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Designing and Constructing an Educational Apparatus for Control of Motion

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in partial fulfillment of the requirements for the
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بسم الله الرحمن الرحيم

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at Palestine Polytechnic University

Jan 2021

Abstract

Control science is one of the most important sciences to understand well, one of the most important problems facing students at the university is not supporting the theoretical information with Practical experiments because these experiments are limited to laboratories due to the difficulty of moving devices from one place to another place. The purpose of this project is to build an educational control apparatus that fulfills several requirements as light weight, modularity, and easy to assembly. Components of the model, its mechanism and applications have been examined to meet the desired specifications. It was designed to consist of a servo motor, servo driver, Data acquisition (DAQ) and computer. Programming a controller by Simulink Matlab and then sends the digital signal to the DAQ to convert a digital signal to an analog signal, the driver received the signal and provide signal for the servo motor and the servo motor provides motion for the applications as pendulum, inverted pendulum, and crank slider. In this graduation project we designed and constructed the educational apparatus and applied the pendulum application and document its results. Where Several requirements have been achieved such as light weight and flexible for applications (can add another applications) while the apparatus is not portable because we faced many problems to interface between servo driver and Matlab software.

Dedication

We dedicate our research project to our beloved parents for their continuous invaluable support and encouragement all through the years and to our dear siblings for providing us with a comfortable environment for study and research.

Acknowledgement

We would like to express our gratitude to our supervisor, Prof. Karim Tahboub, for his full support and guidance and remarkable suggestions. We would also like to thank our teachers for all the efforts they have exerted to make us qualified engineers who can assume-with confidence-our role in building our community. Thanks, are also due to our classmates and friends for their cooperation and encouragement.

We would also like to thank the mechanical engineering department at Palestine Polytechnic University for their financial support.

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Chapter 1: Introduction

Control science is one of the most important things to understand well because it is important and essential to technological progress and development and to facilitate human life. One of the most important problems facing students at the university is not supporting the theoretical information with real applications, which represents a gap in the correct understanding of the science of control. There are many companies as Quanser and Educational Control Product were produced and developed educational control devices such as spring mass model. The devices that exist in the university are limited, big, not portable, and requires laboratories with dedicated tools to operate and apply to it.

1.1 Background:

This section shows the existing educational apparatus. It will be presented in terms of design, application and function specification.

1.2.1 Classical spring mass damper system

This classical plant is readily transformed into the variety of configurations. It serves to vividly demonstrate both lumped parameter dynamics and generic control issues. This system appears commonly in dynamics and controls text books and serves as a benchmark for control method evaluation [1].

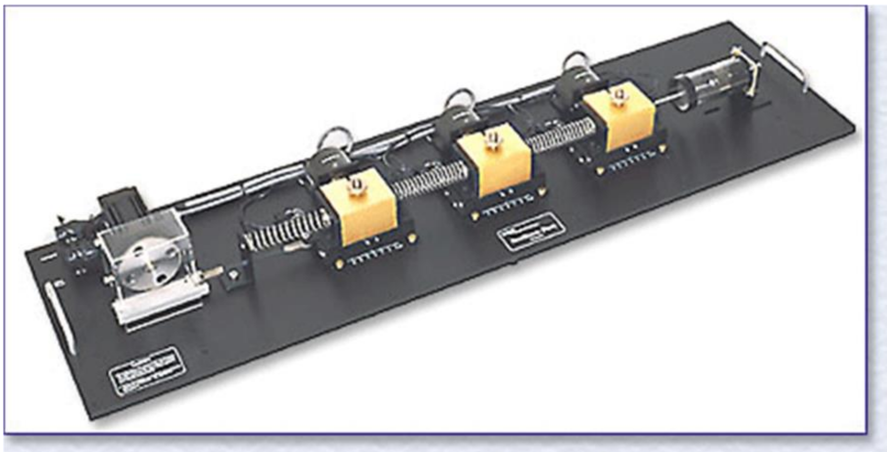


Figure 1.1 Spring mass damper system rectilinear motion [1].

1.2.2 The Educational Control Product 505 model “Inverted pendulum”

The inverted pendulum is a pendulum that has its center of mass above its pivot point. The ECP 505 model “inverted pendulum” includes removable and adjustable moment-arm counter-weights on the vertical and horizontal rods for easy adjustment of plant dynamics. It features linear and rotary ball bearings at the joints for low friction and repeatable dynamic properties. The inverted pendulum is a classic problem in dynamics and control theory and is used as a benchmark for testing control strategies [2].

Figure 1.2 Rotational inverted pendulum [2].

1.2.3 Internet-based control of double inverted pendulum on a cart (DIPC)

The function of control system in DIPC is to stabilize both links as shown in figure 1.3 in their vertical position, while tracing a desired position of the cart along the rail, in addition to reject disturbance that may act on the system. The controller is designed and simulated using Matlab and Simulink. In order to meet the hard-real time requirements of such a system, the controller is implemented on a desktop computer equipped with DAQ using xPC target technique with either local or global host-to-target connections [3].

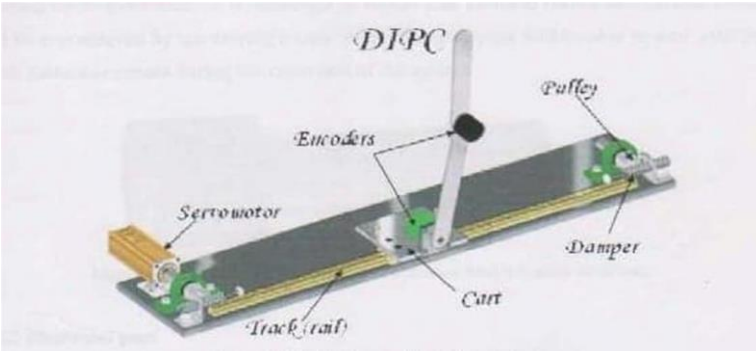


Figure 1.3: Double inverted pendulum on a cart [3].

1.2.4 Linear flexible inverted pendulum

The linear flexible inverted pendulum module augments the classic inverted pendulum challenge by including a flexible link that requires balancing. The linear flexible inverted pendulum module attaches to the linear servo base unit and has both a rigid long rod and a flexible link. The goal is to balance both pendulums using the base angle measurement as well the deflection angle of the flexible link [4].

Figure 1.4: Linear flexible inverted pendulum [4].

1.2 Problem statement:

Due to the development of technology, the importance of control theory increases. It has become necessary to understand the effect and rules of the controllers correctly to close up it to students.

The instructor which face some difficulties to link a theoretical concept with real applications. The existing educational devices have many challenges such as:

- They exist at university labs only.
- They are not portable.

- They need a special hardware and computers to work.

1.3 Methodology:

Building the project through several steps:

- Understanding the requirement.
- Proposing solution (design).
- Evaluate the designs and choosing a suitable one.
- Designing the subsystems.
- Building the subsystem and testing them.
- Assembling the whole system and testing it.
- Preparing experiments and documenting them.

1.4 Budget:

The total estimated budget for project for implements this price is 3100NIS, As detailed in table below:

No	Description	Quantity	Amount	Price	Unit	Contributor
1	Servo motor and driver	1	1	3500	NIS	Mech. Eng. Department
2	Build the structure	1	1	800	NIS	students
3	DAQ 6221	1	1	4000	NIS	Mech. Eng. Department

Table 1.1 Budget

Chapter 2: Conceptual design

This chapter describes the educational apparatus system, including the system components (subsystems), parts, functions and relations between elements.

2.1 Requirement of Project

We will build an educational device to apply different controllers that were learned at university with the following requirement:

- Reliable and durable for a long time.
- Be portable and easy to move from a place to another place.
- Ability to connect with a laptop.
- Flexible device to apply many applications.
- Operated by 220 AC voltage source.
- Its application should be clear and easy to mount and operate.
- Operates using Matlab software.

2.2 Proposing solutions

1. Proposed Software solutions

   Many softwares are used to build control on it, such as Matlab and LabVIEW.

   Matlab software will be used because of the following points:

   - It is a computing language for control algorithm development and simulation.
   - It is used for simulations because of additional libraries that contain higher-level functions.
   - It has co-software and is called Simulink which is used to design a controller.
   - It is flexible and easier than LabVIEW and students have a good knowledge of it.

2. Proposed hardware solutions

   The proposed apparatus solution consists of a computer having a Matlab software, DAQ, AC servo motor with its driver and encoders.

   Many applications were studied to be used with our apparatus to apply the controllers and two ideas are supposed for way of connecting the servo motor with applications. The first one is to use servo motor for each application and the second is to use a flexible design mechanism for all applications with a single servo motor.

The second reason for selected is to have a small size and weight,

and the electric component should save power.

