L\sin(2x_1)\cos(x_2))\dot{x}_8$$
$$+Lx_8x_5(\cos(x_2)\cos(x_1)^2) - Lx_8x_5\sin(2x_1) + L(x_6)^2\cos(x_2)\sin x_1$$
$$+g\cos(x_1\sin(x_2) = 0 \qquad (3.13.b)$$

$$(m+M)\dot{x}_7 - (mL\sin(x_1))\dot{x}_8 + (mL\sin(x_1)\sin(x_2))\dot{x}_5$$
$$-(mL\cos(x_1)\cos(x_2))\dot{x}_6 - -m(x_8)^2Lx_5\cos(x_1)$$
$$(M+m)x_3(x_8)^2 - 2mx_8x_5L\cos(x_1) + 2mx_5x_6L\sin(x_1)\cos(x_2)$$
$$+m(x_5)^2L\cos(x_1)\sin(x_2) + mL(x_6)^2\cos(x_1)\cos(x_2) = F_x \qquad (3.14.b)$$

$$((m+M)(x_3)^2 + J_0 + mL^2\sin(x_1)^2 - 2mx_3L\cos(x_1)\sin(x_1)$$
$$+mL^2\cos(x_1)^2\sin(x_1)^2)\dot{x}_8 - mL\sin(x_1)\dot{x}_7 + (-mL^2\sin(x_1)^2\sin(x_2)$$
$$-mL^2\cos(x_1)^2\sin(x_1) + mL\cos(x_1)x_3)\dot{x}_5 - ((1/2)mL^2\sin(2x_1)\cos(x_2))\dot{x}_6$$
$$+2mL^2(x_5)^2\cos(x_1)\sin(2x_1) - mLx_3(x_5)^2\sin(x_1) - mL^2(x_5)^2\sin(2x_1)\sin(x_2)$$
$$-2mLx_3x_8x_5\sin(2x_1) + (1/2)mL^2x_8x_5\sin(4x_1) - mL^2x_8x_5\sin(2x_1)$$
$$-2mL^2x_5x_6\sin(x_1)^2\cos(x_2) + mL^2x_6x_5\cos(x_1)^2\cos(x_2)$$
$$(1/2)mL^2(x_6)^2\sin(2x_1)\sin(x_2) + 2(M+m)x_3x_7x_8 - mLx_7x_8\cos(x_1)\sin(x_1) = T_y \qquad (3.15.b)$$

From these equations the following states-derivatives: $\dot{x}_5, \dot{x}_6, \dot{x}_7, \dot{x}_8$, can be obtained, since they are of long and non-linear terms the state space representation will be shown after the linearization.

3.2.1 Linearization

In order to use the equations of motion to obtain the transfer functions, and designing the controller which will control the crane motions, they must be linearized. The method that is used to linearize these Equations is the Taylor series method. However, before showing the linear state space model of the system, this principle will be expressed.

3.2.1.1 Linearization Around the Stationary State

If it is assumed that x (t) changes only in the neighborhood of the stationary state by Δx, the Taylor series expansion is used to calculate $\Delta\dot{x} = f(\Delta x, \Delta u)$, so the Taylor series can be expressed as:

$$f(\Delta x, \Delta u) = f(\Delta \bar{x}, \Delta \bar{u}) + \left.\frac{\partial f}{\partial x}\right|_{x=\bar{x}, u=\bar{u}} \Delta x + \left.\frac{\partial f}{\partial u}\right|_{x=\bar{x}, u=\bar{u}} \Delta u + \ldots,$$

$$\Delta \dot{x}_i = \sum_{k=1}^{n} \left.\left(\frac{\partial f_i}{\partial x_k}\right)\right|_{x=\bar{x}, u=\bar{u}} \Delta x_k + \sum_{j=1}^{m} \left.\left(\frac{\partial f_i}{\partial u_j}\right)\right|_{x=\bar{x}, u=\bar{u}} \Delta u_j$$

$$\Delta \dot{x} = A \, \Delta x + B \, \Delta u. \tag{3.18}$$

Where A is System matrix, and B is the Input matrix, and they are given as:

$$A = \sum_{k=1}^{n} \left.\left(\frac{\partial f_i}{\partial x_k}\right)\right|_{x=\bar{x}, u=\bar{u}} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & \vdots & \cdots & \vdots \\ \frac{\partial f_m}{\partial x_1} & \frac{\partial f_m}{\partial x_2} & \cdots & \frac{\partial f_m}{\partial x_n} \end{bmatrix}_{x_0, u_0} \tag{3.19}$$

$$B = \sum_{j=1}^{m} \left.\left(\frac{\partial f_i}{\partial u_j}\right)\right|_{x=\bar{x}, u=\bar{u}} = \begin{bmatrix} \frac{\partial f_1}{\partial u_1} & \frac{\partial f_1}{\partial u_2} & \cdots & \frac{\partial f_1}{\partial u_n} \\ \frac{\partial f_2}{\partial u_1} & \frac{\partial f_2}{\partial u_2} & \cdots & \frac{\partial f_1}{\partial u_n} \\ \vdots & \vdots & \cdots & \vdots \\ \frac{\partial f_m}{\partial u_1} & \frac{\partial f_m}{\partial u_2} & \cdots & \frac{\partial f_m}{\partial u_n} \end{bmatrix}_{x_0, u_0} \tag{3.20}$$

Where the system matrix is called Jacobean, and it is equal to Equation (3.19). [16]

Also the matrices of the output equation which are C and D matrices can be given as:

$$C = \sum_{k=1}^{n} \left(\frac{\partial y_i}{\partial x_k} \right)\bigg|_{x=\bar{x},\, u=\bar{u}} = \begin{bmatrix} \frac{\partial y_1}{\partial x_1} & \frac{\partial y_1}{\partial x_2} & \cdots & \frac{\partial y_1}{\partial x_n} \\ \frac{\partial y_2}{\partial x_1} & \frac{\partial y_2}{\partial x_2} & \cdots & \frac{\partial y_1}{\partial x_n} \\ \vdots & \vdots & \cdots & \vdots \\ \frac{\partial y_m}{\partial x_1} & \frac{\partial y_m}{\partial x_2} & \cdots & \frac{\partial y_m}{\partial x_n} \end{bmatrix}_{x_0, u_0} \tag{3.21}$$

$$D = \sum_{j=1}^{m} \left(\frac{\partial y_i}{\partial u_j} \right)\bigg|_{x=\bar{x},\, u=\bar{u}} = \begin{bmatrix} \frac{\partial y_1}{\partial u_1} & \frac{\partial y_1}{\partial u_2} & \cdots & \frac{\partial y_1}{\partial u_n} \\ \frac{\partial y_2}{\partial u_1} & \frac{\partial y_2}{\partial u_2} & \cdots & \frac{\partial y_1}{\partial u_n} \\ \vdots & \vdots & \cdots & \vdots \\ \frac{\partial y_m}{\partial u_1} & \frac{\partial y_m}{\partial u_2} & \cdots & \frac{\partial y_m}{\partial u_n} \end{bmatrix}_{x_0, u_0} \tag{3.22}$$

Where $\bar{x} = x_0, \bar{u} = u_0$ are the operating points of the physical system. So these points must be determined. There is a definition must be known, that is a point, $x_0 = \bar{x}$ is an *equilibrium point* of:

$$\dot{x} = f(x, u)$$

If $$f(x_0, u_0) = 0$$

Where the zero on the right hand side of the equation is a vector of zeros, that is the same dimension as x and f(x,u). [16]

The Equation $\dot{x} = f(x, u)$ can be written in vector form as:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} f_1(x_1, x_2, \dots\dots, x_n, u_1, u_2, \dots\dots, u_n) \\ f_2(x_1, x_2, \dots\dots, x_n, u_1, u_2, \dots\dots, u_n) \\ \vdots \\ \vdots \\ \vdots \\ fm(x_1, x_2, \dots\dots, x_n, u_1, u_2, \dots\dots, u_n) \end{bmatrix} \tag{3.23}$$

So in order to solve for the operating or equilibrium points, Equation (3.23) must be equated with zero.

3.2.1.2 Calculating the Operating Points of the Jib Crane System

As previously mentioned, that the Jacobean matrix represents the system matrix. This matrix and the input matrix are evaluated at the operating or equilibrium points for both; the states, and the interested inputs. These points are evaluated as expressed in section 3.3.1.1.

The Equation $f(x_0, u_0) = 0$ is a non-linear algebraic equation, so in order to solve it, the *Newton-Raphson* and *Levenberg-Marquardt* methods must be used to solve for its roots, which are the operating points (x_0, u_0). These methods are numerical methods where they need numerical iteration, so this method needs a computer to do these numerical iterations with supporting software. MATLAB software has a toolbox for optimization, which is a process depends on the numerical iteration in order to get the most optimal and perfect values of the operating points at which the system will be linearized. The optimization term can be defined as

"In mathematics, the term **optimization**, or **mathematical programming**, refers to the study of problems in which one seeks to minimize or maximize a real function by systematically choosing the values of real or integer variables from within an allowed set". [13]

In our problem we are seeking for minimizing the effect of the states that represent the in-of-plane and out-of-plane angles and their derivatives.

Commonly, the Newton-Raphson method is used to solve for the roots of complicated functions, and it uses the numerical iteration method. This iterative process follows a set of guidelines to approximate one root, considering the function, its derivative, and an initial x-value. The Newton-Raphson method uses an iterative process to approach one root of a function. The specific root that the process locates depends on the initial, arbitrarily chosen x-value. The following equation is used in the iteration process:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \tag{3.24}$$

Here, x_n is the current known x-value, $f(x_n)$ represents the value of the function at x_n, and $f'(x_n)$ is the derivative (slope) at x_n . x_{n+1} represents the next x-value that must be found. Essentially, $f'(x)$, the derivative represents $f(x)/dx$, $(dx = \Delta x)$. Therefore, the term $f(x)/f'(x)$ represents a value of dx. So:

$$\frac{f(x)}{f'(x)} = \frac{f(x)}{f(x)/\Delta x} = \Delta x \tag{3.25}$$

As more iteration is done, dx will be closer to zero. [15]

The operating points in Jib crane problem are determined for all states and for the two inputs, which represents the input torque T_y from the motor that provides the rotational motion, and the input force from the translational motion. Since x is variable according to the desired position, the matrices A, B, C, and D are determined at the critical point, which is the maximum distance from the center of rotation, and it is assumed to be equal 1m.

The orders that are used to solve for the operating points using the optimal-toolbox under MATLAB, are:

- ***fsolve***: this order is used to solve for the roots of nonlinear equations.[15]

- ***fminsearch***: finds the minimum of a scalar function of several variables, starting at an initial estimate. This is generally referred to as unconstrained nonlinear optimization. [15]

This operation (numerical iteration) needs starting values for the operating points. These points are selected arbitrarily with considering the physical nature of the system or through out an information from the system. According to these starting values, the operating points can be convergent or divergent.

In the Jib crane system, as mentioned previously, it is assumed that the linearization will be at the critical point which is at x = 1m, and the in-of-plane and out-of-plane angles are needed to be minimized. Other point must be mentioned that is the angular displacement $\gamma(t)$, which is represented by x_4 state, doesn't resulted in the equations which are Equations (3.7),(3.8),(3.9),(3.10), so any starting point of this state will be the same value of the operating point for this state, so it is assumed to be equal 1. From this information the starting values are assumed to be:

$$\begin{Bmatrix} x_1 = 0.1 & x_5 = 0.1 \\ x_2 = 0.1 & x_6 = 0.1 \\ x_3 = 0.98 & x_7 = 0.1 \\ x_4 = 0.1 & x_8 = 0.1 \end{Bmatrix} \text{ and } \begin{Bmatrix} u_1 = 0 \\ u_2 = 0 \end{Bmatrix}, \text{ where} \begin{Bmatrix} u_1 = F_x \\ u_2 = T_r \end{Bmatrix}$$

So the calculated operating points are as follow:

$$\begin{Bmatrix} x_1 = 0 & x_5 = 0 \\ x_2 = 0 & x_6 = 0 \\ x_3 = 0.98 & x_7 = 0 \\ x_4 = 0.1 & x_8 = 0 \end{Bmatrix} \text{ and } \begin{Bmatrix} u_1 = 0 \\ u_2 = 0 \end{Bmatrix}, \text{ where} \begin{Bmatrix} u_1 = F_z \\ u_2 = T_r \end{Bmatrix}$$

According to these equilibrium points the system matrix (A), and the input matrix (B), are found to be:

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -101.496 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -117.72 & 0 & 0 & 10 & 0 & 0 & 0 \\ 0 & -49.05 & 0 & 0 & 5 & 0 & 0 & 0 \\ 13.646 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad (3.26)$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -0.0093 \\ 0.0333 & 0 \\ 0.0167 & 0 \\ 0 & 0.0015 \end{bmatrix} \qquad (3.27)$$

Now the system and the input matrices are obtained; so from these matrices, and those of the output, which are matrices C, and D, we can get the transfer functions of the system depending on the considered states to be the outputs.

3.2.2 Transfer Functions of the Jib Crane System

As mentioned before, there are three input motors, which are rotation, translation, and hoisting, but two inputs are considered in the modeling, which are the rotation, and translation. In this system, all states can be considered as outputs, or at least the states that represent the displacements which are in-of-plane, out-of-plane, translational, and rotational displacements, so we can get the transfer functions between the inputs and these outputs. However there are two physical outputs that are interested in the system, which are the translational displacement represented in state x_3, and rotational velocity represented in state x_8 so these outputs:

$$\begin{aligned} y_1 &= x_3 \\ y_2 &= x_4 \end{aligned} \tag{3.28}$$

Accordingly there are two output matrices, which they are:

$$C = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{3.29}$$

Since the outputs are just the translational displacement, and rotational speed (Equation (3.28)), the feed-forward matrix (matrix D) is zero for the both outputs.

Now the state space model for the system can be written by substitution the matrices A, B, C, and D in Equation (3.30). [1]

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned} \tag{3.30}$$

According to these two state space models (considering two output matrices), we can get the transfer functions in this system, using the following Equation:

$$G(s) = \frac{Y(s)}{U(s)} = C(sI - A)^{-1}B + D \tag{3.31}$$

According to the dimensions of the matrix A which is 8×8, and that of C, which is 1×8, and of B, which is 8×2; it can be found that the output transfer functions will be four, which they are shown in Equations (3.32), (3.33), (3.34), and (3.35). [1]

$$G_1(s) = \frac{X(s)}{F(s)} = \frac{0.01667\,s^2 + 0.327}{s^4 + 117.7\,s^2} \tag{3.32}$$

$$G_2(s) = \frac{X(s)}{T_y(s)} = \frac{-0.04637\,s^3 - 0.9097\,s}{s^6 + 219.2\,s^4 + 1.195e004\,s^2} \tag{3.33}$$

$$G_3(s)=\frac{\gamma(s)}{T_\gamma(s)}=\frac{0.001546\ s^2+0.03032}{s^3+101.5\ s} \tag{3.34}$$

$$G_4(s)=\frac{\gamma(s)}{F(s)}=0 \tag{3.35}$$

Where:

- $G_1(s)$: is the transfer function between the input from the motor of translation, and the linear displacement output.
- $G_2(s)$: is the transfer function between the input from the motor of rotation, and the linear displacement output.
- $G_3(s)$: is the transfer function between the input from the motor of rotation, and the rotational speed output.
- $G_4(s)$: is the transfer function between the input from the motor of translation, and the rotational speed output, and it is zero, this is logic because there is no effect from the motor of translational motion on the rotational speed.

In order to have an idea about the effect of the rotational speed, and the trolley position as inputs, on the in-of-plane, and out-of-plane angles as outputs which are $\phi(t)$ and $\theta(t)$ respectively, the transfer functions that relates these outputs to those inputs must found. The output matrices will be:

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{3.36}$$

And as previously mentioned, D matrix is zero, and depending on the dimensions of the matrices, A, B, and C, there will be four transfer functions. So using Equation (3.31) these transfer functions will be:

$$G_1(s) = \frac{\theta(s)}{F(s)} = 0 \tag{3.37}$$

$$G_2(s) = \frac{\theta(s)}{T_r(s)} = \frac{-0.009274}{s^2 + 101.5} \tag{3.38}$$

$$G_3(s) = \frac{\phi(s)}{F(s)} = \frac{0.03333}{s^2 + 117.7} \tag{3.39}$$

$$G_4(s) = \frac{\phi(s)}{T_r(s)} = \frac{-0.09274\ s}{s^4 + 219.2\ s^2 + 1.195e004} \tag{3.40}$$

As shown in Equations (3.37), through to (3.40), the input from the motor of translational motion don't affect the out-of-plane angle, it just affect on the in-of-plane angle, whereas the input from the motor of rotational motion affects both of them, and this agrees with our consideration, that is the control of the rotational motion motor can be enough to decrease these angles, if and only if a discrete motions is applied to the system.

Chapter Four

Mechanical Design

- Introduction
- Design Analysis
- Bearings Selection
- Jib Crane as a Complete System

Chapter Four

Mechanical Design

4.1 Introduction

In addition to the manufacturing purposes, the teaching purposes also are taken under consideration in designing the desired jib crane. So the dimensions of the whole parts of the crane, and the allowable load are designed to satisfy these purposes also to be used for lab operating processes.

This chapter contains the mechanical analysis for parts of the jib crane that will be built up, where most of the parts will be analyzed in order to obtain stresses and deflections that will be resulted due to the applied load. The stresses that will be studied are the first and second principal stresses and the Von-Mises stress, which will be the core of study; since it is the result of the first and second principal stresses. However, two software programs will be used in the analysis, which are the CATIA for designing, and ANSYS for analyzing.

Finally the whole system of the jib crane will be built up using the dimensions which are depended on, and it will be shown as a complete system.

4.2 Design Analysis

The design that is dependent for the crane is shown in Figure (2.2) with all of its details except some enhancements in some parts to facilitate the machining operations and provide better performance.

As mentioned previously the parts of the crane are designed to be used inside the lab, and for teaching purposes, so the crane will be a prototype but with quietly large size. However, the specifications of the designed crane are shown in table (4.1).

Table (4.1): Prototype specifications

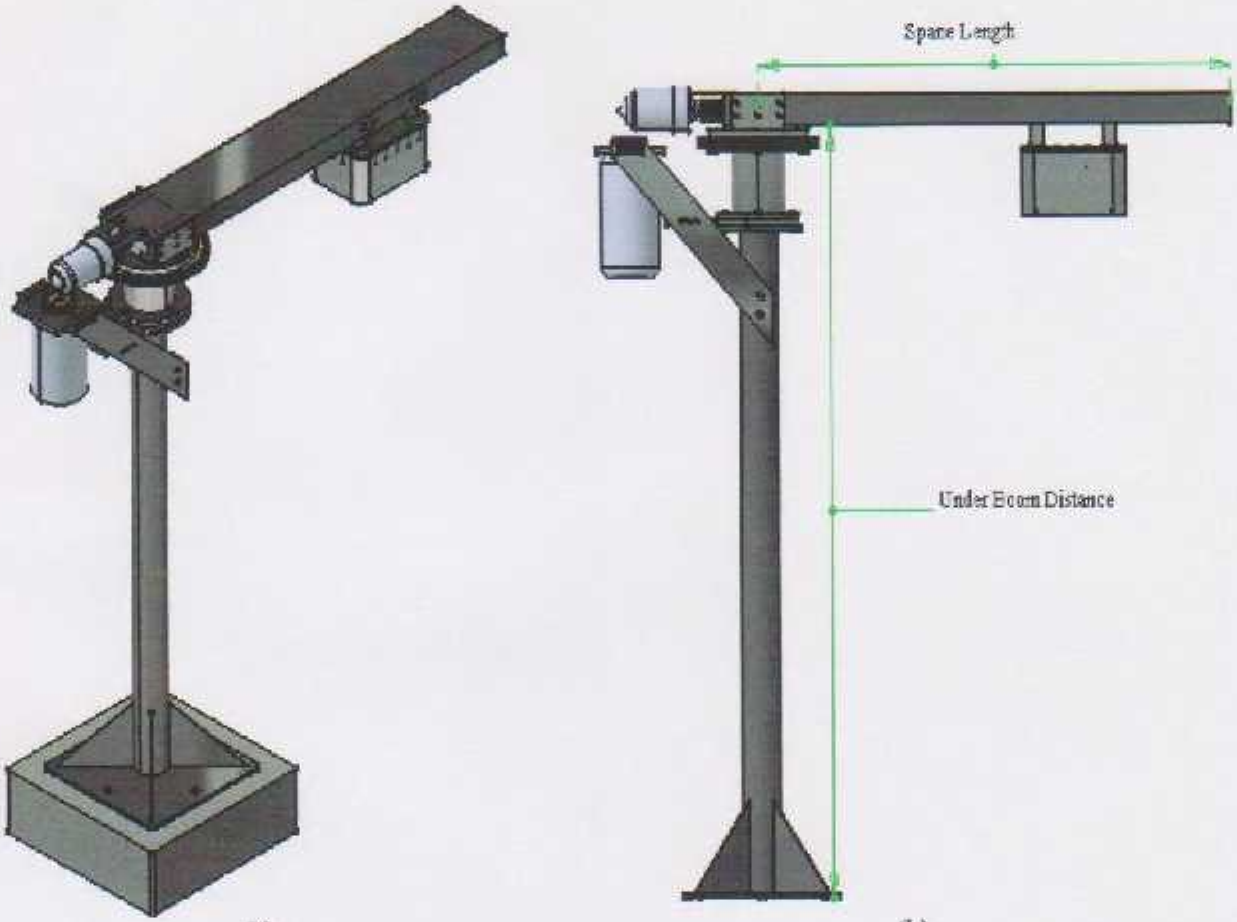
Specification	Value
Under boom distance	1263 mm
Span	915 mm
Load	6 kg
Trolley load	4 kg

The whole design of the prototype crane is done using CATIA drawing program and it is shown in Figure (4.1), where the parts layout is also shown in Figure (2.2), but some differences or enhancements are done in the prototype design, especially in the boom mount, and the lower flange which is the basic part in the rotational joint. These new parts are shown in Figure (4.2). The new design for the lower flange depends on the bearings which forms the rotational joint; where two bearings are used with equal inner diameters but different outer diameters so their cups also as shown in Figure (4.2.a).

This design will decrease the friction force between the upper and lower flanges down to very small value. For the new design of the boom mount, it will provide more rigidity for the connection of the boom with rotational part of the crane, and also it will prevent the boom from letting down.

The mechanism that is dependent to translate the trolley along the boom is power screw, and the trolley will be connected to the screw throughout a bolt as shown in figure (4.3.a). The translational motor is connected to the boom throughout a base, and to provide the alignment, a coupling shaft is used; this is clear in Figure (4.3.b). However, this mechanism is selected in the prototype because the translational distance is small and the load also small; so the deflection in the boom will be very small, and accordingly the deflection in the screw will be small, so the corrosion in the screw will not be worth mentioned.

Generally the most critical parts in the jib crane are: boom, boom mount, lower flange, and the mast, where these parts will be affected critically by the load and its value. So these parts will be designed according to the load values and the speed to be moved by. However, since the prototype that is desired to be designed as mentioned in Table (4.1) is for load 6 kg, and the rotational speed is 0.314 rad/s.



(a) (b)

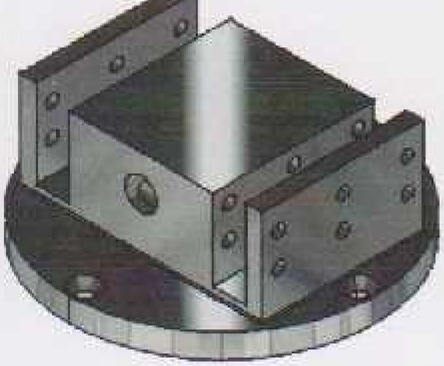
Figure (4.1): Jib crane prototype design by CATIA.

a) Whole design.

b) Prototype specifications.

These parts will be analyzed for the static loads, while the dynamic loads will not be included. The effect of the dynamic loads is critical and of high importance to be considered, so the factor of safety is increased to a value that compensates for this effect.

(a)

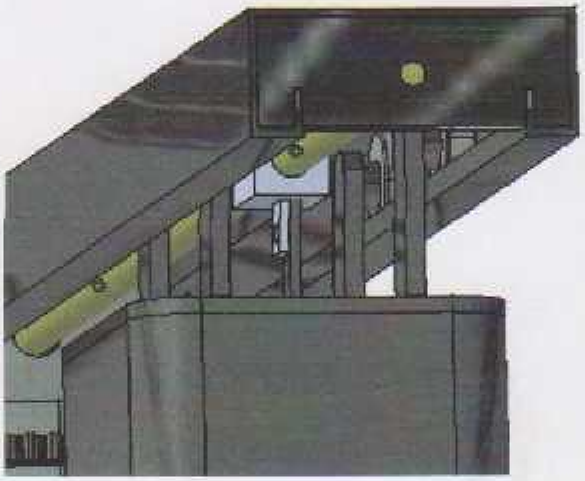


(b)

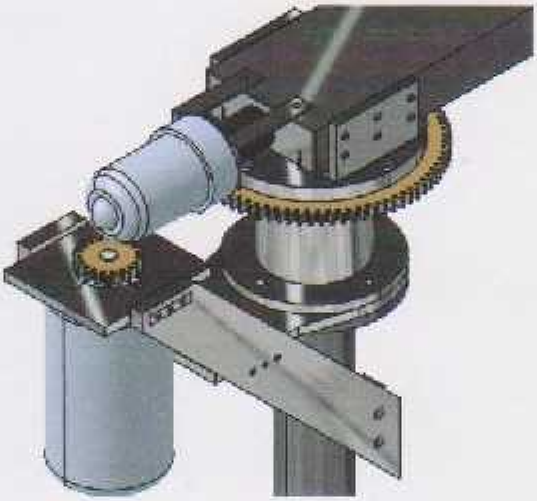
Figure (4.2): Enhancements of the mechanical parts of the prototype.

a) Lower flange

b) Boom's mount.



(a)



(b)

Figure (4.3): Mechanism of trolley translation.

a) Screw and bolt.

b) Translational motor mount and coupling shaft.

4.2.1 Boom Design

The profile of the boom is shown in Figure (4.4). Design of the boom will be used to specify the thickness of the metallic sheet that will be used to form this profile. However this will be done through determining the deflection of the beam for different thicknesses. This will be done using the ANSYS analysis program.

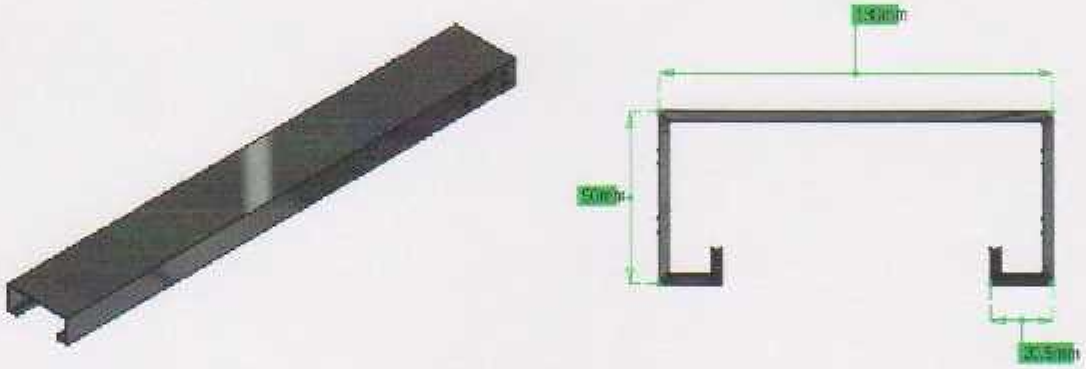


Figure (4.4): Boom and its Profile.

The length of the boom is 980 mm, and the dimensions of its profile are shown in Figure (4.4), except the thickness, which must be designed to minimize the deflection as possible as we can. The boom can modeled as a cantilever with a force applied on it, and the maximum deflection will be when the force applied at its end point. However the deflection in the beam is given as:

$$Y = \frac{Fx^2}{6EI}(x - 3L) \qquad (4.1)$$

Where:

Y: is the deflection in the beam.

F: is the applied force.

E: modulus of elasticity of the material.

I: is the second moment of area.

L: is the length of the beam.

x: is the distance between the point where the force is applied and the original point.

Y is maximum at x =L. So throughout specifying the needed deflection, the profile thickness can be determined through determining the moment of area I. The material of the boom is steal_37, which is of 207 GPas modulus of elasticity and 1520 MP of yield strength. Let the maximum deflection in the beam due to load is about 2 mm, so the second moment of area will be:

$$I = \frac{-FL^3}{3EY} = 0.7578 \times 10^{-6} m \tag{4.2}$$

However I of the profile can be computed as considering it a group of rectangular profile beams which each of them has the moment of area given as:

$$I = \frac{bh^3}{12} \tag{4.3}$$

Where b is the width and h is the height of the cross section of the beam. Accordingly, the thickness can determined, and since the load in addition to the weight of the trolley is relatively small which is about 100 N, the thickness of the metallic sheet is found and it is relatively small, so it is not practical to be used in forming the boom. However the metallic sheet that is used is of 3mm thickness. The following analysis are done on the boom and the maximum deflection, first principal, second principal, and Von-Mises stresses (which are given in Equations (4.4), (4.5), (4.6) respectively for the plane stresses) are determined as shown in Figure (4.5).

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x + \sigma_y}{2}\right)^2 + \tau_{xy}^2} \qquad (4.4)$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x + \sigma_y}{2}\right)^2 + \tau_{xy}^2} \qquad (4.5)$$

$$\sigma' = (\sigma_x^2 - \sigma_x \sigma_y + \sigma_y^2 + 3\tau_{xy}^2)^{1/2} \qquad (4.6)$$

Where σ_x, and σ_y are plane stresses, and τ_{xy} is plane shear stress. These stresses can be determined from the applied loads on the working area. σ' is the Von-Mises stress, which is the effective stress for the entire general state of stress given by σ_x, and σ_y. [3]

The maximum deflections and critically affected points by the applied load can be specified throughout calculating these stresses, which promotes to design the mechanical parts according their outputs and the desired deflections.

The units of the stresses and deflection are related to mm metric unit. As in Figure (4.5) the maximum deflection (DMX) is 0.015751 mm, which doesn't worth to be mentioned.

The maximum stress (SMX) is 1.294 KPas, which is less than the yield strength of the material, and it is concentrated at the screw holes, and this is logic, where the stresses concentrate at the weakest point which is of least area. The minimum stress (SMN) is 0.479e-3 KPas, which is at end of the beam.

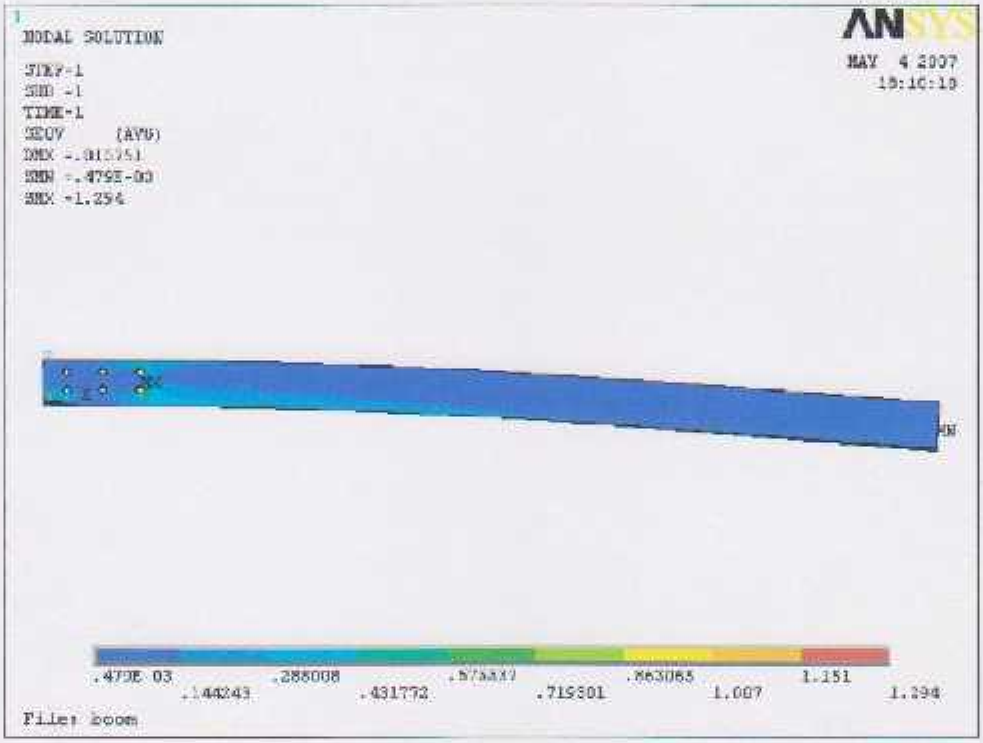


Figure (4.5): Von-Mises stress and the boom deflection.

The deflection in the Figures don't reflect the real value of it, it is just to provide a good clearance to the users.

According to the response of the beam to the applied load (deflection and stresses), it can be strictly depended with its current design, including the thickness.

4.2.2 Boom Mount Design

Boom mount has the mechanical shape that is shown in Figure (4.2.b), this design as mentioned before helps in fixing the boom rigidly. The material of the boom mount is aluminum which has 71.7 GPas modulus of elasticity and 170 MPas yield strength.

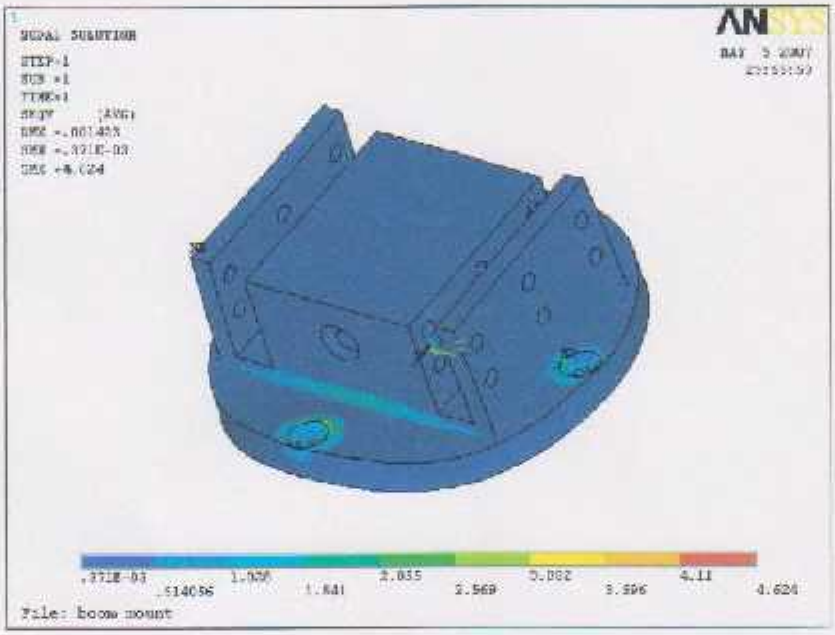
However, boom mount is loaded by the reactions due to the applied load and the weight of the boom itself. So the total load is about 170 N as shown in Figure (4.6). As in the Figure, the pressure will be compression at the first 4 side-holes (boom-fixing screw-holes), and will be tension at the last 2 holes. According to the dimensions that are dependent for this part, the deflections and deformations will be as in Figure (4.7).

Figure (4.6): Load distribution among the boom mount.

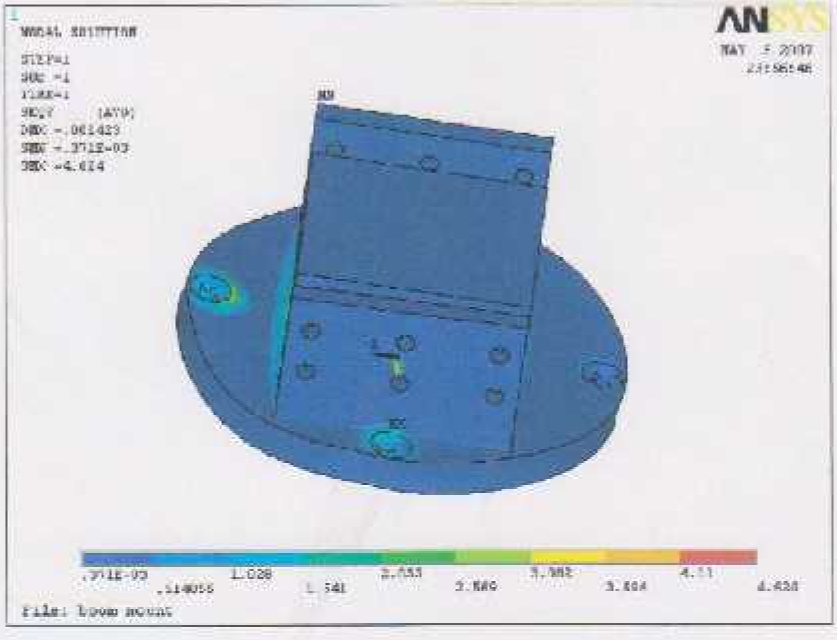
Figure (4.7): Deformations in the boom mount.

Figure (4.7) shows the deformations that will occur due to the applied loads on the boom, and accordingly reflect on the boom mount. The maximum value of the deflection is 0.001423 mm, which is very small and can't be noticed.

The stress concentration is always at the minimum area, which at the screw holes that fixes the boom with boom mount and fixes the boom mount to the gear, which is clearly shown in Figure (4.8) that shows the Von-Mises stress. Another point must be mentioned that the screw-hole which is opposite to the boom direction is largely deformed than its parallel hole where the deformation is small. This is logic; since the load will let the boom down which will dislocate the boom mount.



(a)



(b)

Figure (4.8): Von-Mises stress among the boom mount.