Many challenges were faced for the way of interfacing the servo motor driver with Matlab software.

Some hardware does not support analog out as Arduino, and some of it do not support from matlab software as myrio.

2.3 Conceptual design schematic:

Conceptual design schematic shown in Figure 2.1, there the system has a 220 AC power supply that apply power to both servo driver and computer. The computer with Simulink software is connect with DAQ 6221 through PCI adapter to send the control commands. The DAQ 6221 send analog control command from it is I/O port to the driver directly. The servo driver receives the analog signal then transform it to a motion commands based on a programmed motion mode (torque mode). The motion commands are sent to the servo motor and the motor start rotating. The encoder measures the motor speed and position then feed it back to the driver, the encoder signal out from driver is differential and used IC to convert the signal to pulses then this signal enter to the I/O port of DAQ 6221, after that it sent the counting read to the matlab by PCI connection.

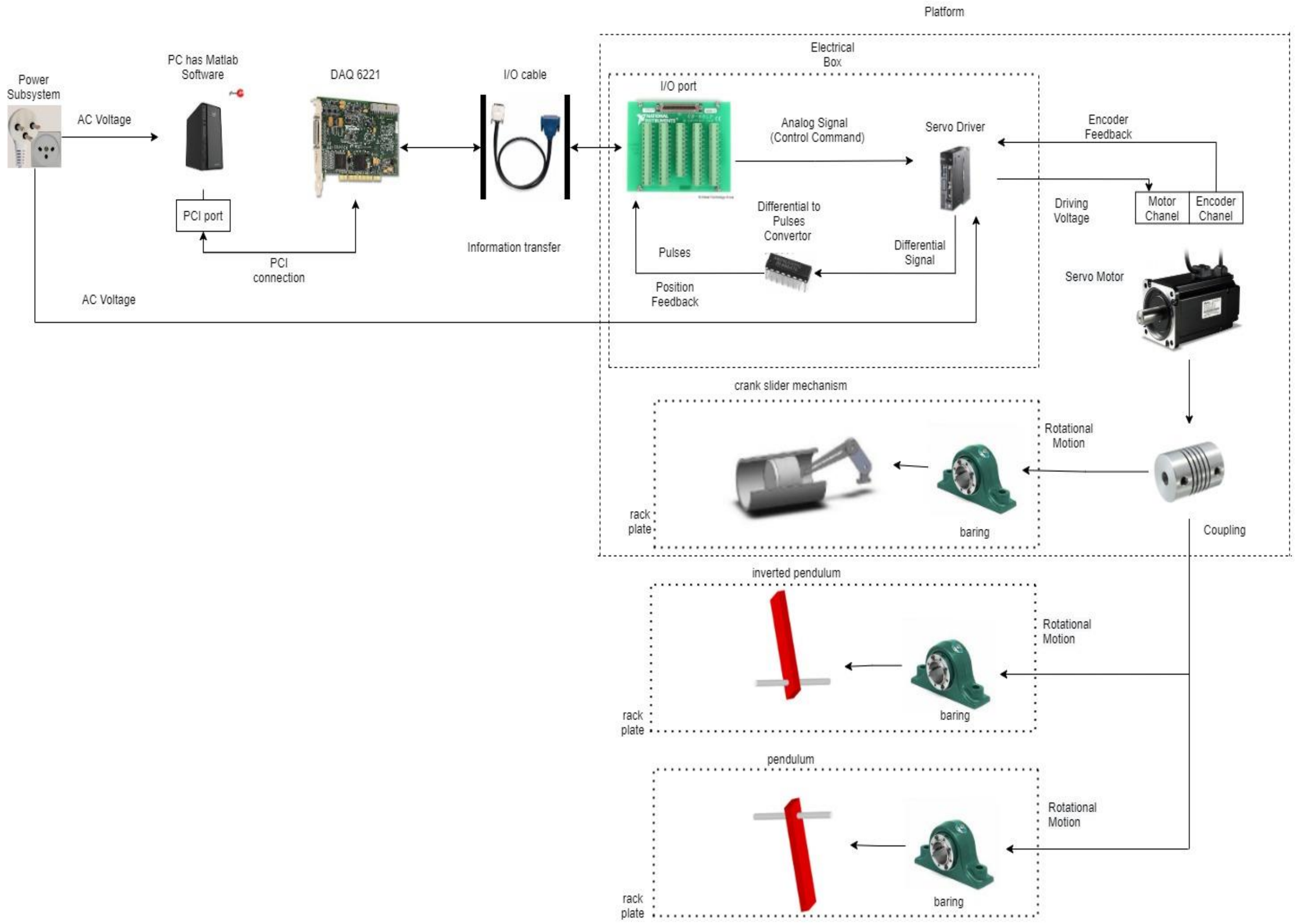


Figure 2.1: Conceptual Design Schematic

2.4 Components:

2.4.1 Servo motor and driver

A servomotor is a rotary actuator or linear actuator that allows for precise control of angular or linear position, velocity and acceleration. It consists of a suitable motor coupled to a sensor for position feedback. It also requires a relatively sophisticated controller, often a dedicated module designed specifically for use with servomotors.

Servomotors are not a specific class of motor, although the term servomotor is often used to refer to a motor suitable for use in a closed-loop control system.

In this project we will use delta ASDA-A2-M series; it is more advanced than other series of servo drives. And it has many applications useful for this project.

2.4.2 Applications

We will use flexible applications that are mean easy to installing and removing.

1. **Pendulum:**

   Pendulums are in common used. Some have crucial uses, such as in clocks; some are for fun, such as a child's swing.

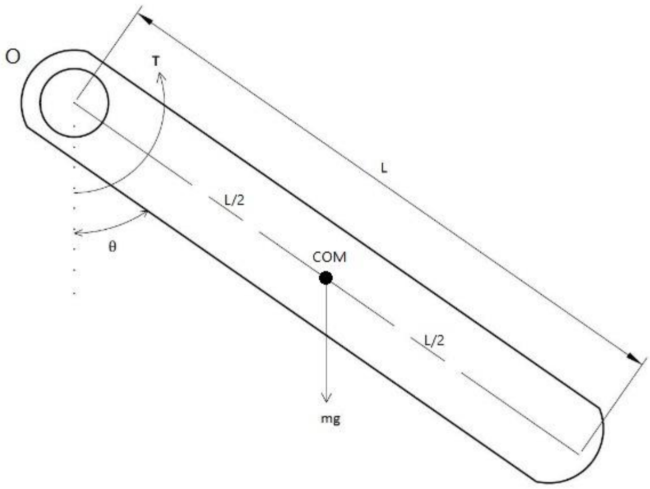


Figure 2.2 Pendulum

A pendulum consists of a mass m hanging from a string of length d and fixed at a pivot point P. When displaced to an initial angle and released, the pendulum will swing back and forth with periodic motion. [5].

2. Inverted pendulum:

Inverted pendulum is a pendulum that has its center of mass above its pivot point. It is unstable and without additional help will fall over. It can be suspended stably in this inverted position by using a control system to monitor the angle of the pole and move the pivot point horizontally back under the center of mass when it starts to fall over, keeping it balanced. The inverted pendulum is a classic problem in dynamics and control theory and is used as a benchmark for testing control strategy[6].

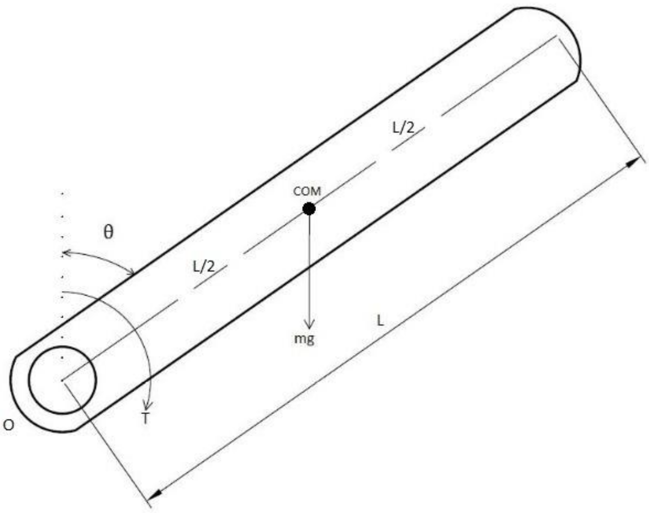


Figure 2.3: Inverted Pendulum

3. Crank Slider mechanism

Slider-crank mechanism, arrangement of mechanical parts designed to convert straight-line motion to rotary motion, as in a piston reciprocating engine, or to convert rotary motion to straight-line motion, as in a reciprocating piston pump. The slider- crank mechanism is a particular four-bar linkage configuration that exhibits both linear and rotational motion simultaneously.