Good solutions for the weakest points can be suggested especially at the corners and the holes where the stresses are concentrated as shown in Figure (4.8). For the case of the corners, the fillets can be made at these locations, either in the forming stage or by adding the welding. For the case of holes, the thickness of the part can be increased at the meant locations. Since the maximum stress is 4.624 KPas which is less than the yield strength of the material of the boom mount, so the factor of safety will be large, and since the deflection also very small, so the design of the boom mount and its current dimensions will be depended.

4.2.3 Upper Flange Design

The rotational joint consists of two parts, the upper flange and lower flange. The upper flange is just a male that is fitted into the bearings which are also fitted into the lower flange. The material of the upper flange is aluminum. Due to the applied load, and the weight of the boom, the upper flange is loaded as shown in Figure (4.9), where the reaction forces are distributed among the areas that are fitted into the bearings. The reactions at these contact areas are determined and found as 6928.6 N which is represented by the blue mesh, and 1478.4 N, which is represented by the red mesh.

The deformations and the stresses that occur due to load are shown in Figure (4.10). The maximum deflection as in the figure is 0.21073 mm, and the maximum stress is 200.7 KPas. This stress is concentrated at the screw hole that is opposite to the boom direction as shown in Figure (4.10). Also the stress has a high values close to its maximum concentrated at the beginning of the flange neck. These stress concentrations are logic; since the area has its minimum values at these points.

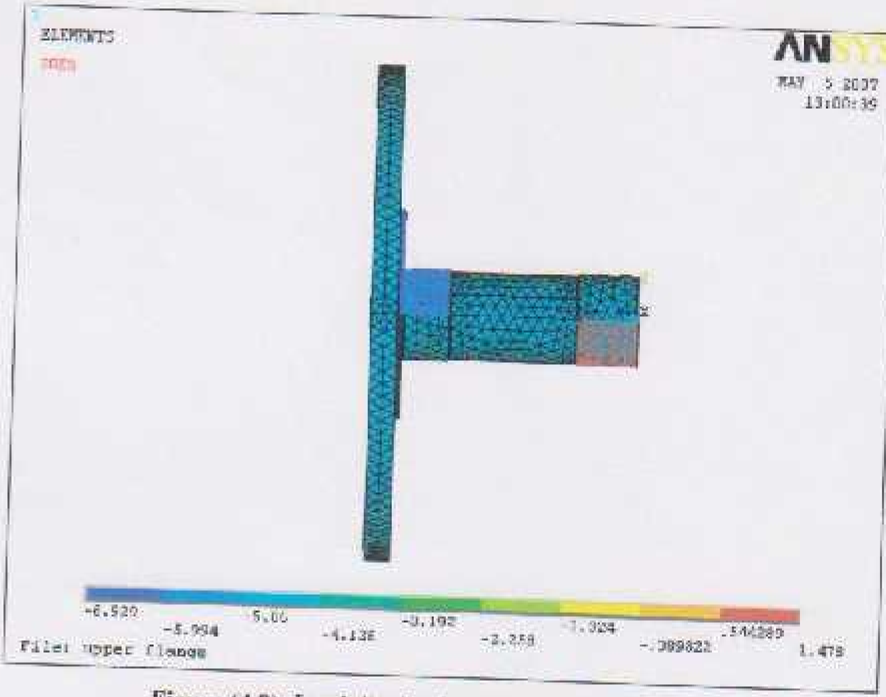


Figure (4.9): Load distribution among the upper flange.

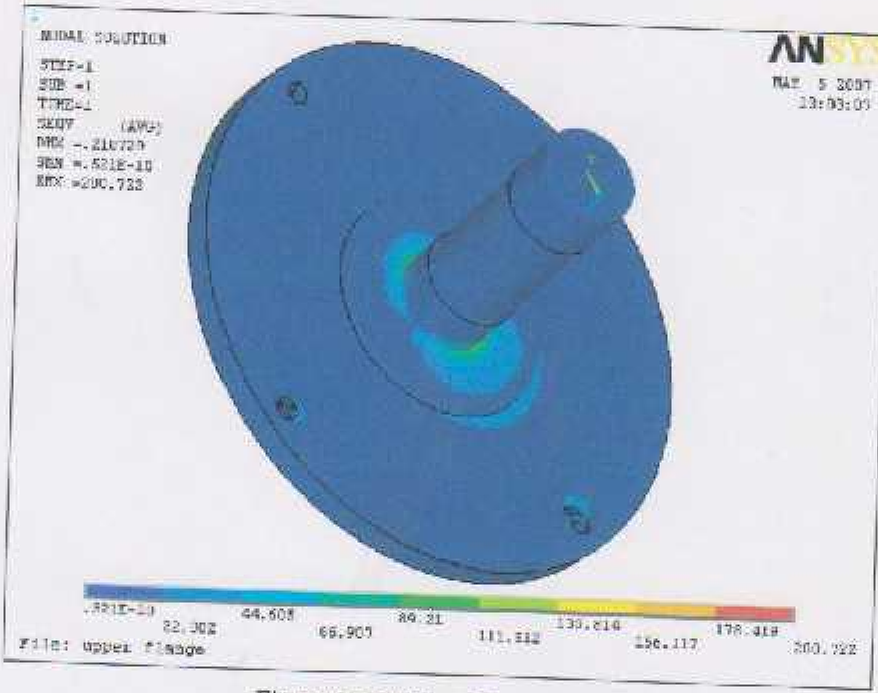


Figure (4.10): Von-Mises stresses

As previously mentioned, the effect of these stresses can be decreased throughout increasing the thickness of the part at the holes and in creasing the fillets at the beginning of the neck. Since the deflection is small or doesn't worth to be cared of it, and the stresses are enough small to be a way from the yield strength of the material of the lower flange which is 170 MPas (aluminum), this design will be depended on with its current dimensions.

4.2.4 Lower Flange Design

It represents the other part of the rotational joint of the system, where the bearings are fitted. Its design is shown in Figure (4.2.a), and the material that is made of it is aluminum. The load is distributed among its internal areas as shown in Figure (4.11). These loads are produced from reactions due to the normal forces among the bearings and equal the normal forces that act on the upper flange which equal 6928.6 N for blue area, and 1478.4 for red region.

The deformations due to the acting loads, and accordingly the deflections are shown in Figure (4.12). The stresses are concentrated at the screw-holes and corners which are the intuitively critical points. The stress that is shown in Figure (4.12) is the first principal stress, which has maximum and minimum values of 3342 KPas and -1592 KPas respectively at the left side screw-hole. This is logic, since the hole will be affected by a compression at the surface (red color) and tension at the bottom (blue color). Also, the stress is concentrated at the beginning of the neck as shown, which is about 610 KPas.

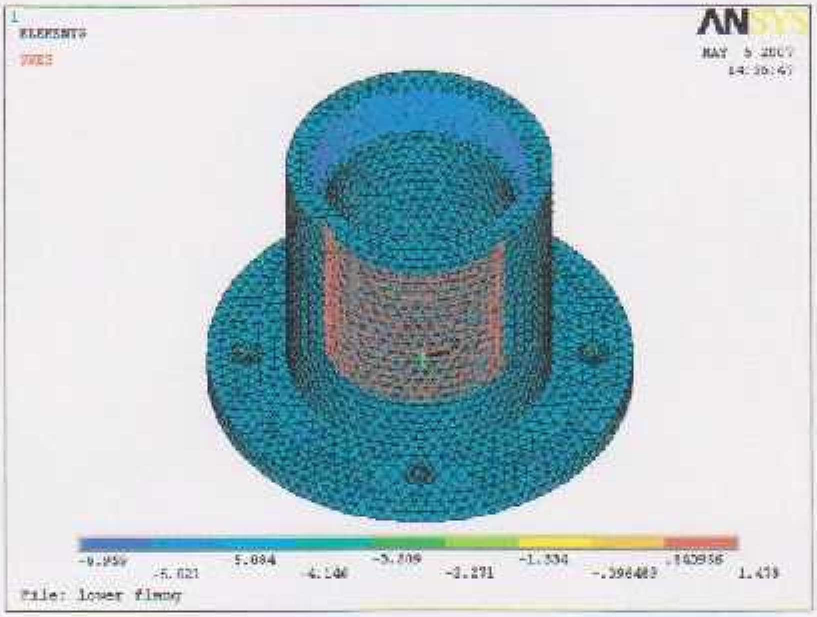


Figure (4.11): Load distribution over the lower flange

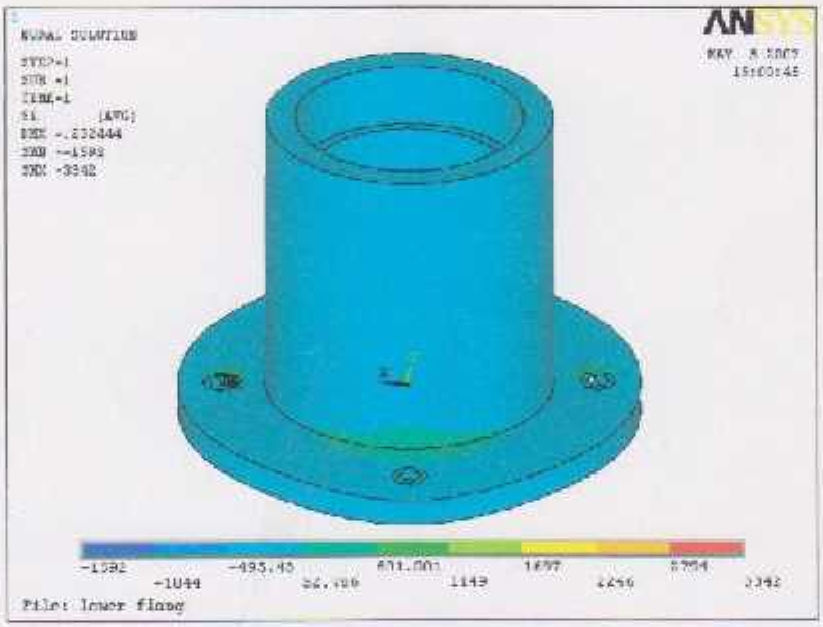


Figure (4.12): Deformations and stress concentration among the lower flange.

The base of the flange will be deflected with a maximum value 0.2324 mm, as shown in Figure (4.12). According to these values of stresses and deflections, the design of the flange will be depended on; since the stresses much less than the yield strength of the material, and the deflections are very small.

4.2.5 Mast and Base Design

The mast is a hollow cylinder of 3mm thickness of steal-37 steal material which in addition is welded to the base that is $30\times30cm^2$ area, and 4mm thickness, and of steal-37 material. In order to reinforce the structure of the mast and base, a four triangular supports are welded between the base and the mast, each of them is of 30mm width, 30mm height, 4mm thickness, and of steal-37 material. This design is clearly shown in Figure (4.1.a). Since the mast, base and the supports are welded together, they will be manipulated together as a one unit, so it will be named simply as 'mast'. The load that is applied to the mast is compression as shown in Figure (4.13), where the load is acting on the half of the cylinder (mast), which is in the direction of the boom. However, the Figure shows also the reactions on the part which are in pink color. The green color is the fixing points that fix the mast to the ground.

The mast will be deflected due to the sided load in a form of buckling, which is shown in Figure (4.14). This type of deflections is seriously danger, and affects directly to the whole system parts. Such a problem can be reduced either through increasing the thickness of the beams, or increasing the height of supports.

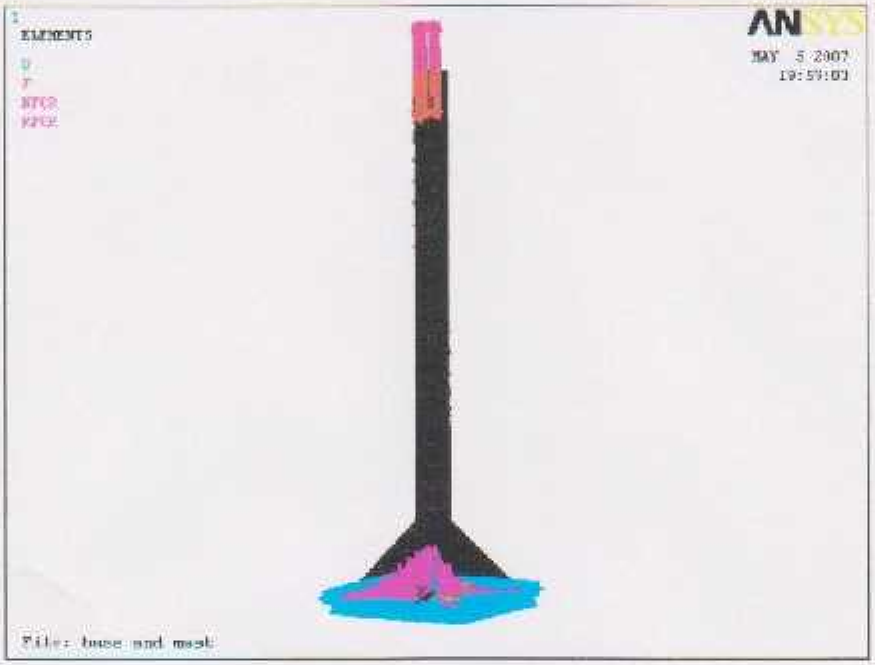


Figure (4.13): Load distribution among the mast

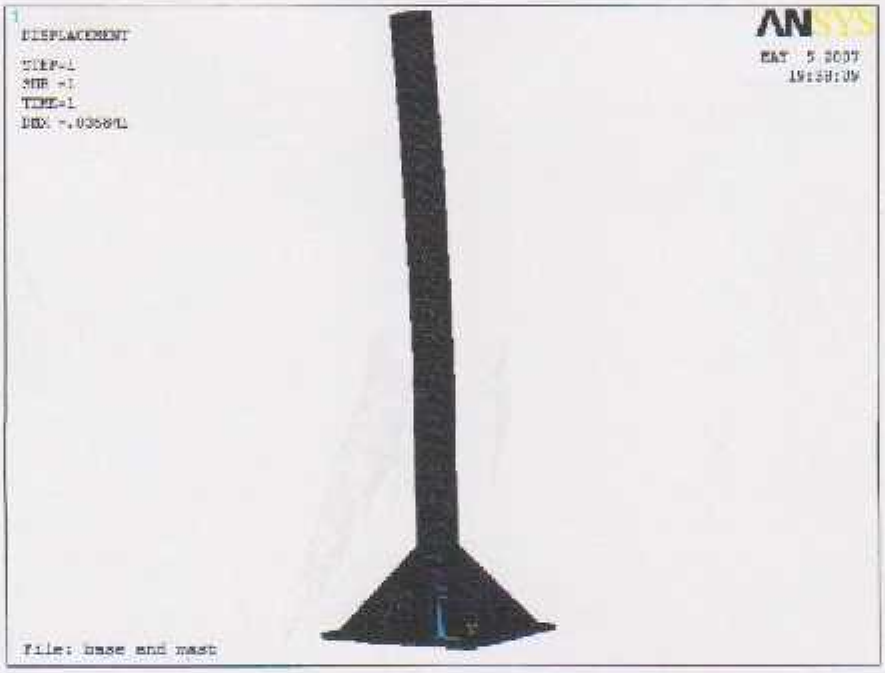


Figure (4.14): Buckling deflection in the mast.

However, since the maximum value of the deflection is 0.0368 mm at middle of the cylinder, which is very small, these dimensions of the cylinder and supports can be depended on. The stresses that are resulted due to loading are shown in Figure (4.15), where the stresses are distributed along the mast, and the supports, also they are concentrated at meeting between the head points of supports, and the mast.

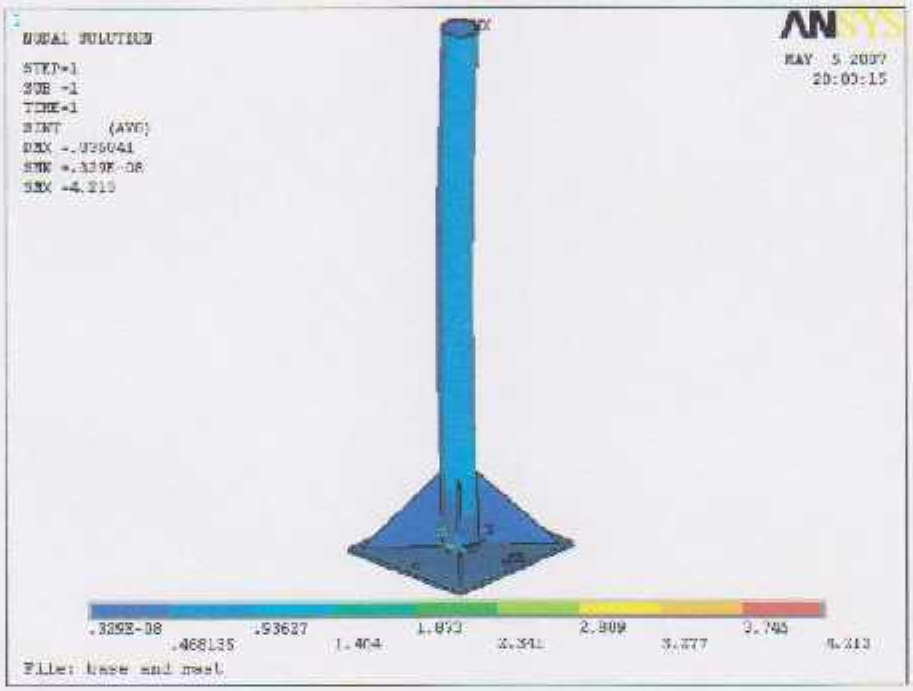


Figure (4.15): Von-Mises stress concentration and deflection among the mast.

As in the Figure, the maximum stress value is 4.213 KPas (compression), at the upper surface of the cylinder, and the minimum value of the stress is 0.329E-8 KPas (tension) at a point in the screw-hole. The solution for the points of high stress concentration is to increase the area on contact, which means increasing the thickness of the material at those points. Because of the small values of the stresses which is much less than the yield strength of the material which is 1520 MPas, the current dimensions of the whole parts in the mast will be depended on.

4.2.6 Trolley Design

The design of the trolley that is depended on to be used in the prototype is the same that is shown in Figure (2.7). The metallic plates that are used to connect the cover of the trolley with the rollers are of sectional area $3 \times 16mm^2$ and steal-37 material. These plates are loaded axially with tension load, and since they are four, the load will be divided among them equally, so the load for each of them will be 25 N, and distributed as shown in Figure (4.16), where the large hole is to be connected to the roller through a shaft, and the small hole is to be connected to the cover of the trolley through a screw.

The maximum elongation that occurs due to the applied load is 0.345E-4 mm, which is nothing to be cared. The stress concentrations and its values are shown in Figure (4.17), where the maximum value of the stress is 0.0536 KPas at the upper hole where the plate in fixed to the roller. Also the deflections and stress paths can be shown in the Figure, where it concentrates extremely at the holed, where the least areas exist.

Since the stress values much less than the yield strength of the material and the maximum deformation value is very small, the current dimensions of these plates will be depended on.

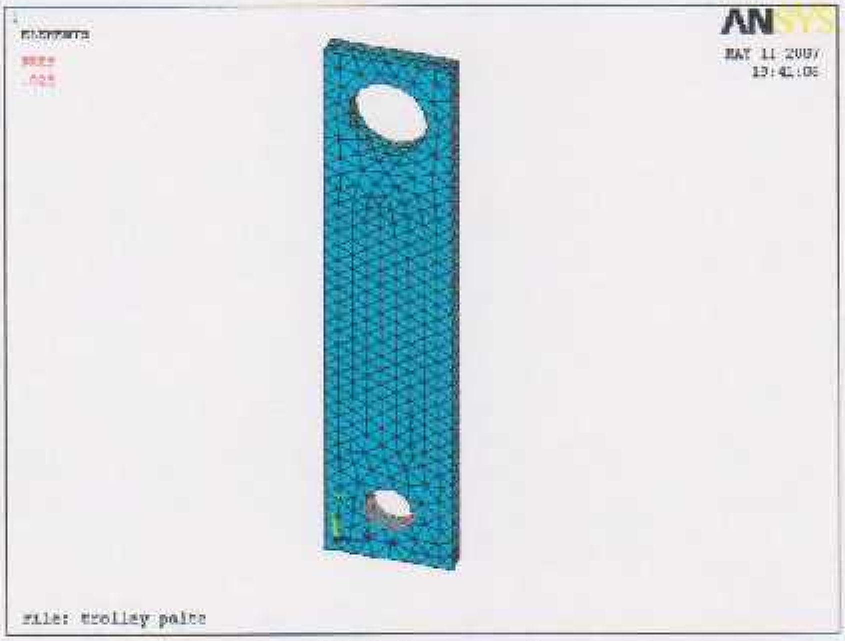


Figure (4.16): Load distribution among the connecting plate.

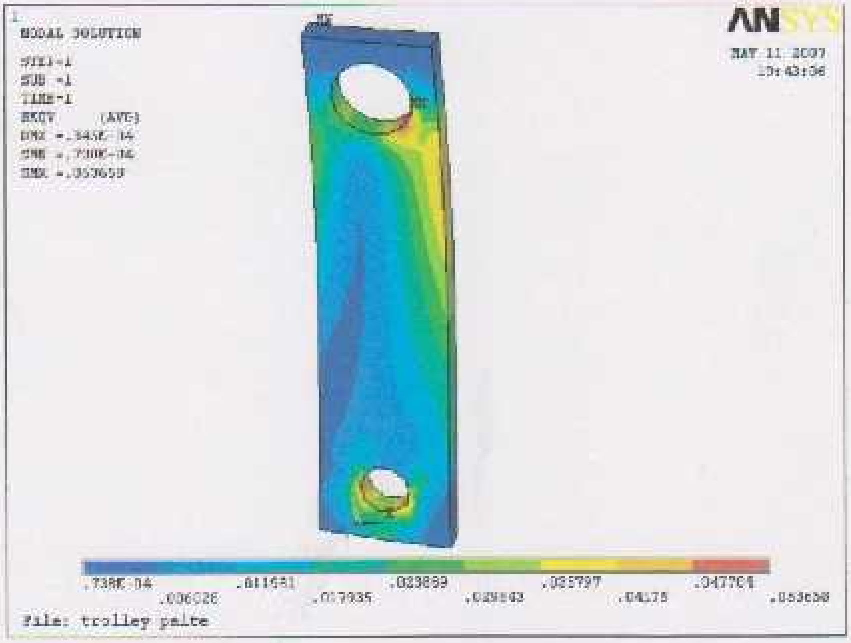


Figure (4.17): Von-Mises stress distribution over the connecting plate

The mechanical analysis for the critical parts in the crane is done. For the rest parts such as the parts of the rotational motor mount (shown in Figure (2.6)), which are: horizontal motor mount, diagonal motor mount, and the motor neck plate as detailed in Figure (2.2), will not be analyzed, since they are not loaded except by the load of the rotational motor, which is of small fixed value and permanent load. Also the applied load, doesn't affect those parts directly.

4.3 Bearings Selection

Commonly, the bearings selection is based on the friction, heat, lubricant, corrosion resistance, kinematic loading (motion speed), its material properties, and machining tolerances. [3]

In the case of the rotational joint, the bearings are selected according to the rotational speed, the lubricant material, the thrust load, and its dimensions. However, since the load is relatively small, and the rotational speed is also very small, a suitable bearings are used which are of manufacturing numbers: 6207, and 6307.

4.4 Jib Crane as a Complete System

After analyzing the parts of the crane and manipulating the stresses that affecting each of them, and finding the maximum deflection in each of them, the mechanical parts can be formed and manufactured. All parts are assembled together and the whole system is built up, and it will be shown later.

Chapter Five

Electrical Design

- Introduction
- Selection of the Motors
- Control and Power Circuits

Chapter Five

Electrical Design

5.1 Introduction

This chapter will discuss the process of designing the needed electrical parts to operate the system. These parts include the actuators in the system which are electrical motors, the power circuits, the protection circuits, and control circuits.

The system includes three rotary electrical motors to provide the needed motions or the degree of freedoms. The first motor is to provide the rotational motion, second one is to provide the translational motion, and the third is for hoisting the load. These motions are constrained by a group of limit switches to specify the limits of rotation, translation, and hoisting.

The selection of the motors, in addition to their controlling and power circuits will be discussed, where the needed torques to rotate, translate, and hoist the load will be determined to select the required motors.

5.2 Selection of Motors

As mentioned previously, the system axes require 3 motors, for rotation, translation, and hoisting. These motors will be selected according to the needed torques and speeds.

5.2.1 Rotation Motion Motor

The parts that will rotate in the system are: boom, boom mount, large sprocket, upper flange, and the trolley. In addition to the needed torque to override theses loads, also the torque provided by the motor must override the friction forces produced at the rotational joint. Here a point must be mentioned, that the bearings which are used in the joint providing a very small coefficient of friction, the friction forces will be very small compared to the rotary loads, so these forces will be neglected.

The torque that is needed to be provided from the motor is given as

$$T = J_{equ}\alpha + T_f \tag{5.1}$$

Where J_{equ} is the reflected moment of inertia to the motor's shaft, α is the angular acceleration, and T_f is the needed torque to come over the coulomb friction force produced at the rotational joint. The rotational joint consists of bearings, so the coefficient of friction is very small, and accordingly the friction force will be very small. In addition, the length of the torque arm produced from the friction force is very small, which is a bout 1.8 cm, so the friction torque will decrease, as a result this torque will be neglected, and just the first term of Equation (5.1) will be considered. Since the motor will rotate the whole system throughout a sprocket and chain (design recommendation), the transmission ratio must taken under consideration throughout determining J_{equ}, so either specifying the transmission ration or the motor torque. Since the design recommends a sprocket with specific diameter, the diameter of the driven sprocket is 23 cm, with 78 teeth and the driver one is of 19 teeth, so the transmission ratio (a) is about 1:4. The system can be modeled as shown in Figure (5.1), where the load block represents all the rotating parts other than the large sprocket.

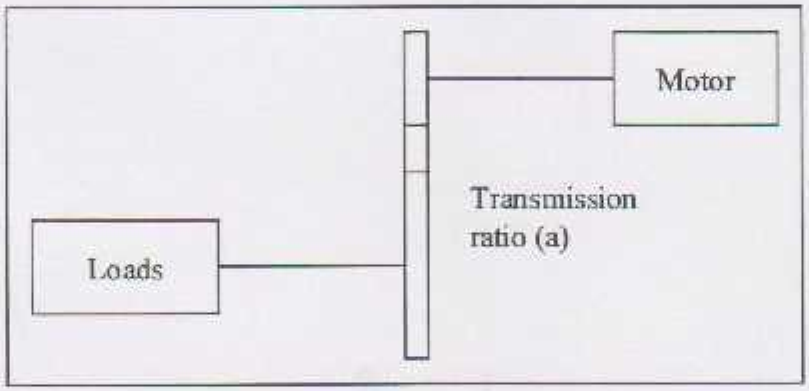


Figure (5.1): Model of the driving system.

The second moments of inertia for the rotating parts are found using the designing program CATIA, and are given in Table (5.1). The equivalent moment of inertia is

$$J_{equ} = a^2(J_{sprocket} + J_{upper\ flange} + J_{boom\ mount} + J_{boom} + J_{trolley} + J_{load}) \\ = 0.25^2(0.615) = 0.0384\ Kg.m^2 \tag{5.2}$$

Table (5.1): Inertias of rotating parts.

Part	Inertia ($Kg.m^2$)
Sprocket	0.02
Upper flange	0.003
Boom mount	0.009
Boom	0.518
Trolley	0.062
Load	0.003

The angular acceleration can be specified through out the user, where the acceleration and deceleration in the rotation motion can be programmed throughout either the reference rotational signal or by the ac-inverter that will be used in the system. However it is specified to be 1rad/s^2. So the torque of the motor will become using Equation (5.1)

$$T = 0.0384 \times 1 = 0.0384\ N.m \tag{5.3}$$

The size of the motor, also considered one of the constraints to be suitable and homogenous with whole crane size. Such a motor is available, where a 3 phase induction motor is used, and its rated torque is 1.26 N.m, and its rated speed is 2800 rpm, provided from its name plate. However, it is of high speed, but it will be controlled using an ac-inverter, and getting the desired speed. So this motor will provide the needed torque.

5.2.2 Translational Motion Motor

The translational motor will translate the trolley throughout power screw, and the trolley will move along the boom using its rollers which are bearings. Since there are 4 rollers, and a fifth hanger is used to connect the trolley to the bolt of the screw, the load and trolley weights are distributed among these five supports equally. The value of these weights is 10 Kg. so the reaction at each support is 2 Kg.

The force that must be produced from the motor must come over the friction force that exists between the teeth of the screw which is of steel material, and that of the bolt which is cast iron material. The static coefficient of friction between these materials is found 0.18, and since there are 33 teeth meshed together at the same time; so the friction force is given as

$$\begin{aligned} F_f &= 33N\,\mu \\ &= 33\times 2\times 9.81\times 0.18 \\ &= 116.54N \end{aligned} \tag{5.4}$$

The needed output torque from the motor is

$$\begin{aligned} T &= F_f r \\ &= 116.54 \times 0.008 \\ &= 0.932 \text{ N.m} \end{aligned} \tag{5.5}$$

Where r is the radius of the power screw, which is 8 mm.

A suitable dc motor with a gear box is used to provide the needed torque, and its rated speed is 60 rpm and rated input voltage is 14 V. This is good to reduce the oscillations that may occur due to motion.

5.2.3 Hoisting Motor

Hoisting motion is done through rotating a pulley which is fixed to the shaft of the motor, so the motor must provide a torque capable to override the load and its acceleration forces. The load value is 6 Kg, and the desired acceleration at starting point is 0.05 m/s^2 and the diameter of the pulley where the cable will be spin is 2 cm, so the torque that is needed to be produced from the motor is

$$\begin{aligned} T &= (g + a)mr \\ &= (9.81 + 0.05)2 \times 0.01 \\ &= 0.2 \text{ N.m} \end{aligned} \tag{5.6}$$

A suitable dc motor with a gear box is used to provide the needed torque, and its rated speed is 60 rpm, and rated input voltage is 14 V.

5.3 Control and Power Circuits

In order to operate the electrical motors properly, control and power circuits are needed to be designed to provide them the required power, preventing the electrical parts from the electrical faults, and providing the needed constraints for the safety purposes.

5.3.1 Controlling the Motor of Rotational Motion

The rotational motor, as mentioned previously is a three phase induction motor, and it will be operated throughout a signal provided from the data acquisition card (DAQ), this signal will be transmitted to the motor throughout an AC-Inverter, where the speed of the motor will be controlled according to the input signal to the inverter, which will be provided from the controller of the whole system.

The power circuit of the rotational motion motor is shown in Figure (5.2). As in the Figure, the inverter is used here also to convert from single phase supply into 3-phase. The direction of rotation will be inverted throughout 2 relays (R1, R2), where their supply will be from the DAC according to the desired direction, and accordingly the inverter will change the direction of rotation of the motor. To prevent the electrical faults that may destroy the inverter, two fuses are used for these possibilities as shown in the Figure. An emergency button is used to allow the labor to stop the whole system if danger actions are occurred during the operation.

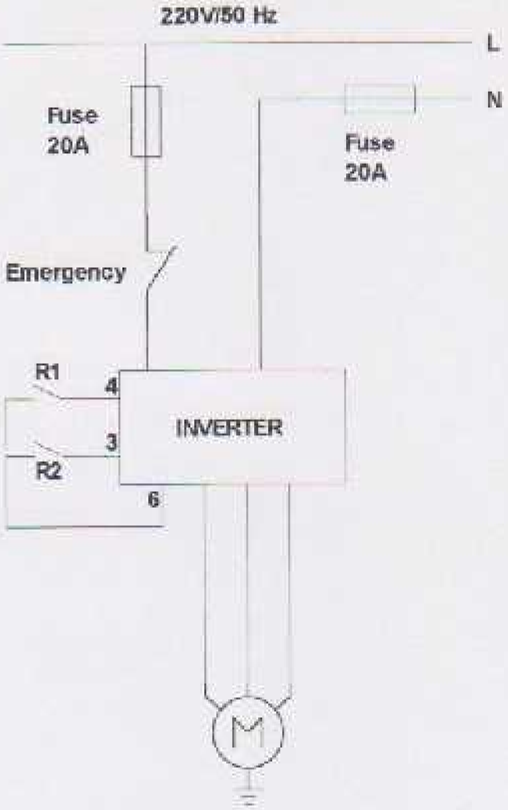


Figure (5.2): Power circuit of rotational motion motor.

The relays which are used to change the direction of rotation throughout the inverter are supplied through the DAC, and since the DAC can't provide the required currents and voltage to the relay to work properly, an isolation circuit must be used between the DAC and the relays. Opt-couplers are used in the control circuit of the motor, which is shown in Figure (5.3). The input of the opt-couplers is supplied from the DAC, and according to its signals, the 12 V will be transmitted to the relays. To prevent the operation in both directions at the same moment, the mutual protection is used throughout the normally closed terminals (R1, R2) for the both relays. The limit switches (L.S1, L.S2) are also provided in the figure, where they are connected to be normally close terminals, and as the end limits of rotation in either of the direction are reached, the power will be disconnected from the inverter in that direction.

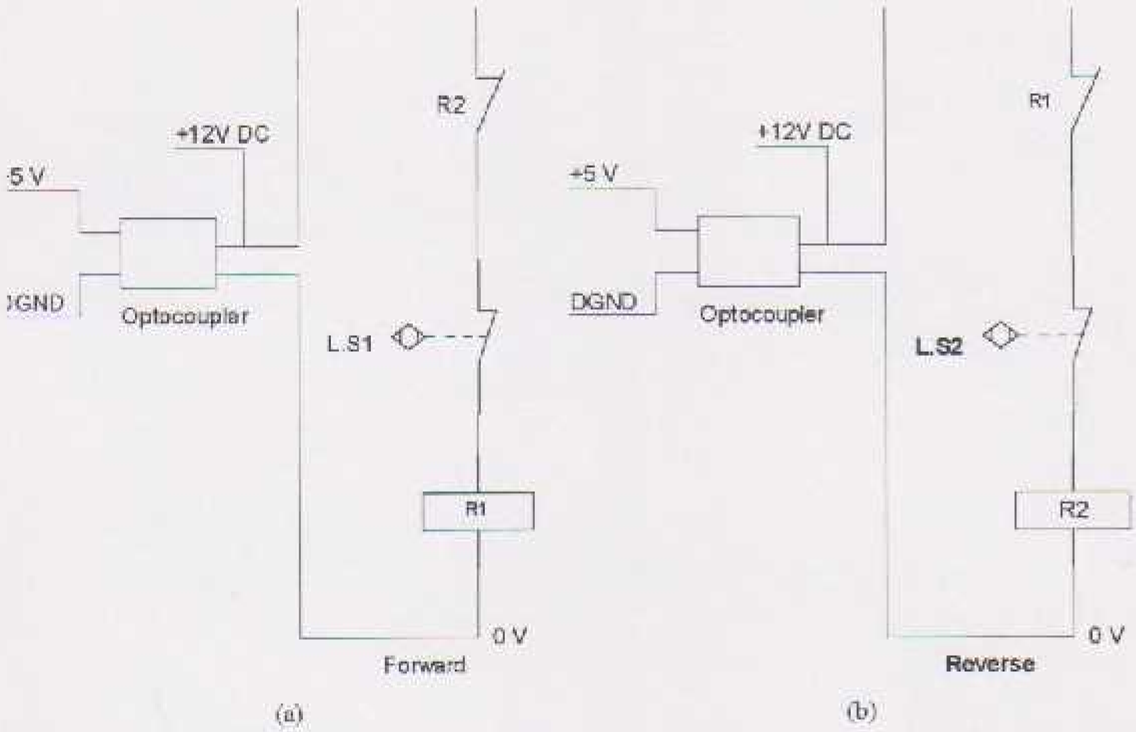


Figure (5.3): Control circuit for the rotational motion motor.

a- Forward direction rotation.

b- Reverse direction rotation.

5.3.2 Controlling the Translational Motion Motor

The translational DC-motor will be controlled throughout its input voltage. The input to the motor will be supplied from the DAC, according to the controller signal, so a power circuit must be designed to generate the needed current to the motor which prevent the damage of the DAC. Such a circuit is shown in Figure (5.4), the circuit consists of operational amplifier connected as shown in the figure. This circuit amplifies the current while maintaining the output voltage value as its input value. A multi stages can be built up in series to provide the needed output current.

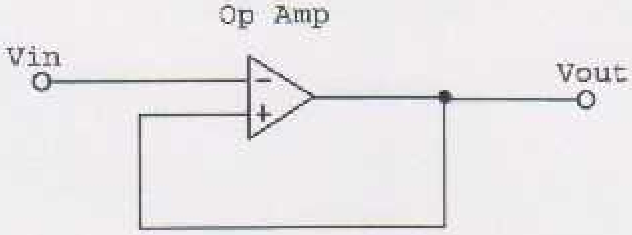


Figure (5.4): Power circuit of translational motion motor.

The control circuit for the translational motion motor is given in Figure (5.5), where the direction of translation is reversed through reversing the direction of rotation of the motor, which done just by changing the input's polarity, as shown in the figure. The circuit contains also the limit switches, which stop the motor as the trolley reached the limits of the boom. Here the motor also can be operated manually.