This mechanism is frequently utilized in undergraduate engineering courses to investigate machine kinematics and resulting dynamic forces. The position, velocity, acceleration and shaking forces generated by a slider-crank mechanism during operation can be determined analytically [7].

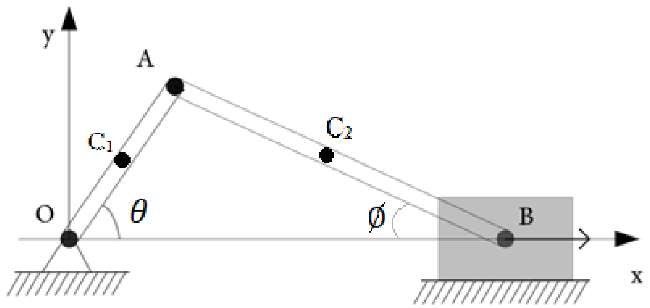


Figure 2.4: Crank slider mechanism

2.4.3 Position sensor

Encoder sensors are a type of mechanical motion sensor that creates a digital signal from a motion. It is an electro-mechanical device that provides users (commonly those in a motion control capacity) with information on position, velocity and direction

2.4.4 DAQ PCI 6221:

The PCI-6221 offers analog I/O, correlated digital I/O, two 32-bit counters/timers, and digital triggering. The device delivers low-cost, reliable DAQ capabilities in a wide range of applications from simple applications in laboratory automation, research, design verification/test, and manufacturing test. You can add sensor and high-voltage measurement capability to your device with signal conditioning modules. The included NI-DAQmx driver and configuration utility simplify configuration and measurements[8].

2.4.5 Matlab:

Millions of engineers and scientists worldwide use Matlab to analyze and design the systems and products transforming our world. Matlab is in automobile active safety systems, interplanetary spacecraft, and health monitoring devices, smart power grids, and LTE cellular networks. It is used for machine learning, signal processing, image processing, computer vision, communications, computational finance, control design, robotics, and much more.

The Matlab platform is optimized for solving engineering and scientific problems. The matrix-based Matlab language is the world's most natural way to express computational mathematics. Built-in graphics make it easy to visualize and gain insights from data. A vast library of pre-built toolboxes lets you get started right away with algorithms essential to your domain. The desktop environment invites experimentation, exploration, and discovery. These Matlab tools and capabilities are all rigorously tested and designed to work together.

Matlab helps you take your ideas beyond the desktop. You can run your analyses on larger data sets, and scale up to clusters and clouds. Matlab code can be integrated with other languages, enabling you to deploy algorithms and applications within web, enterprise, and production systems.

Control system engineers use Matlab and Simulink at all stages of development from plant modeling to designing and tuning control algorithms and supervisory logic, all the way to deployment with automatic code generation and system verification and validation [9].

Chapter 3: Design of structure and mechanical component

In this chapter we discuss the mechanical design of selected apparatus shown in Figure (3.1a) and (3.1b), calculation of each component in terms of strength, geometry, durability and material properties will be explained. Thus, these components will operate in the system without failure or defect.

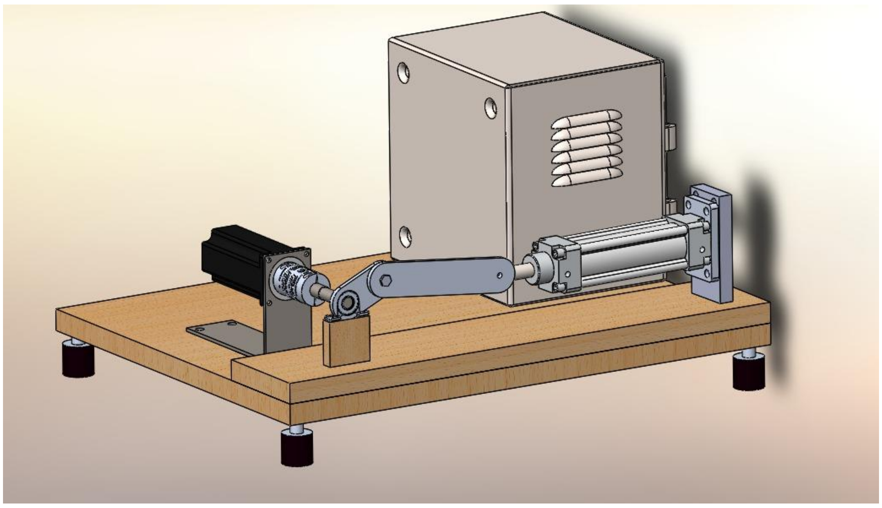


Figure 3.1a: Structure for apparatus with crank slider mechanism

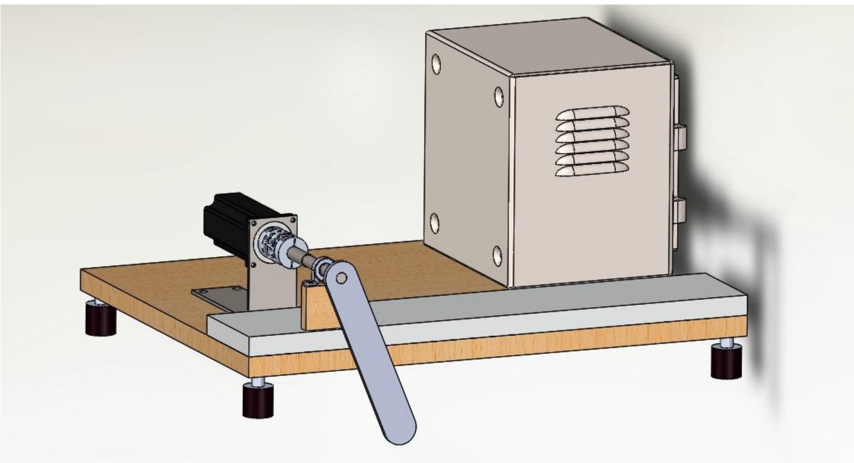


Figure 3.1b: Structure for apparatus with pendulum application

3.1 Design through SOLIDWORKS software and material selection

The word SOLIDWORKS is a Three-dimensional Interactive Application, and it is one of the “best” software program that are used for CAD, CAE and CAM, and it is able to make part design for every part of any machine or project, and it is able to estimate stress and strain and load analysis for the machine project.

3.2 Mechanical frame

The structure mechanical frame consists from wood frame with dimensions 60× 45 cm which forms the body of the structure which carries all mechanisms, mechanical and electrical parts. The mechanical frame must be light enough as shown in Figure (3.2), a wood material is selected to manufacture for the mechanical frame.

Figure (3.2): Mechanical frame

3.2.1 Structure and load analysis

In this section we will make the stress and deflection analysis to find the maximum stress and maximum deflection, which acts when the maximum possible load placed on the structure, the maximum load is 12 Kg which is about120 N distributed as shown in Figure (3.3). The stress and deflection analysis are as follow.

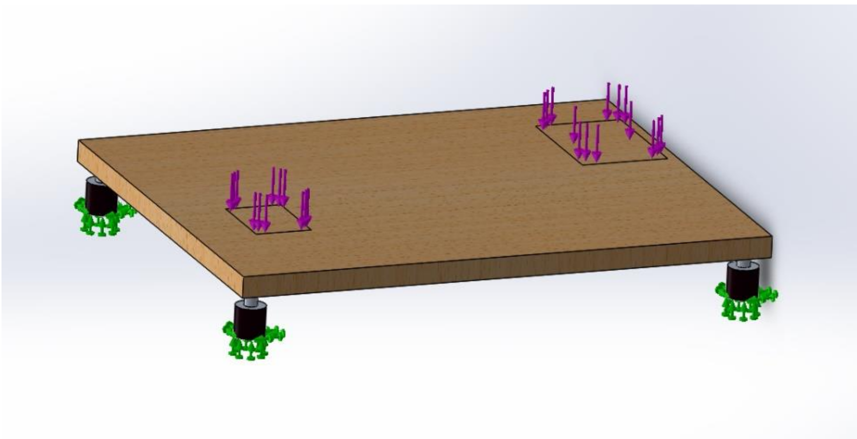


Figure 3.3: Distributed force in the frame

3.2.2 Stress and deflection analysis

A wood material is selected to be the frame, because of it has a light weight, the yield strength is 20 MPa. The stress analysis will be perform using SolidThinking by applying 120N vertically downwards at the storing area. The maximum stress equal to 2.1 MPa as shown in the following Figure (3.5).

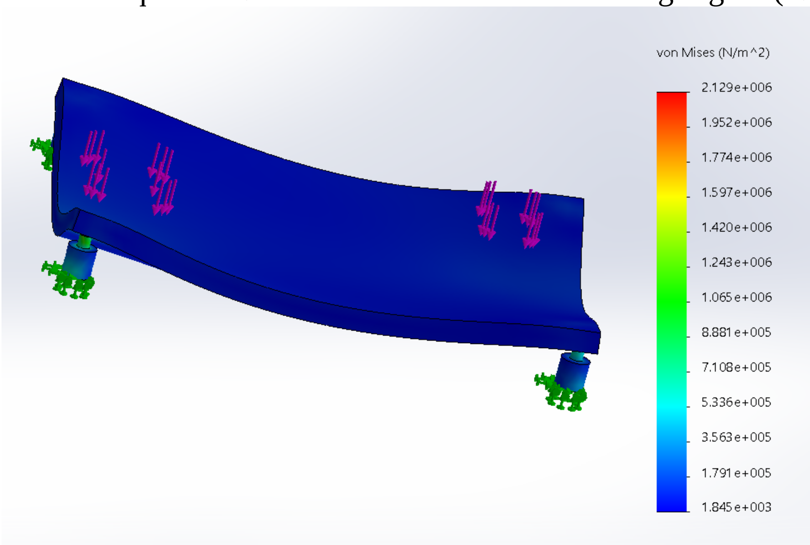


Figure 3.4: Stress result for mechanical frame

$$\text{Factor of safety} = \frac{\text{Yield strength}}{\text{von Mises Stress}} = \frac{20}{2.1} = 9.5$$

The maximum deflection due to bending as shown in the Figure (3.5) is 0.7 mm. This deflection is acceptable and does not contradict

with assumption criteria of the design, so can use this design without any hazard.

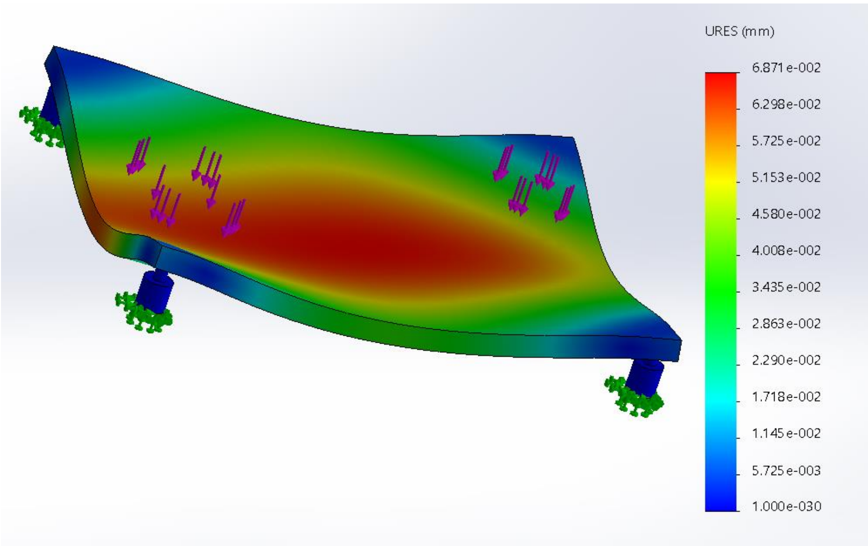


Figure 3.5: The maximum deflection

3.3 Mechanical applications

This project consists from two basic applications pendulum and crank slider mechanism as shown in the Figures (3.6) and (3.9). The stress and deflection analysis are as follow.

3.3.1 Pendulum application

The pendulum consists from two-part, shaft from steel and the compound pendulum from aluminum alloy connect at the end of the shaft as shown in Figure (3.6), the lengths of the shaft is 18 cm and the compound pendulum is 20 cm.

Figure 3.6: Pendulum application

3.3.1.1 Stress and deflection analysis for pendulum application

In this section will make the stress and deflection analysis for pendulum to find the maximum stress and maximum deflection, which act under gravity. As shown in Figure (3.7), the maximum stress is 0.34 MPa and the yield strength for aluminum alloy 27 MPa.

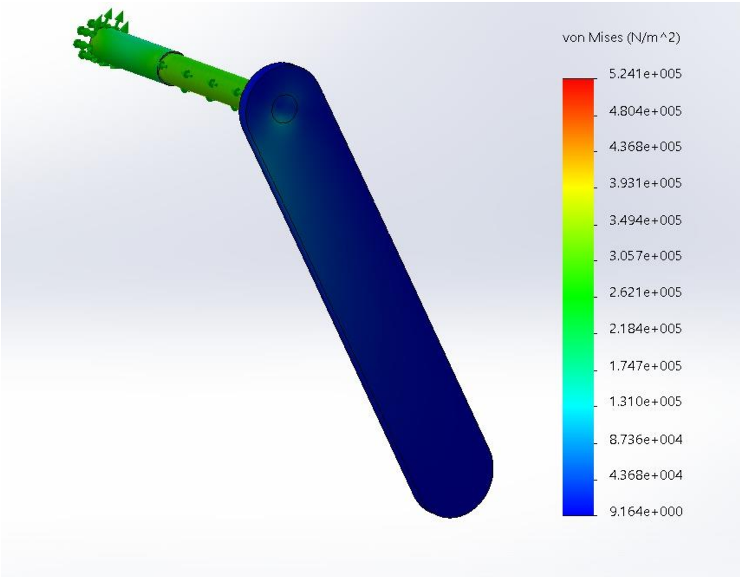


Figure 3.7: Stress analysis for Pendulum application

The maximum deflection due to gravity is 0.006 mm as shown in Figure (3.8), so can use this design without any hazard.

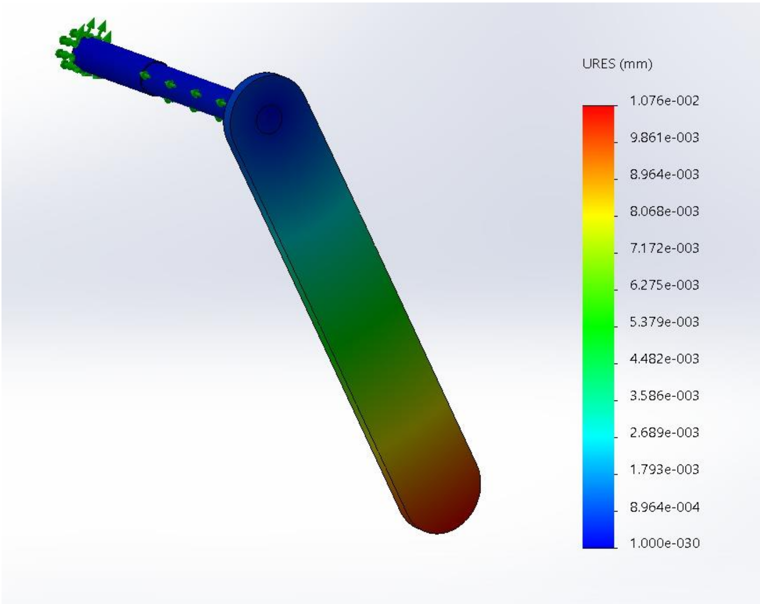


Figure 3.8: Deflection analysis for Pendulum application

3.3.2 Crank slider mechanism

This application consists from two-parts, shaft from steel and the crank slider mechanism from aluminum alloy connect at the end of the shaft as shown in Figure (3.9), the lengths of the shaft is 9 cm, the crank is 5 cm, the connecting rod is 15 cm and the slider stoke is 10 cm.

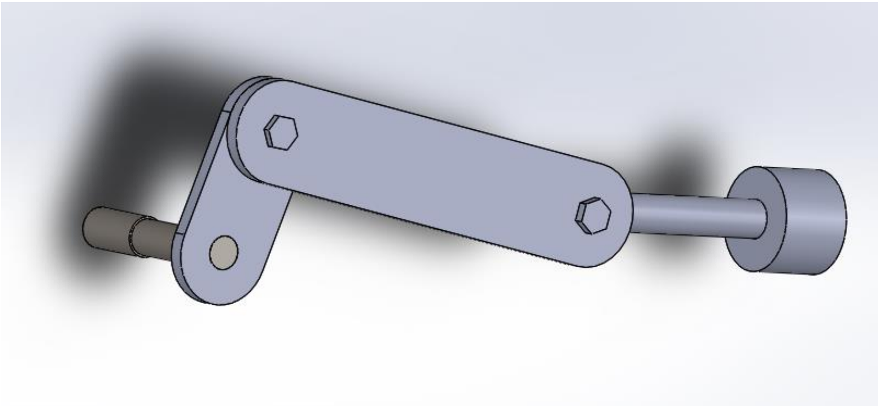


Figure 3.9: Crank slider mechanism

3.3.2.1 Stress and deflection analysis for crank slider mechanism

In this section will make the stress and deflection analysis for crank slider mechanism to find the maximum stress and maximum deflection, which act under gravity. As shown in the Figure (3.10) the maximum stress is 2.3 MPa and the yield strength for aluminum alloy 27 MPa.