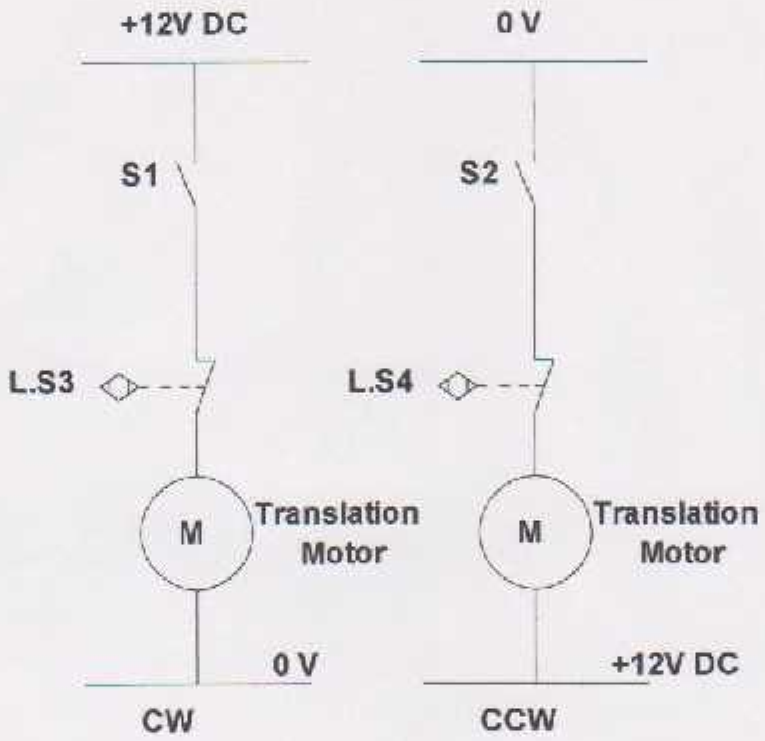


Figure (5.5): Control circuit of the translational motion motor

5.3.3 Controlling the Hoisting Motor

The hoisting motor as previously mentioned will be operated manually, so its power supply will be independent, and so, no power amplification will be used. The control circuit of the hoisting motor is shown in Figure (5.6). The motor will be operated in two directions, and a limit switch also will be used in one direction of operation which is the rising stroke, to prevent the load to hit the trolley and accordingly damaging it, in addition to the motor.

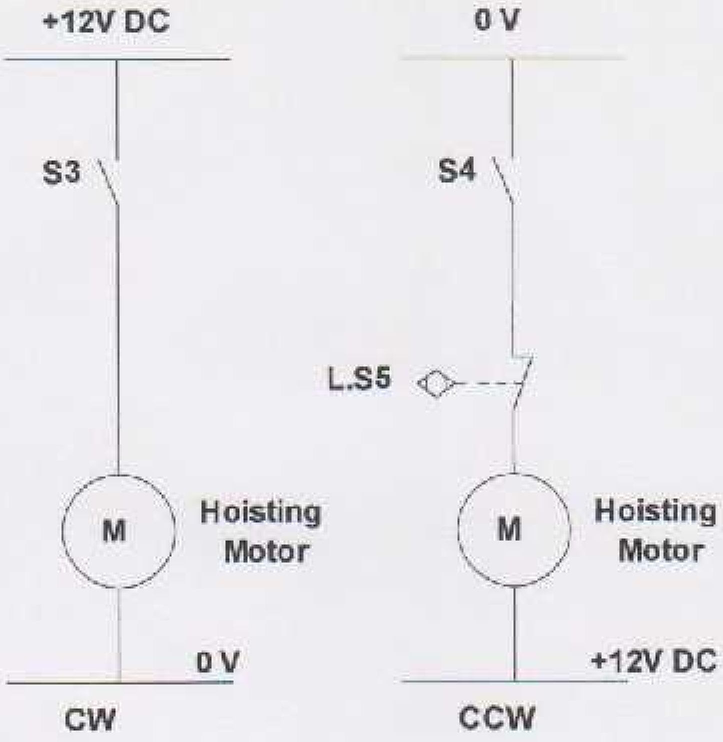


Figure (5.6): Control circuit of the hoisting motor.

However, the crane now can be operated in a complete controlling and protection systems, which will provide the proper operation and high protection for the system itself, and the human in its surrounding.

Chapter Six

Controller Design and Simulation Results

- Introduction
- Mathematical Model Manipulation for Control
- Estimation Unmeasured States
- Quadratic Optimal Systems
- Controller Design
- Simulation Results

Chapter Six

Controller Design and Simulation Results

6.1 Introduction

The idea of the jib crane is to hoist the load vertically, rotates it around the origin then translates it along the boom. The structure of the jib crane is given in Figure (2.2). The desired is to track the rotational and translational positions of the load (trolley) throughout controlling the rotational angle ($\gamma(t)$), and the translational displacement ($x(t)$), with minimum vibration in the load, that is the in-of-plane angle ($\phi(t)$), and out-of-pale angle ($\theta(t)$) are dying with time. Also any disturbance acts on the system such as outer acting forces must be compensated and prevented from acting the performance of the system. So any controller that is applied for the Jib crane must guarantee two main functions:

1- Tracks the desired position of the trolley ($x(t)$) along the boom, and tracks the desired angular position ($\gamma(t)$) of the boom.
2- Performs the previous point robustly, thus it will be able to compensate for *system parameter changes.*

These functions should be achieved and meet the following basic constraints:

1- Limited driving torques those are generated by the motors of translation, and rotation.
2- Limited boom length, which means that motion beyond this limit, is not possible.

6.2 Mathematical Model Manipulation for Control

Based on the results of chapter three, the mathematical model which is obtained there, will be used to design the controller. As a first step, that model needs to be manipulated into various forms including linearized state space representation for state feed-back controller design, and augmented state space model in order to get a robust controller.

6.2.1 State Space Model

In control engineering, a state space representation is a mathematical model of a physical system as a set of inputs, outputs and state variables, represented by first-order differential equations. The state space representation (also known as the "time-domain approach") provides a convenient and compact form to model and analyze systems with multiple inputs and multiple outputs. Unlike the frequency domain approach, the use of the state space representation is not limited to time-invariant systems with linear components and zero initial conditions. "State space" refers to the space whose axes are the state variables. The state of the system can be represented as a vector within that space.

The linearized state space model for the jib crane is derived as mentioned previously in chapter 3 according to the values of the system parameters that are given in Table 6.1, where a prototype for the jib crane is built up, with 1.5 meters height and 1m boom length.

Table (6.1): Parameters of the designed jib crane.

Element	Parameter and its Symbol	Value
Boom	Second moment of inertia (J_0)	0.7 $Kg.m^2$
	Thickness of the profile (t)	3 mm
Load	Mass of load (m)	6 Kg
Trolley	Mass of the trolley (M)	4 Kg
Cable	The length of hoisting cable is constant during the rotation and translation (L)	0.2 m

So the state space representation of the jib crane system is given in Equation (6.1) as:

$$
\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \\ \dot{x}_7 \\ \dot{x}_8 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -101.496 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -117.72 & 0 & 0 & 10 & 0 & 0 & 0 \\ 0 & -49.05 & 0 & 0 & 5 & 0 & 0 & 0 \\ 13.646 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \phi \\ x \\ \gamma \\ \dot{\theta} \\ \dot{\phi} \\ \dot{x} \\ \dot{\gamma} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -0.0093 \\ 0.0333 & 0 \\ 0.0167 & 0 \\ 0 & 0.0015 \end{bmatrix} \begin{bmatrix} F_x \\ T_\gamma \end{bmatrix}
$$

$$
y = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \phi \\ x \\ \gamma \\ \dot{\theta} \\ \dot{\phi} \\ \dot{x} \\ \dot{\gamma} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} F_X \\ T_\gamma \end{bmatrix} \qquad (6.1)
$$

And the state vector is:

$$\begin{Bmatrix} x_1 = \theta(t) \\ x_2 = \phi(t) \\ x_3 = x(t) \\ x_4 = \gamma(t) \\ x_5 = \dot{\theta}(t) \\ x_6 = \dot{\phi}(t) \\ x_7 = \dot{x}(t) \\ x_8 = \dot{\gamma}(t) \end{Bmatrix}$$

6.2.2 Control Strategy

Since the system is a multi input, and multi output (MIMO), a special controller is needed to be designed to guarantee the robust tracking for the input reference signals, and good disturbance rejection. The method that is used is briefed here according to the reference [9].

The following linear time-invariant system is considered:

$$\begin{aligned} \dot{x} &= Ax + Bu_1 + D_x d \\ y &= Cx + D_y d \end{aligned} \qquad (6\text{-}2)$$

$$e = r - y \tag{6-3}$$

Where

x: The plant state vector.

u_1: The plant input vector.

A: System matrix.

B: Input matrix.

C: Output matrix.

y: The output vector to follow the reference signal.

d: The disturbance signal.

r: The reference signal.

e: The tracking error to be regulated.

Furthermore **d** and **r** are assumed to be modeled by the following state equations:

$$\dot{x}_d = A_d x_d \tag{6-4}$$

$$d = C_d x_d \tag{6-5}$$

And

$$\dot{x}_r = A_r x_r \tag{6-6}$$

$$r = C_r x_r \tag{6-7}$$

and (A_d, C_d) and (A_r, C_r) are completely observable. [9]

The controller that is needed to be designed here is of a feedback type, that is, it is driven by the error signal e. Such a controller structure is shown in Figure (6.1).

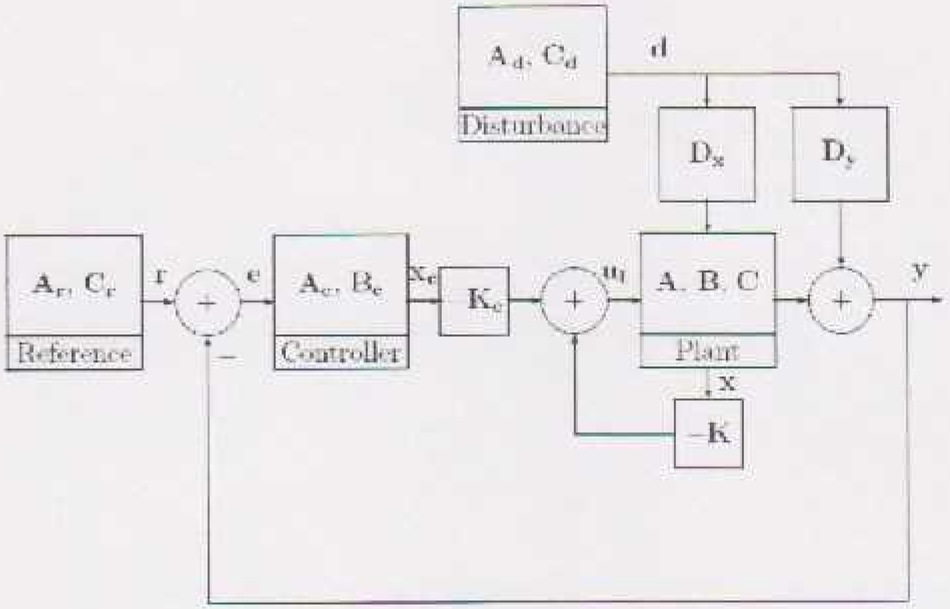


Figure 6.1: Tracking and disturbance rejection by feedback

The system which is described by Equation (6.2) through Equation (6.7), and supposing that **(A, B, C)** is minimal (pair **(A, B)** is controllable, and pair **(A,C)** is observable). Letting

$$\Theta = s^{q} + \alpha_1 s^{q-1} + \ldots + \alpha_{q-1} s + \alpha_q \qquad (6\text{-}8)$$

be the least-common multiple of the minimal polynomials of $\mathbf{A_d}$ and $\mathbf{A_r}$. Then if

$$\text{rank}\begin{bmatrix} sI - A & B \\ -C & 0 \end{bmatrix} = n_s + n_o \qquad (6\text{-}9)$$

for each s, where s is a root of the characteristic polynomial of Equation (6-4) or Equation (6.6), where n_s is number of system's states, and n_o is the number of reference signals. So the controller is given by

$$\dot{x}_c = A_c x_c + B_c e \tag{6-10}$$

Where

$$\mathbf{A_C} = block\ diag.\left[\Lambda, \Lambda, \ldots, \Lambda\right] \tag{6-11}$$

and

$$\mathbf{B_C} = block\ diag\left[\Upsilon, \Upsilon, \ldots, \Upsilon\right] \tag{6-12}$$

with

$$\Lambda = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -\alpha_q & -\alpha_{q-1} & -\alpha_{q-2} & \cdots & -\alpha_1 \end{bmatrix} \quad \in R^{q \times q} \tag{6-13}$$

Where q is the order of the minimal polynomial. And

$$\Upsilon = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} \quad \in R^q \tag{6-14}$$

Hence, the composite (augmented) system (plant followed by the controller)

$$\begin{bmatrix} \dot{x} \\ \dot{x}_c \end{bmatrix} = \begin{bmatrix} A & 0 \\ -B_c C & A_c \end{bmatrix} \begin{bmatrix} x \\ x_c \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u \qquad (6\text{-}15)$$

is completely controllable, that is there exit a control law

$$u = -Kx - K_c x_c \qquad (6\text{-}16)$$

such that the closed-loop system is exponentially stable.

For any such a control law, asymptotic tracking and disturbance rejection hold. Moreover, asymptotic tracking and disturbance rejection are robust (with respect to **A**, **B**, **C**, **Bc**, **K**, and **Kc**) for the class of perturbation such that: **Bc** remains block diagonal and each diagonal is a nonzero vector, and the closed-loop system remains exponentially stable. [9]

6.3 Estimation of Unmeasured States

The synthesis of state-feedback control systems presumes that all state variables are available for feedback. In practice, however, not all state variables are available for feedback. So it is needed to estimate unavailable state variables.

Estimation of unmeasured state variables is commonly called *observation*. A device or a computer program that estimates or observes the state variables is called a *state observer*.

A state observer estimates the state variables based on the measurements of the output and control variables. The state observer can be designed if and only if the observability condition is satisfied.

A system is said to be completely observer if every state $\mathbf{x}(t_0)$ can be determined from the observation of $\mathbf{y}(t)$ over finite interval, $t_0 \leq t \leq t_1$. The system is, therefore, completely observable if every transition of the state eventually affects every element of the output vector. [5]

Referring to Figure (6-2) which shows the basic concept of observer design, the measured outputs of the system will be compared to those estimated, and the error will be fed back to the observer, and this will increase the speed of convergence between the actual and estimated states. The observer must be much faster than the controlled closed system which makes the controller receives the estimated states instantaneously. [1]

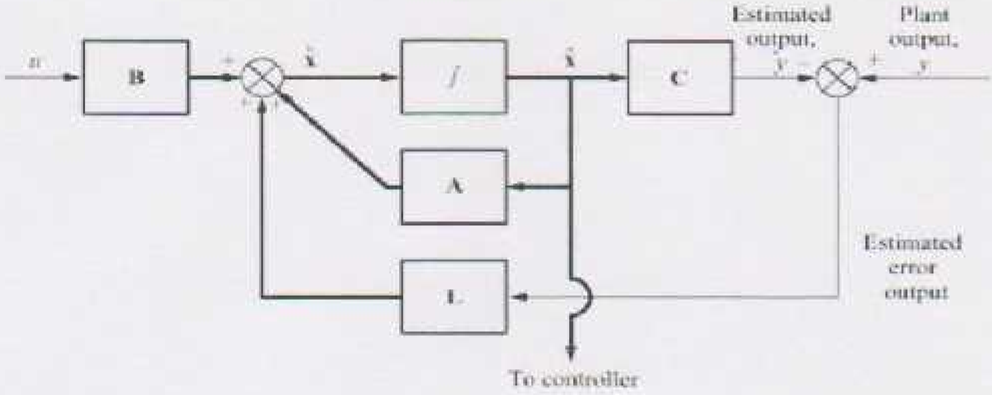


Figure 6-2 Observer design process.

The state equation of the observer is found from Figure (6-2) as follows

$$\dot{\hat{x}} = A\hat{x} + Bu + LC(x - \hat{x}) \qquad (6\text{-}17)$$

The error signal between the measured output and the observer output is defined as

$$\hat{e} = x - \hat{x} \qquad (6\text{-}18)$$

Then the error dynamic equation is

$$\dot{\hat{e}} = (A - LC)\hat{e} \qquad (6\text{-}19)$$

Thus by choosing an appropriate gain vector (L), the poles of the error characteristic equation can be placed to achieve the desired speed of the observer.

6.4 Quadratic Optimal Systems

An advantage of the quadratic optimal control over the pole-placement method is providing the designer a systematic way of computing the state feedback control gain matrix.

The optimal problem is considered here in order to determine the controller gains, where the feedback gains **K** and **K**$_r$ of the control law which is given in Equation (6.16) are determined to minimize the performance index

$$J = \int_0^\infty (x^T Q x + u^T R u) dt \qquad (6\text{-}20)$$

Where **Q** and **R** are a positive-definite (or positive-semidefinite), and x is the state vector of the plant and the controller states. The second term on the right-hand side of Equation (6-20) accounts for the expenditure of the energy of the control signals. [4]

The direct solution for the optimal control gain is the MATLAB statement

$$[\mathbf{K}, \mathbf{P}, \mathbf{E}] = \mathit{lqr}\,(\mathbf{A}, \mathbf{B}, \mathbf{Q}, \mathbf{R}) \qquad (6\text{-}21)$$

This command return the gain matrix **K**, eigenvalues vector **E**, and matrix **P** which is the unique positive-definite solution to the associated matrix Riccati equation, which is given as in Equation (6.22).

$$PA + A'P - PBR^{-1}B'P + Q = 0 \qquad (6\text{-}22)$$

One reasonable method to start the LQR design iteration is suggested by Bryson's rule, where an appropriate choice to obtain acceptable values of **x** and **u** provides an initial choose of the diagonal matrices **Q** and **R** such that

$$Q_{ii} = 1/\,(\text{maximum acceptable value of } x_i)^2$$

$$R_{ii} = 1/(\text{maximum acceptable value of } u_i)^2$$

The weighting matrices are then modified during subsequent iteration to achieve an acceptable trade-off between performance and control effort. [4]

6.5 Controller Design

The first step in control system design process is to build a closed loop controller that is able to track the position of the trolley along the boom and track the desired angular position of the boom, with zero error, while keeping the out of plane and in of plane angle minimum as possible. In this stage of the design, a controller will be designed using the state feedback method.

In order to prevent the oscillations that occur due to the rotation and translation motions in the system and also due to the disturbances, trajectories are designed to be tracked for both inputs, which are the rotational and translational motors. The needed trajectories for both motions are designed to have an acceleration, constant speed, and finally deceleration, then reaching the steady state response.

In order to generate this signal the *cubic polynomials* method is used. The rotational angle ($\gamma(t)$) is assumed to be changed from 0-to-180 deg, and the traveling time is 20s; that means the boom will rotate from zero-to-180 deg in 20 sec, as shown in Figure (6.3). For the translational motion, the displacement of trolley changes from 0-to-1 meter, with 20 sec traveling time, as shown in Figure (6.4).