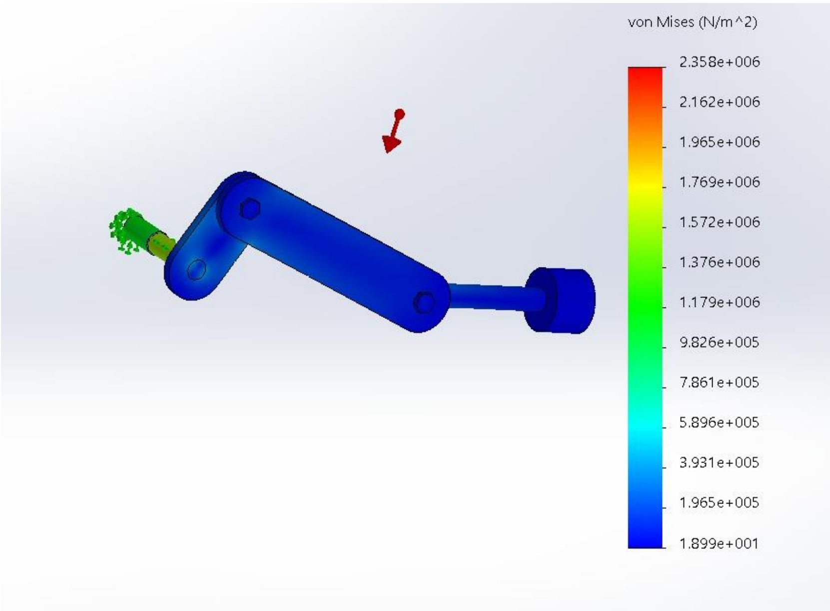


Figure 3.10: Stress analysis for crank slider mechanism

The maximum deflection is out from crank slider mechanism 0.05 mm as shown in Figure (3.11), so we can use this design without any hazard.

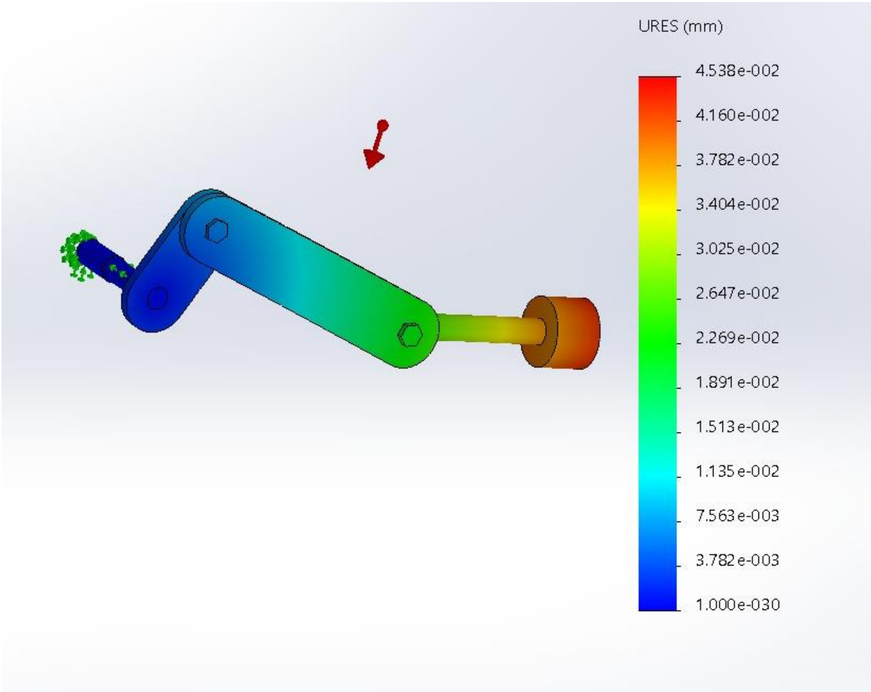


Figure 3.11: Deflection analysis for crank slider mechanism

Chapter 4: Mathematical model

Mathematical modeling of the system represent all important features of the system and describe its behavior in terms of differential equations. Generally, a simplified model is needed to study the main characteristics of the system, while a detailed model is needed for precise simulation and prediction studies.

In general, there are two main purposes for modeling a physical system:

- Develop a mathematical model in order to predict the dynamic behavior of the system as accurately as possible. Using numerical solution methods, such a model serves as a tool for extensive evaluation of system behavior without actually using or building the real system.
- Develop models to gain insight into the behavior of the dynamic system qualitatively instead of exact response prediction, i.e. knowledge of margins, controllability, observability, and the sensitivity of response to parameter changes, such a model needs not to contain all of the details of the actual system, but only the most essential features so as to provide the needed insight from an engineering stand point.

In order to obtain the mathematical model of the system, Newton second law and Lagrange approach are used to derive the basic differential equations that govern system's dynamics.

4.1 Mathematical modeling for mechanisms

4.1.1 Mathematical modeling for the pendulum

In this section will use Newton second law to derive the basic differential equations that govern system dynamics.

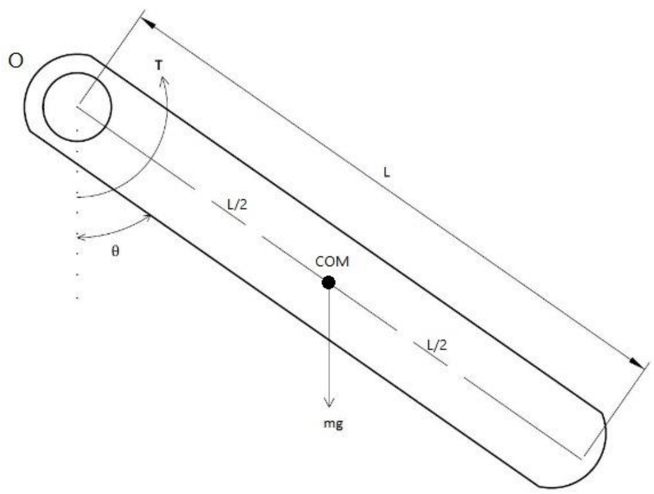


Figure 4.1: Pendulum

Figure 4.1 shows an inverted pendulum in the form of a uniform rod pinned at point O. It has a mass m and length L. Its center of mass is at distance $L/2$ from O. A torque T is applied at the pendulum as shown. Th rotation of the pendulum with an angle θ takes place about point O.

Considering the gravitational force and applied torque acting at the pendulum and applying Newton second law[10] of motion, one obtains

$$\Sigma M_o = J\ddot{\theta}$$

$$-(\mathrm{mg}\sin\theta * \frac{\mathrm{L}}{2}) + \mathrm{T} = \mathrm{J}\ddot{\theta} \qquad (4.1)$$

Where

M_o: moment at point O
J: mass moment of inertia of the pendulum at point O
$\ddot{\theta}$: angular acceleration
T: applied torque
θ: the angle of pendullum

Assuming a small displacement θ, Equation (4.1) can be approximated around $\theta = 0$ as

$$-(\mathrm{mg}\,\theta * \frac{\mathrm{L}}{2}) + \mathrm{T} = \mathrm{J}\ddot{\theta} \qquad (4.2)$$

Applying the Laplace transform, then

$$-(\mathrm{mg}\,\theta(\mathrm{s}) * \frac{\mathrm{L}}{2}) + \mathrm{T(s)} = Js^2\,\theta(\mathrm{s})$$

$$Js^2\,\theta(\mathrm{s}) + (\mathrm{mg}\,\theta(\mathrm{s}) * \frac{\mathrm{L}}{2}) = \mathrm{T(s)}$$

$$\theta(\mathrm{s})\left[Js^2 + \mathrm{mg} * \frac{\mathrm{L}}{2}\right] = \mathrm{T(s)}$$

Then the transfer function will be

$$\frac{\theta(\mathrm{s})}{\mathrm{T(s)}} = \frac{1}{Js^2 + mg*\frac{\mathrm{L}}{2}} \qquad (4.3)$$