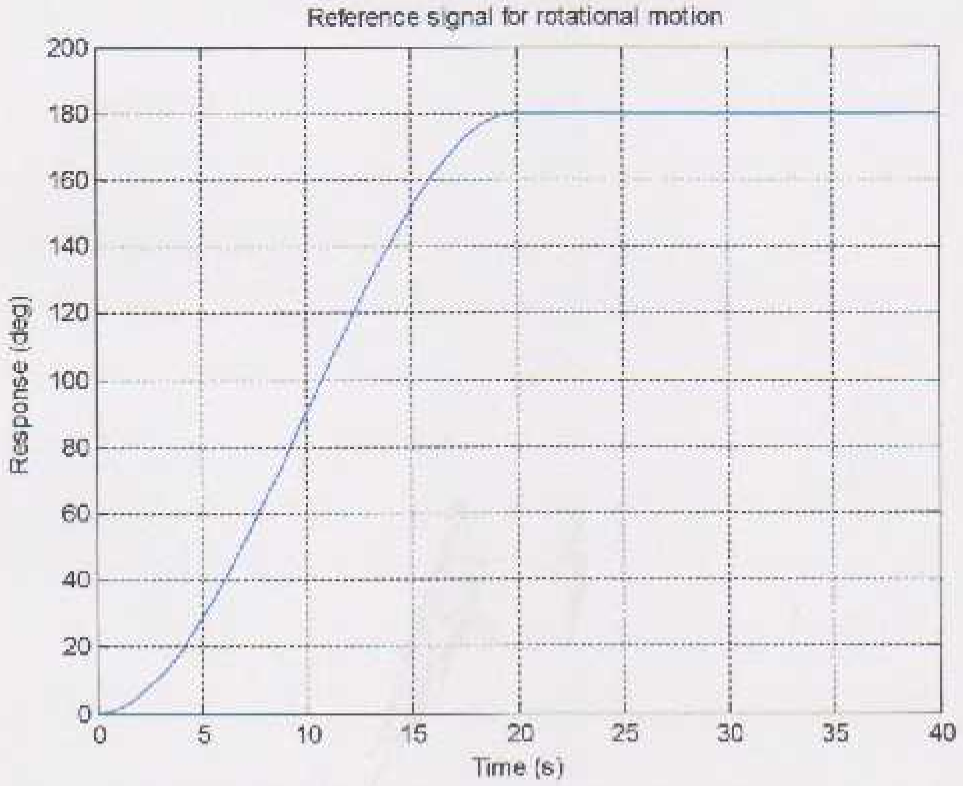


Figure (6.3): Desired reference signal for the rotational motion.

The cubic polynomials equation for the motion from rest to rest is given as:

$$\gamma(t)=\gamma_0(t)+\frac{3(\gamma_f-\gamma_0)}{t_f^2}t^2+\frac{2(\gamma_0-\gamma_f)}{t_f^3}t^3 \quad (6.23)$$
$$0\le t\le t_f$$

For the jib crane system γ_0 is assumed to be zero, $\gamma_f=180^{\circ}$, and $t_f=20\sec$. So the equation becomes:

$$\gamma(t)=1.35t^2-0.045t^3 \quad (6.24)$$
$$0\le t\le 20$$

The signal that is generated from Equation (6.24) is shown in Figure (6.3). But it is needed to determine the initial conditions that are required to generate this signal. However, these conditions are determined and found as follows:

$$x(0)=[0\ 0\ 0\ 1]' \quad (6.25)$$

For translational motion one case is studied, where $x_0=0$, $x_f=1$, and $t_f=20$ sec, so Equation (6.23) becomes:

$$x(t)=0.015t^2-0.0015t^3 \quad (6.26)$$
$$0\le t\le 20$$

Its initial conditions are the same as it given in Equation (6.25).

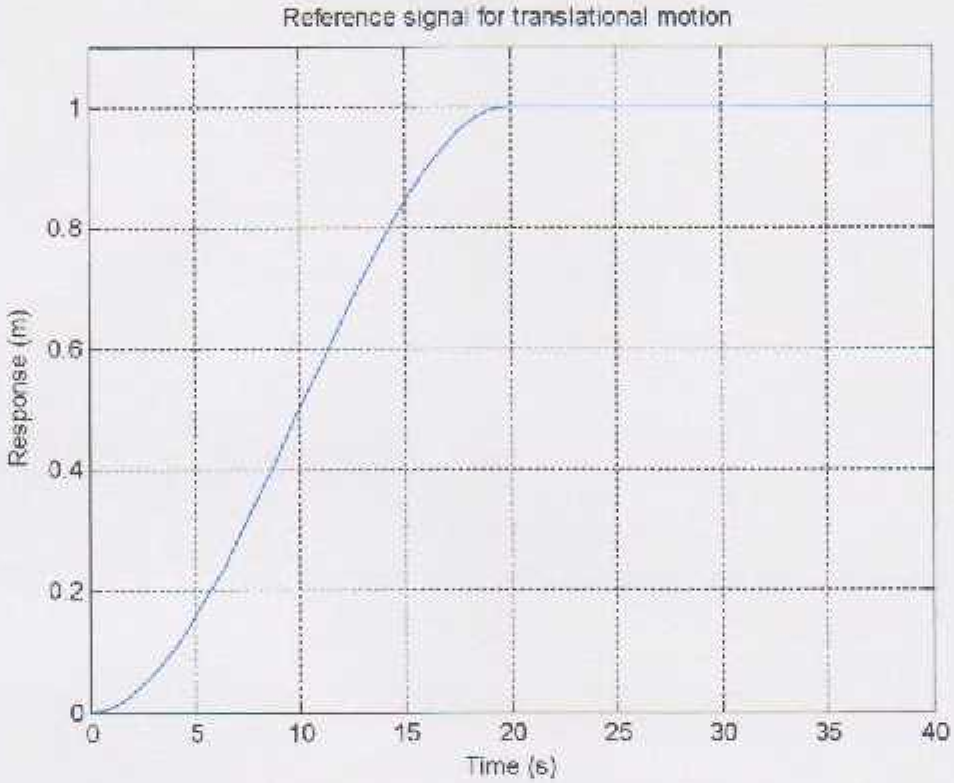


Figure (6.4): Reference signal fro translational motion.

Since the crane system is a multi input, multi output, the reference input signals must be represented into one state space representation, the method used for represent these two signal is:

$$\dot{x}_r = \begin{bmatrix} A_{r1} & 0 \\ 0 & A_{r2} \end{bmatrix} x_r + \begin{bmatrix} B_{r1} & 0 \\ 0 & B_{r2} \end{bmatrix} u_r$$

6-27

$$r = \begin{bmatrix} C_{r1} & 0 \\ 0 & C_{r2} \end{bmatrix} x_r$$

with initial conditions

$$x(0) = [x_1(0) \ \ x_2(0)]$$

So the state space model for the generated trajectories (the reference signals) is:

$$\dot{x}_r = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} x_r \qquad (6.28)$$

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} -0.0015 & 0.015 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.27 & 2.7 & 0 & 0 \end{bmatrix} x_r$$

$$x(0) = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Where the input signal r_1 is the translational input signal, and r_2 is the rotational input signal.

The disturbances that are possible to affect the system are assumed to be step signals, and this is logic since the disturbance will occur if the load is hit by something.

The pair (A_r, C_r) is observable and accordingly the minimal polynomial of the reference rotational signal is:

$$\Theta = s^4 \tag{6.29}$$

The condition that is given in Equation (6.9); so the controller can be constructed as given in Equation (6.10), and its matrices are determined which are

$$A_c = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad B_c = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \tag{6.30}$$

The composite system (plant and the controller)

$$\begin{bmatrix} \dot{x} \\ \dot{x}_{e} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -101.496 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -117.72 & 0 & 0 & 10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -49.05 & 0 & 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 13.646 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ x_{e} \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -0.0093 \\ 0.033 & 0 \\ 0.0167 & 0 \\ 0 & 0.0015 \\ \multicolumn{2}{c}{zeros(8,2)} \end{bmatrix} \begin{bmatrix} F_{x} \\ T_{z} \end{bmatrix} \tag{6.31}$$

is completely controllable, so that, there is a control law

$$u = -Kx - K_{e}x_{e} \tag{6.32}$$

such that the closed loop system is stable. Now the gains K, and K_v must be determined, and this can be done throughout minimizing the quadratic performance index which is given in Equation (6.20). However, this can be obtained using the *MATLAB* as given in Equation (6.21). The matrices Q and R are determined as the approach that is presented previously, where a lot of trials are done in order to obtain the desired response, and the least possible gains, which are always high. The corresponding weighting matrices are found as:

$$R = \begin{bmatrix} \frac{1}{16^2} & 0 \\ 0 & \frac{1}{10^2} \end{bmatrix} \qquad (6\text{-}33)$$

$$Q_x = \text{diag}\begin{bmatrix} \frac{1}{0.2^2} & \frac{1}{25e\text{-}4} & \frac{1}{8^2} & \frac{1}{\pi^2} & \frac{1}{1e\text{-}2} & \frac{1}{144e\text{-}6} & \frac{1}{1e\text{-}4} & \frac{1}{0.316^2} & \frac{0.001}{10^2} & \frac{1}{36e\text{-}6} & \frac{1}{1e2} & \frac{1.5}{56.25e\text{-}8} & \frac{0.001}{1e-4} & \frac{0.5}{9e-10} & \frac{1}{1e\text{-}3} & \frac{1}{81e\text{-}3} \end{bmatrix} \qquad (6.34$$

The first eight elements in Q matrix belong to the plant states, whereas the rest eight elements belong to the controller.

Consequently, the gains are

$$K = \begin{bmatrix} -0.0031 & 0.2024 & 1.8940 & -0.0145 & 0.0282 & -0.0982 & 0.5488 & -0.0030 \\ 0.6187 & -0.0139 & -0.0048 & 5.6169 & 0.1914 & -0.0012 & -0.0039 & 1.6608 \end{bmatrix} *1.0e{+}4 \qquad (6.35)$$

$$K_v = \begin{bmatrix} -0.0000 & -0.0267 & -0.1349 & -0.3412 & -0.0000 & -0.0002 & 0.0019 & 0.0026 \\ 0.0000 & 0.0000 & 0.0000 & 0.0002 & -0.0003 & -0.7459 & -1.2737 & -1.0877 \end{bmatrix} *1.0e{+}5 \qquad (6.36)$$

Unfortunately, the gains are of high value, since as given in Equations (3.23)-to-(3.26), there are always repeated poles at origin in the transfer functions between the inputs and outputs; and so to stabilize such a system, a high gains are needed.

The only available measurement is the angular position state $y(t)$. It is possible to construct an observer to estimate the other states in order to be used in the state feedback controller. Thus by choosing an appropriate gain vector (L), the poles of the error characteristic equation can be placed to achieve the desired speed of the observer. Using the state space model, and based on the fact that the system is observable, and choosing the closed loop poles of the observer to be 10 times greater than those of the real system, the gain vector (L) are

$$L = \begin{bmatrix} 0.0440 & 0.3276 \\ -0.7284 & 0.1677 \\ 0.0033 & -0.0059 \\ 0.0056 & 0.0014 \\ -4.3486 & 0.6654 \\ -3.1340 & -3.3420 \\ 0.5161 & -0.1970 \\ 0.0675 & 0.4219 \end{bmatrix} *1.0e+4 \qquad (6.37)$$

Where the gain vector (L) is found through placing the poles that are 10 times greater than the poles that are found previously using Equation (6.21), which are given as

$$
\begin{aligned}
p_1 &= -0.09 + 10.07i \\
p_2 &= -0.09 - 10.07i \\
p_3 &= -0.38 + 4.42i \\
p_4 &= -0.38 - 4.42i \\
p_5 &= -1.38 + 1.05i \\
p_6 &= -1.38 - 1.05i \\
p_7 &= -0.50 + 1.61i \\
p_8 &= -0.50 - 1.61i
\end{aligned}
\tag{6.37}
$$

So the controller parameters are calculated and determined to be used in the simulation, where its results for both the controller and observer based on the previous calculations are introduced in the next section.

6.6 Simulation Results

The next step in controller design process is simulation. This step is of significant importance to check whether the resulted system response meets the design specifications or not. Using the controller and observer design results obtained in the previous section, MATLAB and Simulink toolbox are used to simulate system performance. The model that is used in simulation process is shown in Figure (6-5).

In order to maintain the rotational angle and the displacement at their steady state values (180 deg, 1 meter), a switch is used to switch the reference signals when the time reaches the 20 s (after finishing the transient response) as shown in Figure (6.5).

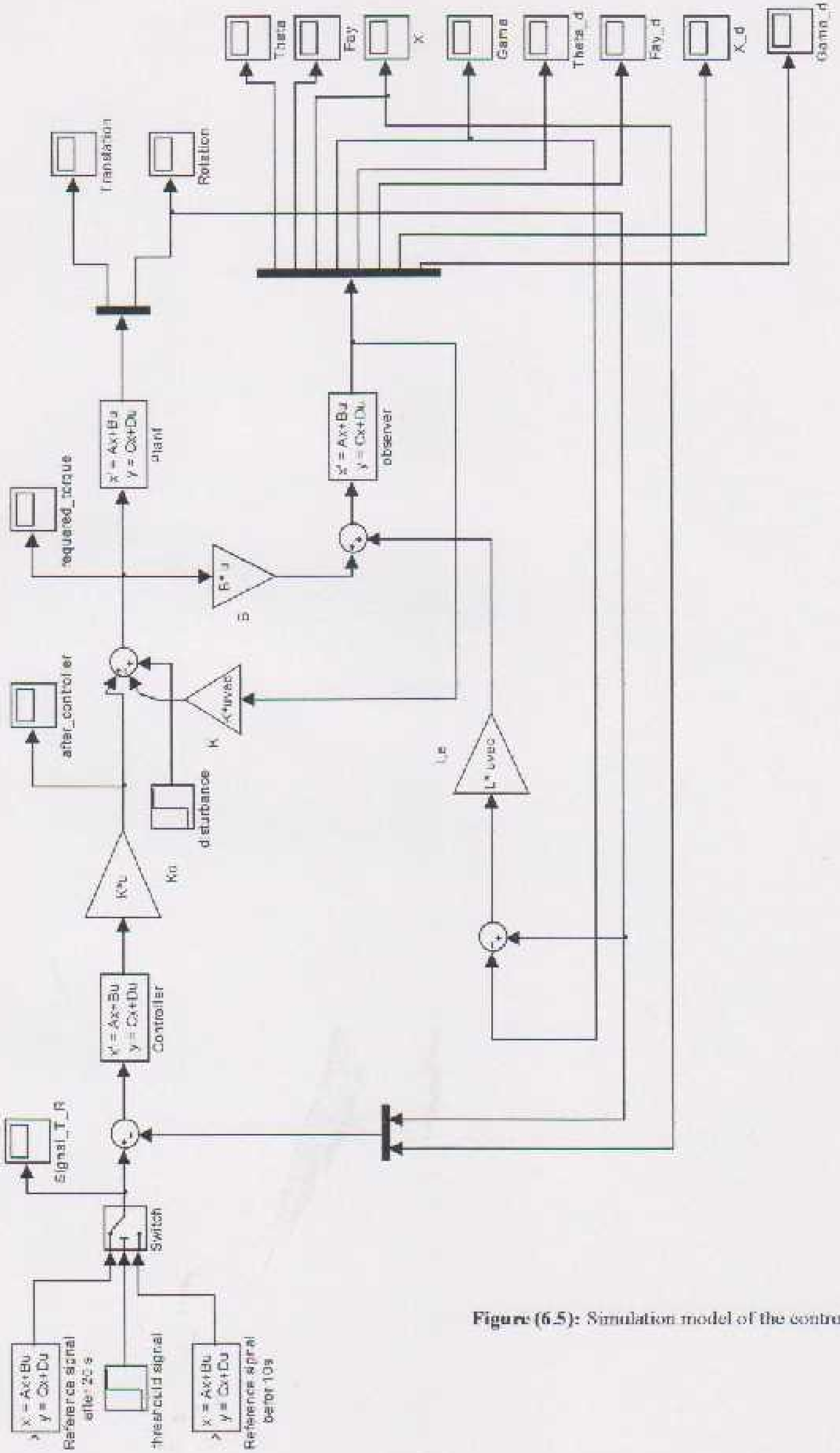


Figure (6.5): Simulation model of the controlling system.

The most important states that must be considered in the simulation are the rotational angle $\gamma(t)$ as an output, translational displacement $x(t)$ as an output, the in-of-plane angle $\phi(t)$, and the out-of-plane angle $\theta(t)$. However, for the output responses, Figure (6.6) shows the rational response for the input reference signal.

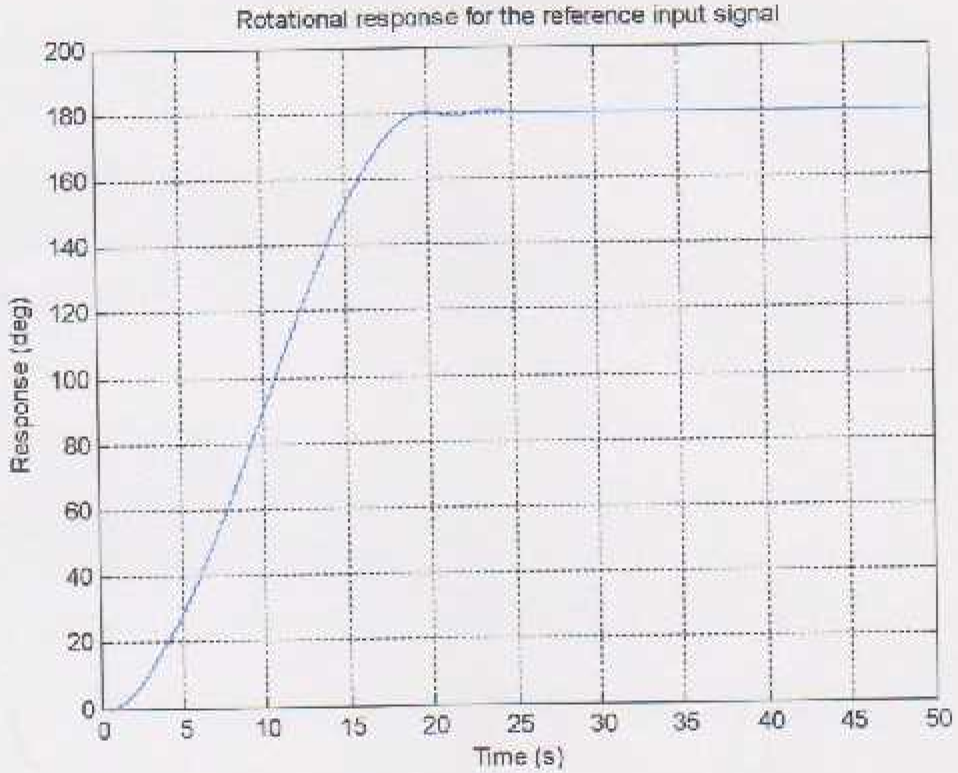


Figure (6.6): Rotational response for the designed controller.

As in the Figure, the reference input signal is fully tracked, and also with zero steady state error. Also a disturbance is applied to the plant, and the response doesn't affect. We can see also that the steady state position will be reached in 23s, while it is designed to be reached within 20s, but an overshoot is produced when the boom reaches the desired angular position, this is caused by the oscillations caused by the out-of-plane angle $\theta(t)$, where the oscillations in this angle are shown in Figure (6.7).