4.1.2 Mathematical modeling for the inverted pendulum

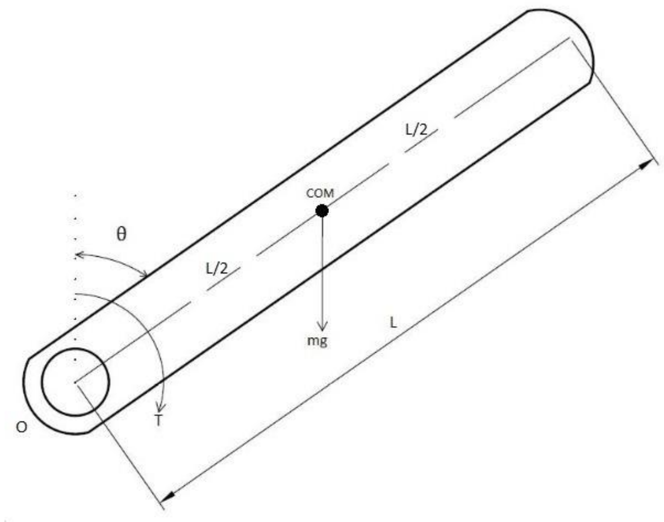


Figure 4.2: inverted Pendulum

Applying the same method, the equations for the inverted pendulum can be

$$\sum M = J\ddot{\theta}(\text{t})$$

$$(\text{mg} \sin \theta(\text{t}) * \text{R}) + \text{T}(\text{t}) = \text{J}\ddot{\theta}(\text{t}) \qquad (4.4)$$

Where

M_o: moment at point O
J: mass moment of inertia of the inverted pendulum at point O
$\ddot{\theta}$: angular acceleration
T: applied torque
θ: the angle of inverted pendullum

Assuming a small displacement θ, Equation (4.1) can be linearized about $\theta = 0$ as

$$(\text{mg}\,\theta * \frac{\text{L}}{2}) + \text{T} = \text{J}\ddot{\theta} \qquad (4.5)$$

Applying the Laplace transform, then

$$(\text{mg}\,\theta(\text{s}) * \frac{\text{L}}{2}) + \text{T}(\text{s}) = Js^2\,\theta(\text{s})$$

$$Js^2\,\theta(\text{s}) - (\text{mg}\,\theta(\text{s}) * \frac{\text{L}}{2}) = \text{T}(\text{s})$$

$$\theta(s)\left[Js^2 - mg * \frac{L}{2}\right] = T(s)$$

Then the transfer function[11] will be

$$\frac{\theta(s)}{T(s)} = \frac{1}{Js^2 - mg*\frac{L}{2}} \quad (4.6)$$

4.1.3 Mathematical modeling for crank slider

A crank slider mechanism is a four-link mechanism with three revolute joined the point O is fixed, link OA is the crank, link OB is the connecting rod and link B is the slider

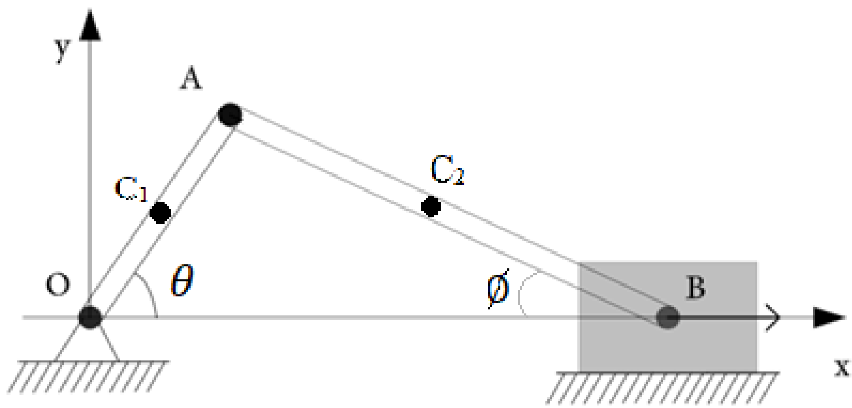


Figure 4.3: crank slider mechanism

Figure 4.3 shows the angle between rod OA and the horizontal direction is θ . The length of the rod OA is R, the length of the rod AB is L and the horizontal length OB is x. The mass of rod OA is m_1, the mass of rod AB is m_2 and the mass of slider is m_3. The center of mass for rod AB is C_1, the length of OC_1 is r, the center of mass for rod AB is C_2, the length of AC_2 is l, the inertia for rod OA is I_1 and the inertia for rod AB is I_2. A torque τ is applied at the rod OA and the direction is CCW.

In order to eliminate Ø from next equation, note that

$$R \sin\theta = L \sin\emptyset$$

$$\sin \emptyset = \frac{R}{L} \sin \theta \qquad (4.7a)$$

$$\cos \emptyset = \sqrt{1 - \frac{R^2}{L^2} sin^2\theta}$$

Using Binomial Theorem, then

$$\cos \emptyset \approx 1 - \frac{1}{2}\frac{R^2}{L^2} sin^2\theta \qquad (4.7b)$$

In order to eliminate $\dot{\emptyset}$ from next equation, note that

$$\frac{d}{dt}(R \sin \theta = L \sin \emptyset) \rightarrow R \cos \theta \, \dot{\theta} = L \cos \emptyset \dot{\emptyset}$$

$$\dot{\emptyset} = \frac{R}{L}\frac{\cos \theta}{\cos \emptyset}\dot{\theta} = \frac{R}{L}\frac{\cos \theta}{1 - \frac{1R^2}{2L^2}sin^2\theta}\dot{\theta} \qquad (4.8a)$$

In order to eliminate $\ddot{\emptyset}$ from next equation, note tha

$$\frac{d^2}{dt^2}(R \sin \theta = L \sin \emptyset) \rightarrow R \cos \theta \, \ddot{\theta} - R \sin \theta \, \dot{\theta}^2 = L \cos \emptyset \, \ddot{\emptyset} - L \sin \emptyset \, \dot{\emptyset}^2$$

$$\ddot{\emptyset} = \frac{R \cos \theta}{L \cos \emptyset}\ddot{\theta} - \frac{R \sin \theta}{L \cos \emptyset}\dot{\theta}^2 + \frac{L \sin \emptyset}{L \cos \emptyset}\dot{\emptyset}^2$$

$$\ddot{\emptyset} = \frac{R \cos \theta}{L\sqrt{1 - \frac{R^2}{L^2}sin^2\theta}}\ddot{\theta} + \frac{R \sin \theta}{L\sqrt{1 - \frac{R^2}{L^2}sin^2\theta}}\left(\frac{R^2 - L^2}{L^2 - R^2 sin^2\theta}\right)\dot{\theta}^2 \qquad (4.8b)$$

- **Kinematics model of crank slider**

Kinematic modeling is the study of the motion of mechanical systems without considering the forces that affect the motion. The rotation of the crank drives the horizontal linear movement the slider.

1. Kinematics for C1

$$x_{c_1} = r \cos \theta \qquad (4.9)$$

$$y_{c_1} = r \sin \theta \qquad (4.10)$$

$$\dot{x}_{c_1} = -\mathrm{r} \sin \theta \, \dot{\theta} \qquad (4.11)$$

$$\dot{y}_{c_1} = \mathrm{r} \cos \theta \, \dot{\theta} \qquad (4.12)$$

$$\ddot{x}_{c_1} = -\mathrm{r} \sin \theta \, \ddot{\theta} - \mathrm{r}\cos \theta \, \dot{\theta}^2 \qquad (4.13)$$

$$\ddot{y}_{c_1} = \mathrm{r} \cos \theta \, \ddot{\theta} - r \sin \theta \, \dot{\theta}^2 \qquad (4.14$$

2. Kinematics for C2

$$x_{c_2} = R\cos\theta + l\cos\emptyset$$

$$x_{c_2} = R\cos\theta + l\sqrt{1-\frac{R^2}{L^2}sin^2\theta} \quad (4.15)$$

Substitution equation (4.7b) in equation (4.15), then

$$x_{c_2} \approx\ R\cos\theta + l\left(1-\frac{1}{2}\frac{R^2}{L^2}sin^2\theta\right) \quad (4.16)$$

$$y_{c_2} = R\sin\theta - l\sin\emptyset$$

Substitution equation (4.7a), then

$$y_{c2} = R\sin\theta - l\frac{R}{L}\sin\theta$$

$$y_{c_2} = R\sin\theta\left(1-\frac{l}{L}\right) \quad (4.17)$$

$$\dot{x}_{c_2} \approx -\mathrm{R}\sin\theta\,\dot{\theta} - \frac{R^2}{2L^2}l\sin\theta\cos\theta\,\dot{\theta}$$

$$\dot{x}_{c_2} \approx -\mathrm{R}\sin\theta\,\dot{\theta} - \frac{R^2}{2L^2}l\sin 2\theta\,\dot{\theta}$$

$$\dot{x}_{c_2} \approx -r\dot{\theta}\left(\sin\theta + \frac{R}{2L^2}l\sin 2\theta\right) \quad (4.18)$$

$$\dot{y}_{c_2} = R\dot{\theta}\cos\theta\left(1-\frac{l}{L}\right) \quad (4.19)$$

$$\ddot{x}_{c_2} \approx -R\ddot{\theta}\left(\sin\theta + \frac{Rl}{2L^2}\sin 2\theta\right) - R\dot{\theta}^2\left(\cos\theta + \frac{Rl}{L^2}\cos 2\theta\right) \quad (4.20)$$

$$\ddot{y}_{c_2} = R\ddot{\theta}\cos\theta\left(1-\frac{l}{L}\right) - R\dot{\theta}^2\sin\theta\left(1-\frac{l}{L}\right) \quad (4.21)$$