Figure (6.7): The oscillations in the angle $\theta(t)$

As shown in the Figure, the value of angle $\theta(t)$ reaches -1.5 deg when the boom starts rotating. It is in negative value since it is the reaction of the rotating, so it will be in its opposite direction, then when the boom reaches the 180 deg of rotation (desired) at t = 20s and stops, the load will oscillates again as a reaction of the stopping; so the value of the angle reaches up to 0.75 deg. To make the oscillations in the load die out, the boom must make overshoot about the steady state position which is clearly shown in Figure (6.6).

For the translation motion of the trolley, its response is shown in Figure (6.8). The steady state error is zero, and the motion trajectory is fully tracked.

Figure (6.8): Translational motion response of the trolley.

The Figure shows also a little bit overshoot in the response of the system which is about 0.95%, as a result, it can be accepted; but since the trolley is connected to the power screw, so this overshoot will not exist, where the system is modeled considering the trolley free to move along the boom. The settling time is about 25 s, which is good, and suitable; since the translational motor is selected to be slow to provide the needed torque to translate the trolley among the boom.

In-of-plane angle $\phi(t)$ response is shown in Figure (6.9), where the angle reaches 0.0053 deg at the starting of the motion, and -0.00095 deg at the stopping then it dies out as the trolley reaches its steady state position. The amplitude of the oscillations is very small, and don't worth to be mentioned.

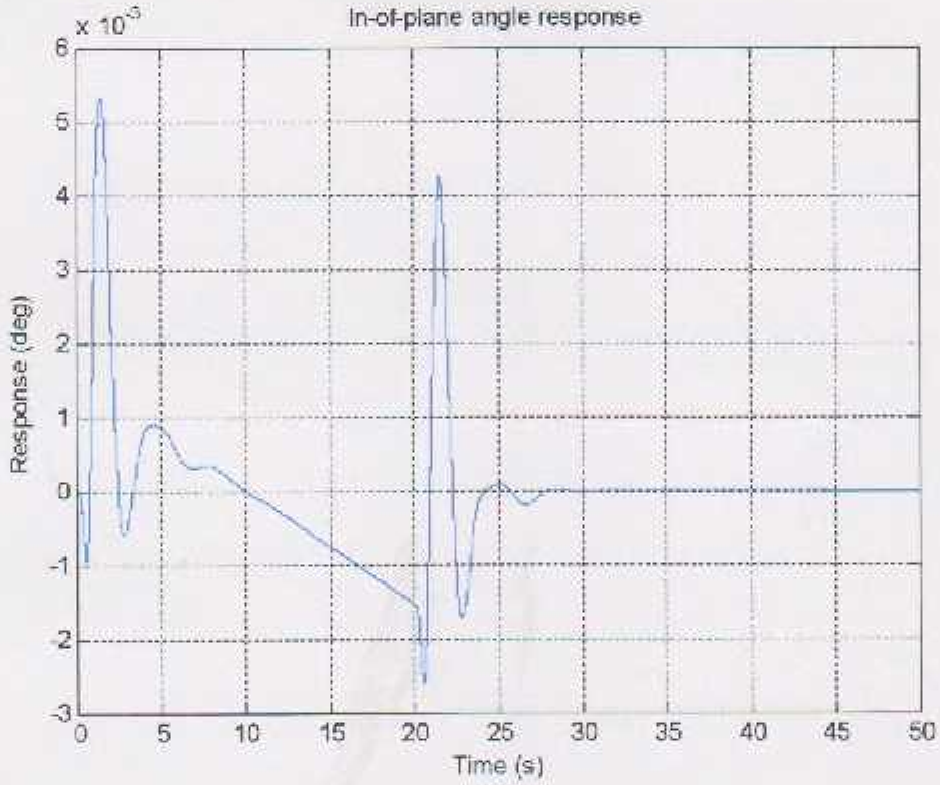


Figure (6.9): Oscillations in the angle $\phi(t)$.

Chapter Seven

Results and Recommendations

- Practical Results
- Recommendations
- The Complete Design of the Jib Crane

Chapter Seven

Results and Recommendations

This chapter contains the results that are obtained from the experiments which are done to verify the theoretical results reached in the previous chapters, where the mechanical, electrical, and control designs are used to be applied in the practical side. The practical results and modifications that are done to the theoretical results are discussed.

A group of suggestions and recommendations are provided to enhance the performance of the system especially at the control field. Also the problems that faced the working team will be expressed.

7.1 Practical Results

7.1.1 Practical Mechanical Structure

As discussed in chapter four, the mechanical parts that are designed and analyzed in order to be used in building up the system are completely formed and used as they are designed, with its true dimensions. The following figures show a group of practical pictures for the assembled parts, which are used in the mechanisms of the project. The rotational motor mount mechanism parts are shown in Figure (7.1), where the aluminum diagonal bars are connected to the motor neck through the horizontal plate as shown.

Figure (7.1): Practical rotation motor mount.

The rotational pivot is also shown in the figure, where the lower flange is clearly shown, which represents the lower part of the pivot. The boom mount also shown in the figure.

Power screw mechanism which is dependent to translate the trolley along the boom is practically applied and it can be shown in Figure (7.2). It can be seen that the bolt over the screw, which is of 1.5 mm teeth pitch and of 16 mm diameter, and it not of square tooth, so the translation operation is slow. This was one of the problems that subtended the working team, where the power screws that are suitable to the application is of very high cost, so it is replaced by a normal screw, where it satisfies the needed application in some manner. The screw is fixed to the boom through a bearings attached to aluminum cups. Also the rollers that transfer the trolley along the boom are shown, which are 4 bearings.

Figure (7.2): Practical power screw-mechanism of trolley translation.

These are the most important mechanical parts that form the most important mechanisms in the system, and provide the concept of theoretical design then translating to the practical work.

7.1.2 Practical Electrical Structure

After finishing the assembly of mechanical parts, the electrical parts are added to provide the safety factor to the system and to operate and control the motors. A group of limit switches are added to the system as designed and given in chapter six, where 2 limit switches are added to limit the rotation motion to be between 0-to-210 deg (not complete cycle). Other 2 limit switches added to limit the motion of the trolley along the boom, where they are positioned at beginning and ending of the boom. For the hoisting motor, a limit switch is added to prevent the motor from hoisting the load up continuously. These limit switches are shown in Figure (7.3).

(a)

(b)

(c)

Figure (7.3): Limit switches in the project.

a- Translation motion limit switches.

b- Rotation motion limit switches.

c- Hoisting motion limit switch.

The controlling unit for the rotation motion and its protection fuses. Figure (7.4) shows the Ac inverter and it's controlling and protection elements assembled in panel.

Figure (7.4): Controlling devices of the rotational motor.

The Ac-inverter receives its input signal from the computer as 0-10 V, and accordingly the speed of the motor controlled according to input signal.

Up to this point, the project is working properly, and it is ready to be operated in the needed applications with manual control, where the operator can position the load in the desired position through the traditional control, so the electromechanical system now is ready.

7.1.3 Practical Controller Design

The controller which is designed and built in chapter six can't be applied to the practical system due to several reasons. First one is technical, where the AC-inverter doesn't contain input for torque signal, or in other words it doesn't provide a controlled torque signal to the motor, and as known, the input to the plant is torque signal. So this represents one of the problems that faced the working team. The second reason is the high gains that are needed to be provided from the actuator to get the needed position. However, the needed control here is position control; that means providing the system with the needed rotational angle, and trolley position along the boom, then the system must respond. So another controller is designed specially to control the angular position of the load. This controller is simple, where its idea is to find the transfer function between the input voltage to the AC-inverter and output angel.

This can be done by finding the closed loop transfer function (T(s)) to the whole subsystem (rotation part), where a factor of translating the needed angle into voltage is determined experimentally and is found as 0.01377 V/deg. then for the required input angle, the actual output angle is measured, and then the closed loop transfer function can be determined. Through the closed loop transfer function, the open loop transfer function (G(s)) can be determined, then designing the needed controller which will be either P, or PI controller. The Simulink model for such a system with PI controller is given in Figure (7.5). Another problem appeared, that the negative error signal cant be read by the inverter, so it doesn't convert the direction of rotation of the motor, so a conditions are provided to the model in order to convert the sign of error signal, and to opposite the direction of rotation of motor through the relays connected to the inverter simultaneously, and this is clearly shown in Figure (7.5) by using the switches Simulink block.

The working team started the work and it still work to perform the mission, where a lot of technical problems facing them every moment related to the safety of the project parts, and interfacing circuits, specially the power amplification circuits.

7.2 Recommendations

During the theoretical and practical working, a lot of problems faced the working team throughout all the project stages. The first problem was in derivation the model of the system, where the operating points are determined in order to linearize the model equations. These operating points are determined using numerical iteration method, which requires a lot of numerical analysis, and so a lot of working time is needed, so the team recommends offering a suitable work station computer to perform such a work.

Other thing requires such a work station that is the mechanical design analysis. This analysis is done using the ANSYS software program. The analysis operations last to long time especially if the mechanical part is of complex design, so it is highly appreciated if this station is offered to the student.

At the practical side, most of the problems that faced the working team in controlling the rotational motion were due to using an Ac-inverter, where the torque control can't be done, so it is recommended to use a servo motor provided with its driver which provide controlled torque signal.

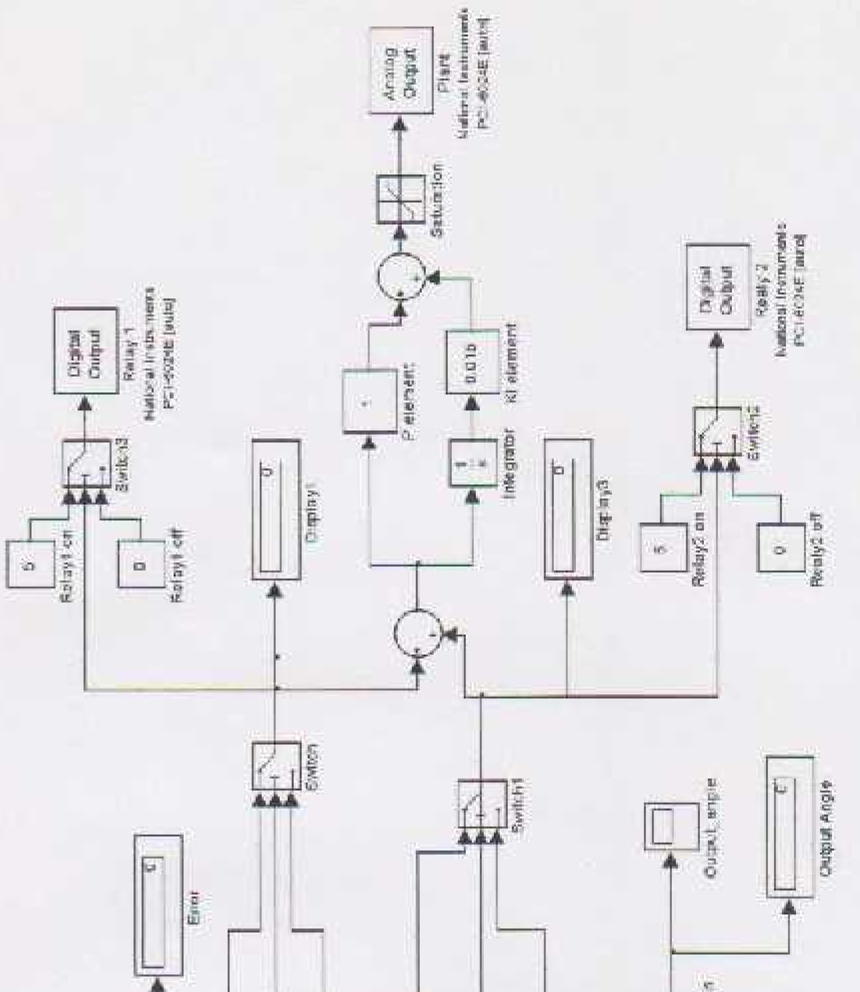


Figure (7.5): Simulink model of the alternative controller.

Working team recommends developing the system to be fully controlled for the three motions: rotation, translation, and hoisting, and also to be performed simultaneously. This can be done using displacement sensor, which provide how far the trolley is from the axial line of the rotating joint. Also a weight sensor is needed to measure the weight of the load. According to these measurements the torque signal is controlled and provided to the actuators which rotate, translate, and hoist the load simultaneously and efficiently with the suitable speed and needed torque.

7.3 The Complete Design of the Jib Crane

The system is completed with all of it parts and units and it is given in Figure (7.6).

Figure (7.6): Complete design of the Power Assisted Jib Crane.

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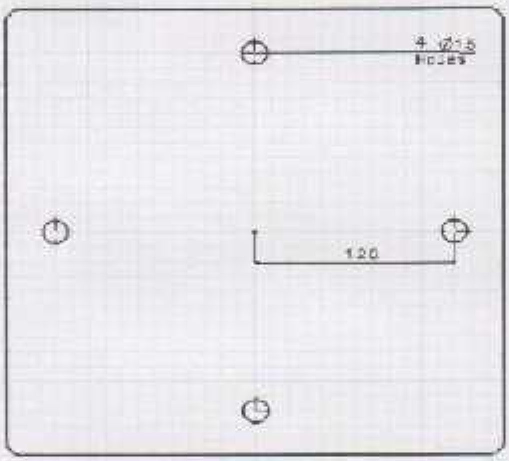
Appendix A

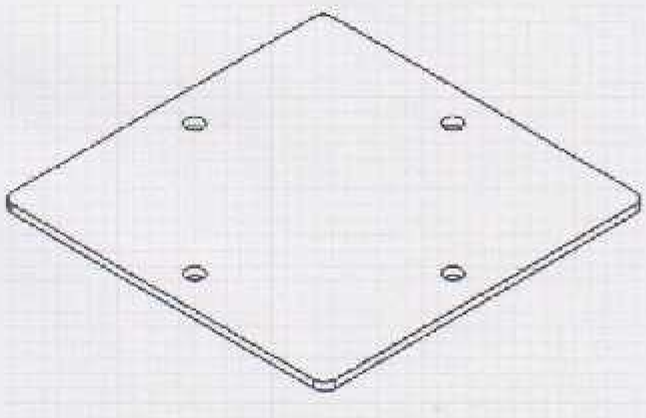
Dimensions of the Mechanical Parts

Note: All dimensions in millimeters (mm).

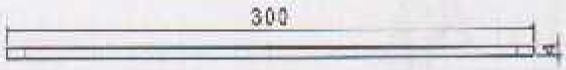
1- **Base**



Front view
Scale: 1:1

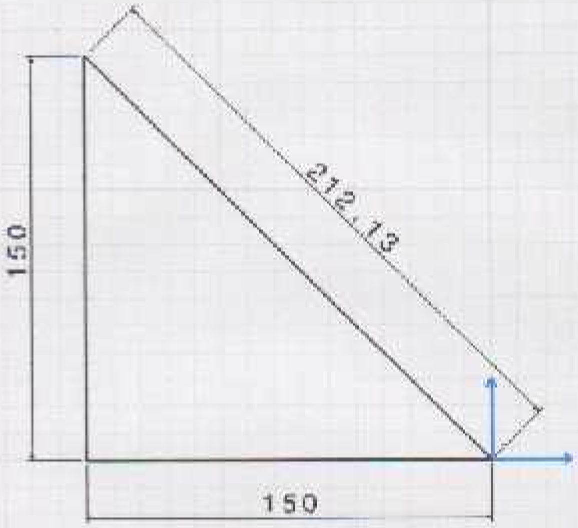


Isometric view
Scale: 1:1

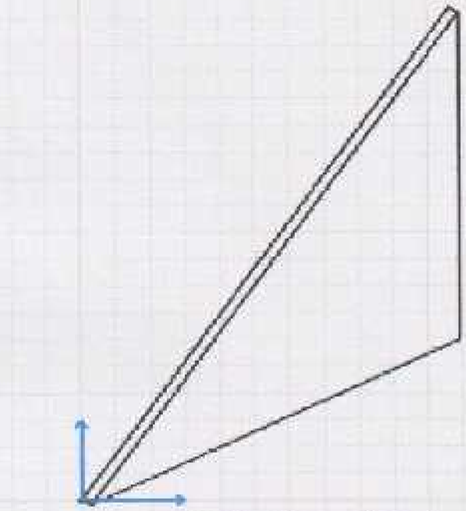


Top view
Scale: 1:1

2- **Mast support**



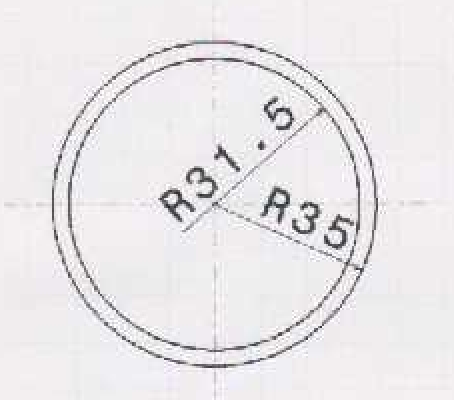
Bottom view
Scale: 1:1



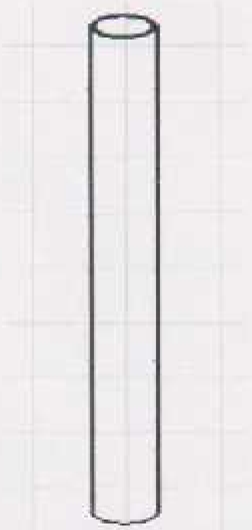
Isometric view
Scale: 1:1

Front view
Scale: 1:1

3- **Mast**



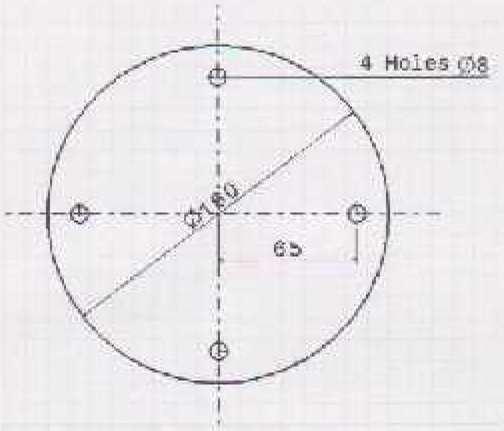
Front view
Scale: 1:1



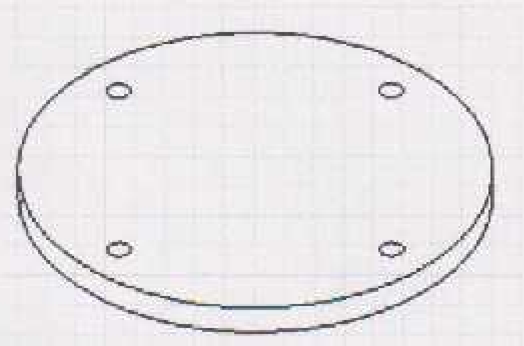
Isometric view
Scale: 1:1

Right view
Scale: 1:1

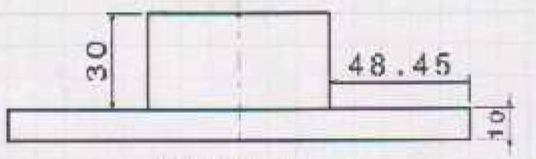
4- Mast upper plate



Front view
Scale: 1:1

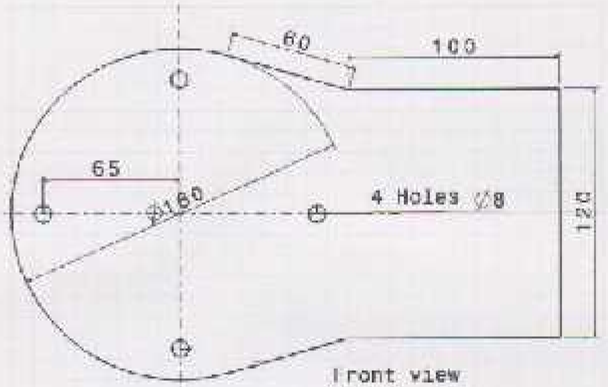


Isometric view
Scale: 1:1

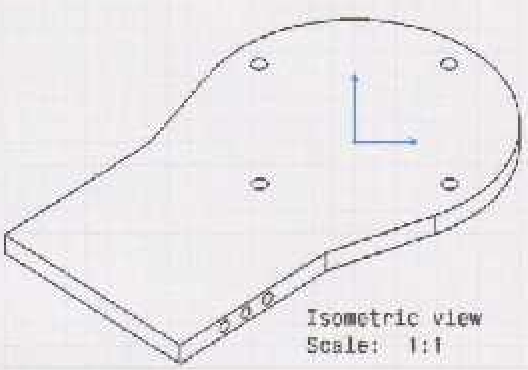


Top view
Scale: 1:1

5- Horizontal motor mount plate



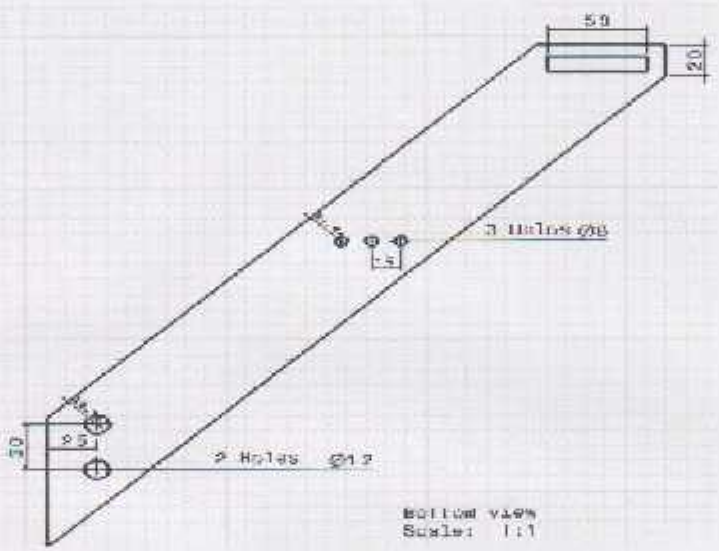
Front view
Scale: 1:1



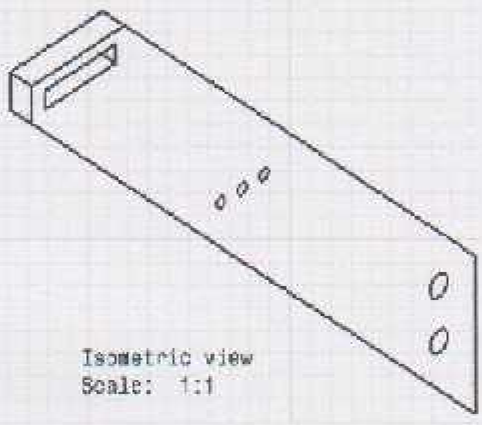
Isometric view
Scale: 1:1

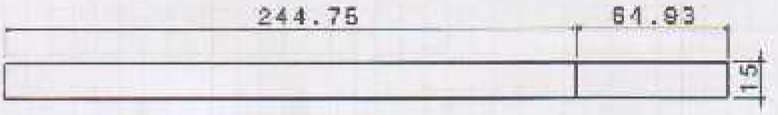
Top view
Scale: 1:1

6- Diagonal supporting motor



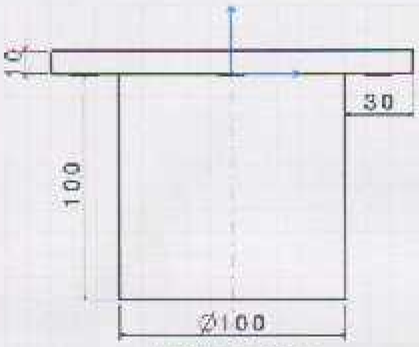
Bottom view
Scale: 1:1


Isometric view
Scale: 1:1

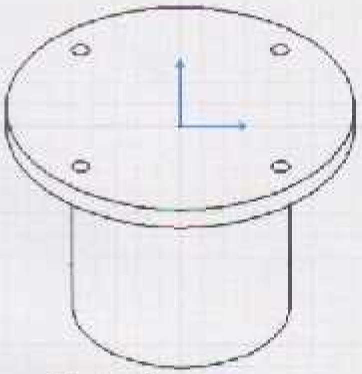


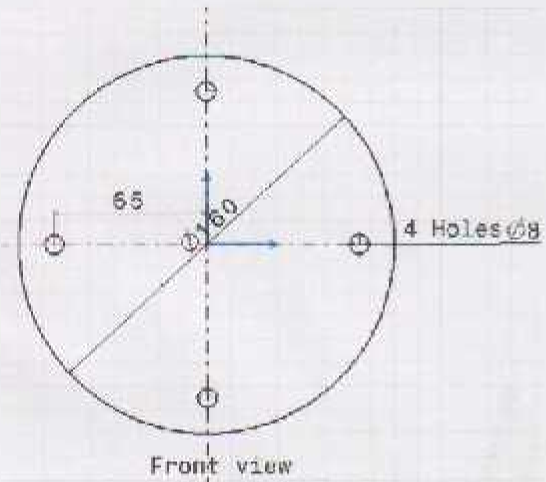
Front view
Scale: 1:1

7- **Lower flange**

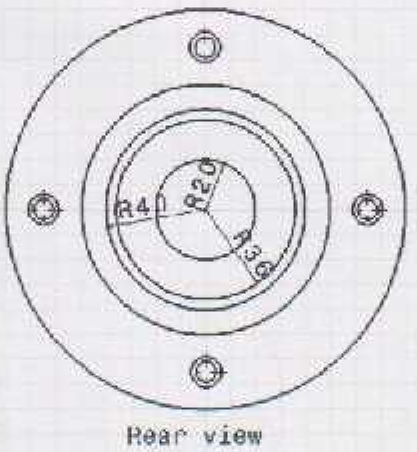


Bottom view
Scale: 1:1


Isometric view
Scale: 1:1

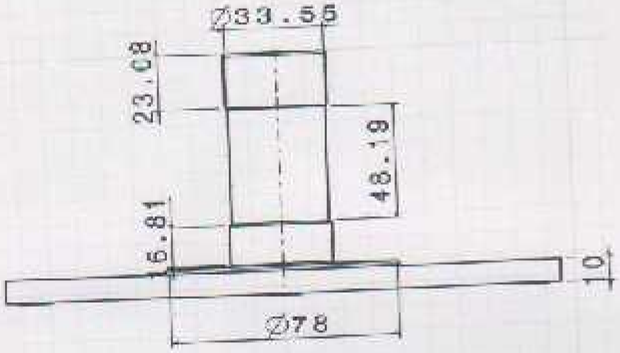


Front view
Scale: 1:1

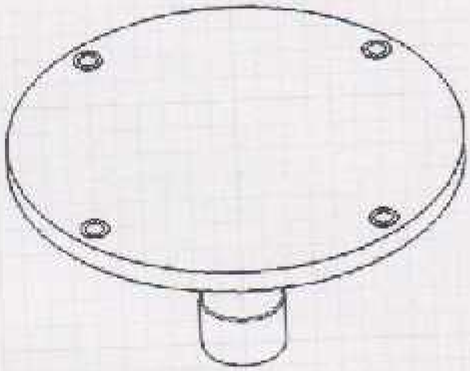


Rear view
Scale: 1:1

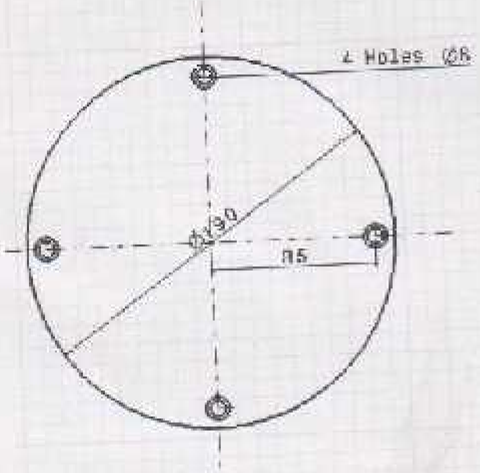
8- Upper flange



Top view
Scale: 1:1

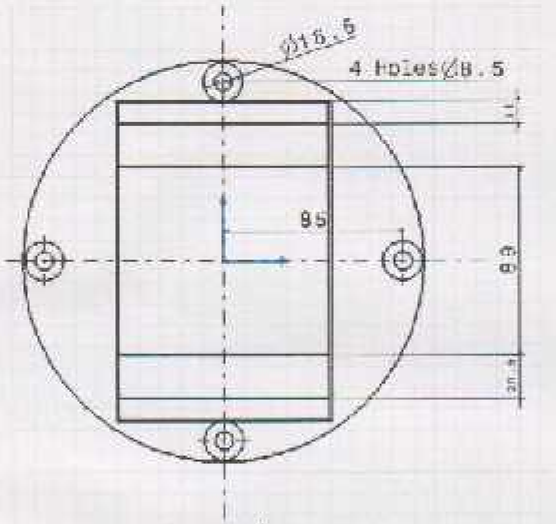


Isometric view
Scale: 1:1

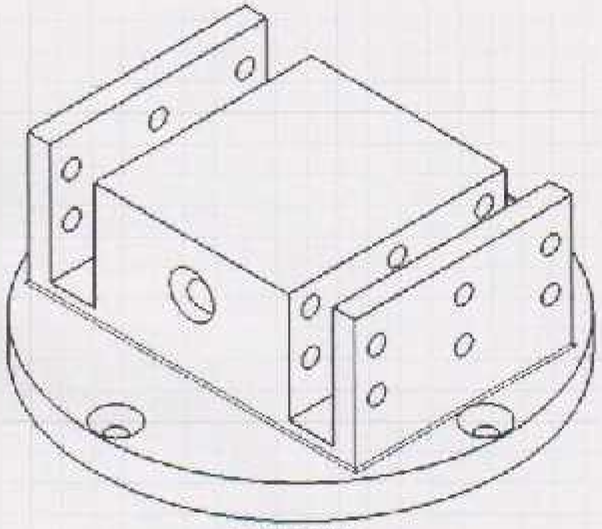


Front view
Scale: 1:1

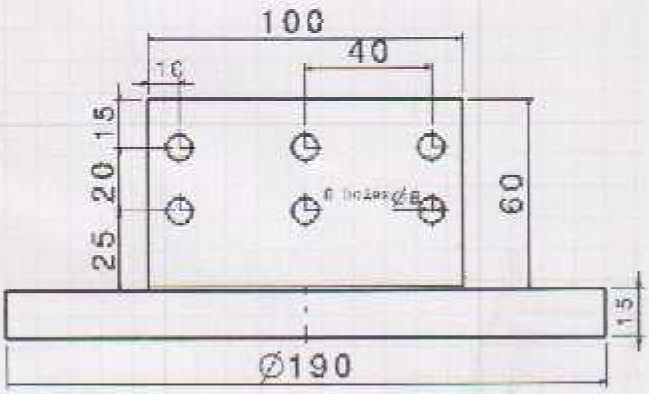
9- Boom Mount



Front view
Scale: 1:1

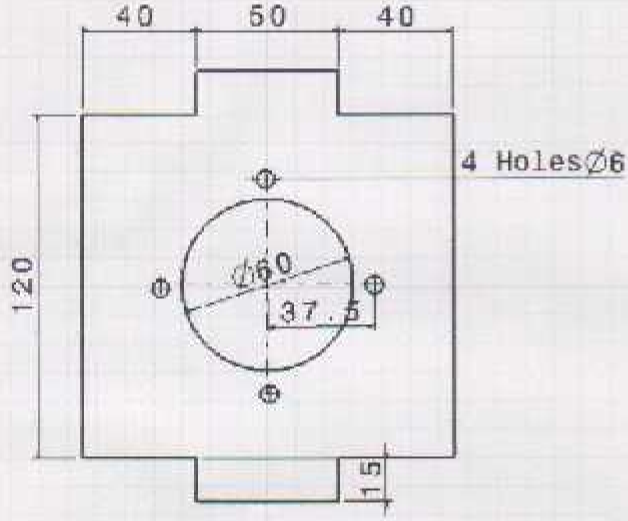


Isometric view
Scale: 1:1

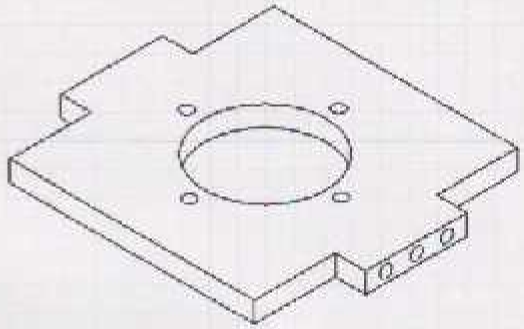


Bottom view
Scale: 1:1
dimentions: mm

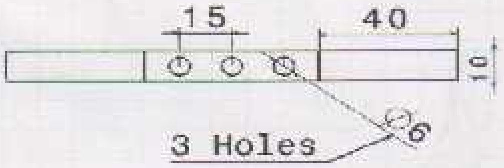
10- **Rotational motor neck mount**



Front view
Scale: 1:1

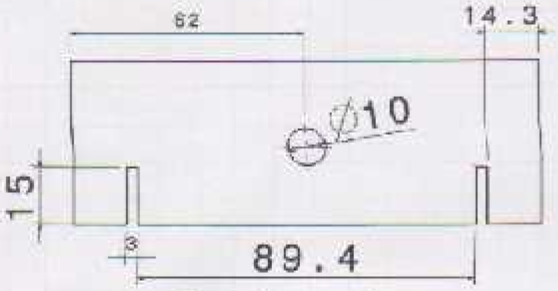


Isometric view
Scale: 1:1

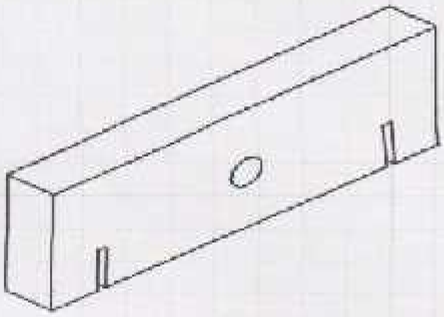


Bottom view
Scale: 1:1

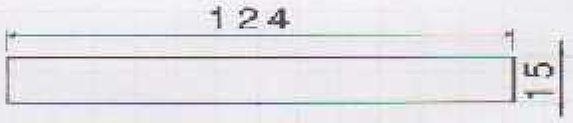
11- **Screw Cup**



Bottom view
Scale: 1:1

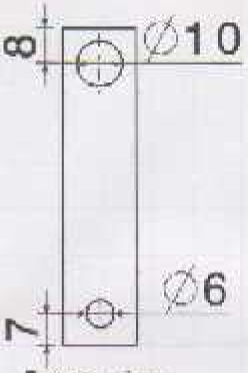


Isometric view
Scale: 1:1

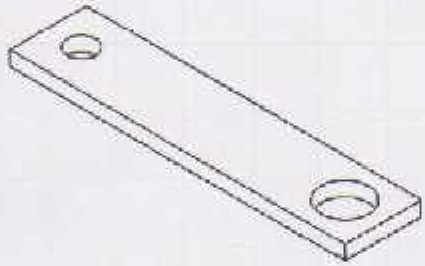


Front view
Scale: 1:1

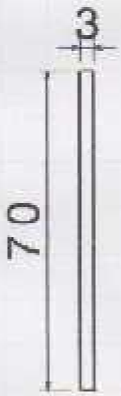
12- Trolley-roller connecting plate



Front view
Scale: 1:1

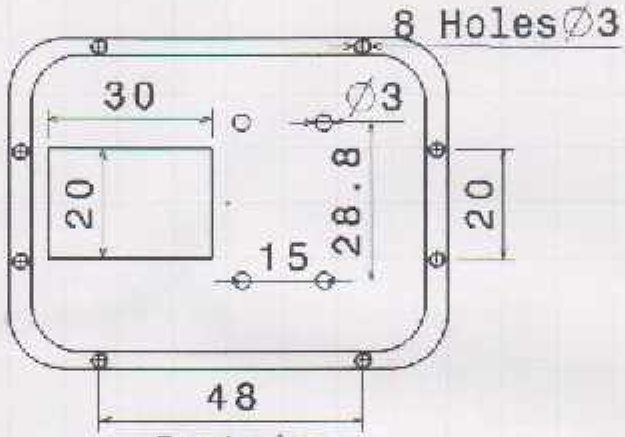


Isometric view
Scale: 1:1

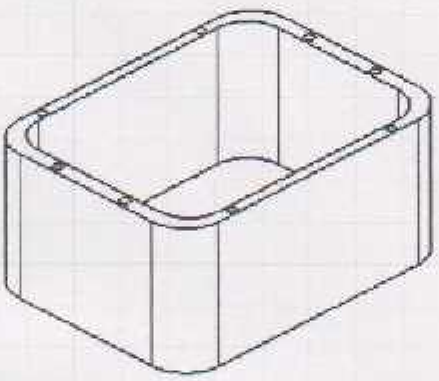


Right view
Scale: 1:1

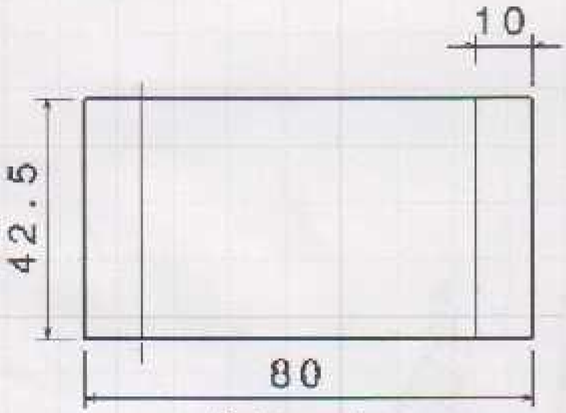
13- Trolley box



Front view
Scale: 1:1



Isometric view
Scale: 1:1



Bottom view
Scale: 1:1