3. Kinematics for B

Replace l with L in the previous equations

$$x_B \approx\ R\cos\theta + L\left(1-\frac{1}{2}\frac{R^2}{L^2}sin^2\theta\right) \quad (4.22)$$

y_B=0

$$\dot{x}_B \approx -\mathrm{R}\sin\theta\,\dot{\theta} - \frac{R^2}{2L^2}L\sin 2\theta\,\dot{\theta} \approx -r\dot{\theta}\left(\sin\theta + \frac{R}{2L}\sin 2\theta\right) \quad (4.23)$$

$\dot{y}_B$=0

$$\ddot{x}_B \approx -R\ddot{\theta}\left(\sin\theta + \frac{R}{2L}\sin 2\theta\right) - R\dot{\theta}^2\left(\cos\theta + \frac{R}{L}\cos 2\theta\right) \quad (4.24)$$

$\ddot{y}_B$=0

- Newton Euler Dynamic

1. Slider

Considering the gravitational force and reaction force from connecting rod acting at the slider and applying Newton's second law of motion, one obtains

$$\Sigma F_y = 0$$

$$F_{23y} + m_3 g = F_{3y} \quad (4.25)$$

$$\Sigma F_x = m_3 \ddot{x}_B$$

$$F_{23x} + f_e = m_3 \ddot{x}_B \quad (4.26)$$

Then,

$$F_{23x} = m_3 \ddot{x}_B - f_e \quad (4.26b)$$

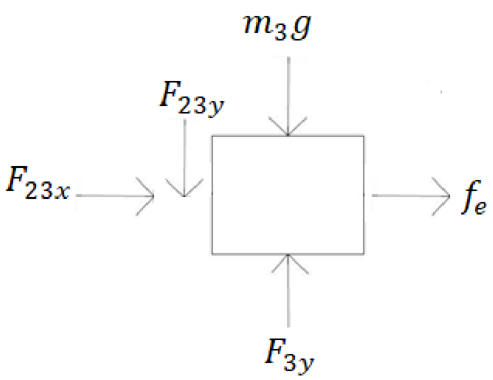


Figure 4.4: Free body diagram for the slider

2. Connecting rod

Considering the gravitational force and reaction force from slider and crank and applied torque acting at crank and applying Newton's second law of motion, one obtains

$$\Sigma F_y = m_2 \ddot{y}_{c_2}$$

$$F_{23Y} - F_{12Y} - m_2 g = m_2 \ddot{y}_{c_2} \quad (4.27)$$

$$\Sigma F_x = m_2 \ddot{x}_{c_2}$$
$$F_{12X} - F_{23X} = m_2 \ddot{x}_{c_2} \qquad (4.28)$$

$$\Sigma M_A = (I_2 + m_2 l^2)\ddot{\emptyset}$$

$$F_{23Y} L \cos\emptyset - F_{23X} L \sin\emptyset - m_2 g L \cos\emptyset - m_2 \ddot{y}_{c_2} L \cos\emptyset - m_2 \ddot{x}_{c_2} L \sin\emptyset = (I_2 + m_2 l^2)\ddot{\emptyset} \qquad (4.29)$$

Substitution equations (4.26b),(4.7a),(4.21), (4.20), (4.7b) and (4.8b) in (4.29), then

$$F_{23Y} L\left(1-\frac{1}{2}\frac{R^2}{L^2}sin^2\theta\right) - (m_3\ddot{x}_B - f_e)L\frac{R}{L}\sin\theta - m_2 g L\left(1-\frac{1}{2}\frac{R^2}{L^2}sin^2\theta\right)$$

$$-m_2 R\ddot{\theta}\cos\theta\left(1-\frac{l}{L}\right) - R\dot{\theta}^2\sin\theta\left(1-\frac{l}{L}\right)L\left(1-\frac{1}{2}\frac{R^2}{L^2}sin^2\theta\right) - m_2\ddot{x}_{c_2}L\frac{R}{L}\sin\theta.$$

$$= (I_2 + m_2 l^2)\left(\frac{R\cos\theta}{L\sqrt{1-\frac{R^2}{L^2}sin^2\theta}}\ddot{\theta} + \frac{R\sin\theta}{L\sqrt{1-\frac{R^2}{L^2}sin^2\theta}}\left(\frac{R^2-L^2}{L^2-R^2sin^2\theta}\right)\dot{\theta}^2\right)$$

Then the

$F_{23Y} =$

$$\frac{(I_2 + m_2 l^2)}{L\left(1-\frac{1}{2}\frac{R^2}{L^2}sin^2\theta\right)}\left(\frac{R\cos\theta}{L\sqrt{1-\frac{R^2}{L^2}sin^2\theta}}\ddot{\theta} + \frac{R\sin\theta}{L\sqrt{1-\frac{R^2}{L^2}sin^2\theta}}\left(\frac{R^2-L^2}{L^2-R^2sin^2\theta}\right)\dot{\theta}^2\right)$$

$$+\frac{(m_3\ddot{x}_B - f_e)L\frac{R}{L}\sin\theta}{L\left(1-\frac{1}{2}\frac{R^2}{L^2}sin^2\theta\right)} + \frac{m_2 g L\left(1-\frac{1}{2}\frac{R^2}{L^2}sin^2\theta\right)}{L\left(1-\frac{1}{2}\frac{R^2}{L^2}sin^2\theta\right)} + \frac{m_2 R\ddot{\theta}\cos\theta\left(1-\frac{l}{L}\right)}{L\left(1-\frac{1}{2}\frac{R^2}{L^2}sin^2\theta\right)}$$
$$+\frac{R\dot{\theta}^2\sin\theta\left(1-\frac{l}{L}\right)L\left(1-\frac{1R^2}{2L^2}sin^2\theta\right)}{L\left(1-\frac{1R^2}{2L^2}sin^2\theta\right)} + \frac{m_2\ddot{x}_{c_2}L\frac{R}{L}\sin\theta.}{L\left(1-\frac{1R^2}{2L^2}sin^2\theta\right)} \qquad (4.29a)$$

Substitution equations (4.21) and (4.29a) in equation (4.27)

$F_{12Y} =$

$$\frac{(I_2+m_2l^2)}{L\left(1-\frac{1}{2}\frac{R^2}{L^2}sin^2\theta\right)}\left(\frac{R\cos\theta}{L\sqrt{1-\frac{R^2}{L^2}sin^2\theta}}\ddot{\theta}+\frac{R\sin\theta}{L\sqrt{1-\frac{R^2}{L^2}sin^2\theta}}\left(\frac{R^2-L^2}{L^2-R^2sin^2\theta}\right)\dot{\theta}^2\right)$$

$$+\frac{(m_3\left(-R\ddot{\theta}\left(\sin\theta+\frac{R}{2L}\sin2\theta\right)-R\dot{\theta}^2\left(\cos\theta+\frac{R}{L}\cos2\theta\right)\right)-f_e)L\frac{R}{L}\sin\theta}{L\left(1-\frac{1}{2}\frac{R^2}{L^2}sin^2\theta\right)}$$

$$+\frac{m_2gl\left(1-\frac{1}{2}\frac{R^2}{L^2}sin^2\theta\right)}{L\left(1-\frac{1}{2}\frac{R^2}{L^2}sin^2\theta\right)}+\frac{m_2(R\ddot{\theta}\cos\theta\left(1-\frac{l}{L}\right)}{L\left(1-\frac{1}{2}\frac{R^2}{L^2}sin^2\theta\right)}$$

$$+\frac{R\dot{\theta}^2\sin\theta\left(1-\frac{l}{L}\right))L\left(1-\frac{1}{2}\frac{R^2}{L^2}sin^2\theta\right)}{L\left(1-\frac{1}{2}\frac{R^2}{L^2}sin^2\theta\right)}+\frac{m_2\ddot{x}_{c_2}L\frac{R}{L}\sin\theta.}{L\left(1-\frac{1}{2}\frac{R^2}{L^2}sin^2\theta\right)}-m_2g$$
$$-m_2R\ddot{\theta}\cos\theta\left(1-\frac{l}{L}\right)-m_2\,R\dot{\theta}^2\sin\theta\left(1-\frac{l}{L}\right) \qquad (4.29b)$$

Substitution equations (4.20) and (4.26b) in equation (4.28)

$$F_{12x}=m_2\left(-R\ddot{\theta}\left(\sin\theta+\frac{Rl}{2L^2}\sin2\theta\right)-R\dot{\theta}^2\left(\cos\theta+\frac{Rl}{L^2}\cos2\theta\right)\right)$$
$$+\left(m_3\left(-R\ddot{\theta}\left(\sin\theta+\frac{R}{2L}\sin2\theta\right)-R\dot{\theta}^2\left(\cos\theta+\frac{R}{L}\cos2\theta\right)\right)-f_e\right) \qquad (4.29c)$$

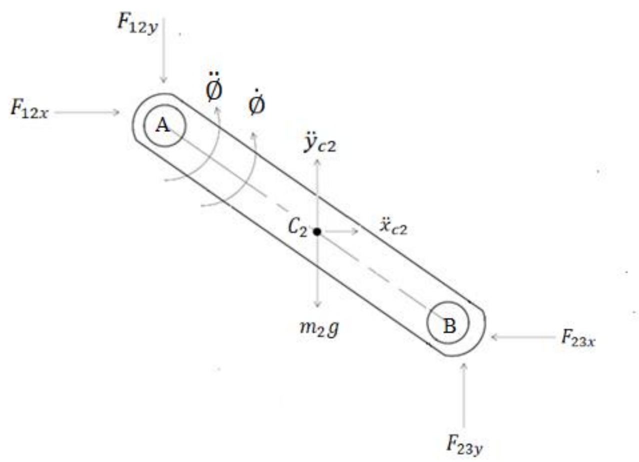


Figure 4.5: Free body diagram for the Connecting rod

3. Crank

Considering the gravitational force and reaction force from connecting rod and crank at connecting rod acting and applying Newton's second law of motion, one obtains

$$\Sigma F_y = m_1 \ddot{y}_{c_1}$$
$$F_{01Y} - F_{12Y} - m_1 g = m_1 \ddot{y}_{c_1} \qquad (4.30)$$

$$\Sigma F_x = m_1 \ddot{x}_{c_1}$$
$$F_{01x} - F_{12X} = m_1 \ddot{x}_{c_1} \qquad (4.31)$$

$$\Sigma M_A = (I_1 + m_1 r^2)\ddot{\theta}$$

$$F_{12x} R \sin\theta + F_{12y} R \cos\theta + m_1 \ddot{x}_{c_1} r \sin\theta - m_1 \ddot{y}_{c_1} r \cos\theta$$

$$-m_1 g r \cos\theta + \tau = (I_1 + m_1 r^2)\ddot{\theta} \qquad (4.32$$

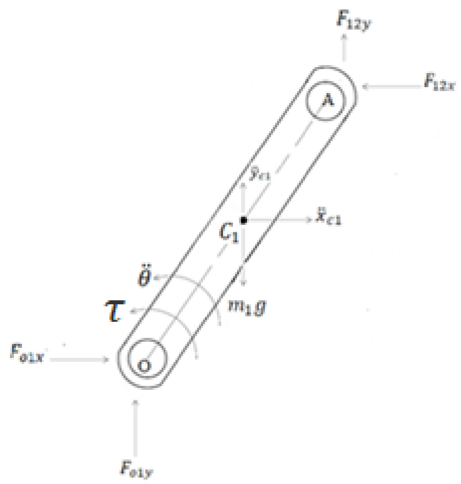


Figure 4.6: Free body diagram for the Crank

From equation (4.29c), (4.29b), (4.13) and (4.14)

$$\Bigg(m_2\left(-R\ddot{\theta}\left(\sin\theta + \frac{Rl}{2L^2}\sin 2\theta\right) - R\dot{\theta}^2\left(\cos\theta + \frac{Rl}{L^2}\cos 2\theta\right)\right) +$$

$$\left(m_3\left(-R\ddot{\theta}\left(\sin\theta + \frac{R}{2L}\sin 2\theta\right) - R\dot{\theta}^2\left(\cos\theta + \frac{R}{L}\cos 2\theta\right)\right) - f_e\right)\Bigg) R\sin\theta +$$

$$\left(\frac{(I_2 + m_2 l^2)}{L\left(1 - \frac{1R^2}{2L^2}sin^2\theta\right)}\left(\frac{R\cos\theta}{L\sqrt{1 - \frac{R^2}{L^2}sin^2\theta}}\ddot{\theta} + \frac{R\sin\theta}{L\sqrt{1 - \frac{R^2}{L^2}sin^2\theta}}\left(\frac{R^2 - L^2}{L^2 - R^2 sin^2\theta}\right)\dot{\theta}^2\right) +\right.$$

$$\frac{(m_3\left(-R\ddot{\theta}\left(\sin\theta+\frac{R}{2L}\sin 2\theta\right)-R\dot{\theta}^2\left(\cos\theta+\frac{R}{L}\cos 2\theta\right)\right)-f_e)L\frac{R}{L}\sin\theta}{L\left(1-\frac{1R^2}{2L^2}sin^2\theta\right)}+\frac{m_2 gL\left(1-\frac{1R^2}{2L^2}sin^2\theta\right)}{L\left(1-\frac{1R^2}{2L^2}sin^2\theta\right)}+$$

$$\frac{m_2(R\ddot{\theta}\cos\theta\left(1-\frac{l}{L}\right)+R\dot{\theta}^2\sin\theta\left(1-\frac{l}{L}\right))L\left(1-\frac{1R^2}{2L^2}sin^2\theta\right)}{L\left(1-\frac{1R^2}{2L^2}sin^2\theta\right)}+$$

$$\frac{m_2\left(-R\ddot{\theta}\left(\sin\theta+\frac{Rl}{2L^2}\sin 2\theta\right)-R\dot{\theta}^2\left(\cos\theta+\frac{Rl}{L^2}\cos 2\theta\right)\right)l\frac{R}{L}\sin\theta.}{L\left(1-\frac{1R^2}{2L^2}sin^2\theta\right)}-m_2 g-$$

$$\left.m_2 R\ddot{\theta}\cos\theta\left(1-\frac{l}{L}\right)-R\dot{\theta}^2\sin\theta\left(1-\frac{l}{L}\right)\right)R\cos\theta+m_1\big(-\mathrm{r}\sin\theta\,\ddot{\theta}-$$

$$\mathrm{r}\cos\theta\,\dot{\theta}^2\big)r\sin\theta+m_1\big(\mathrm{r}\cos\theta\,\ddot{\theta}-r\sin\theta\,\dot{\theta}^2\big)r\cos\theta-m_1 g\,r\cos\theta+\tau=$$
$$(I_1+m_1r^2)\ddot{\theta} \qquad (4.30)$$

$$\ddot{\theta}\left(-m_3R^2\,sin^2\theta-\frac{m_3R^3\sin 2\theta\sin\theta}{2L}-m_2R^2\,sin^2\theta-\frac{m_2R^3l\sin 2\theta\sin\theta}{2L^2}+\frac{(I_2+m_2l^2)R^2\,cos^2\theta}{L^2\left(1-\frac{1}{2}\frac{R^2}{L^2}sin^2\theta\right)\sqrt{1-\frac{R^2}{L^2}sin^2\theta}}\right.$$
$$-\frac{m_3R^3\,sin^2\theta\cos\theta\ .}{L\left(1-\frac{1}{2}\frac{R^2}{L^2}sin^2\theta\right)}-\frac{m_3R^4\sin 2\theta\sin\theta\cos\theta}{2L^2\left(1-\frac{1}{2}\frac{R^2}{L^2}sin^2\theta\right)}+\frac{m_2R^2lcos^2\theta}{L}-m_2R^2\,cos^2\theta\,\frac{l^2}{L^2}$$
$$-\frac{m_2R^3\,sin^2\theta\cos\theta\,l}{L^2\left(1-\frac{1}{2}\frac{R^2}{L^2}sin^2\theta\right)}-\frac{m_2R^4\sin 2\theta\sin\theta\cos\theta l^2}{2L^4\left(1-\frac{1}{2}\frac{R^2}{L^2}sin^2\theta\right)}-m_2R^2cos^2\theta+\frac{m_2R^2lcos^2\theta}{L}-m_1r^2\,sin^2\theta$$
$$\left.+m_1r^2\,cos^2\theta-I_1-m_1r^2\right)$$

$$=\dot{\theta}^2\left(\left(m_3R^2\sin\theta\cos\theta+\frac{m_3R^3\cos 2\theta\sin\theta}{L}+m_2R^2\cos\theta\sin\theta+\frac{m_2R^3\cos 2\theta\sin\theta l}{2L^3}\right)-\right.$$
$$\frac{(I_2+m_2l^2)R^2\sin\theta\cos\theta(R^2-L^2)}{L^2\left(1-\frac{1R^2}{2L^2}sin^2\theta\right)(L^2-R^2sin^2\theta)\sqrt{1-\frac{R^2}{L^2}sin^2\theta}}+\frac{m_3R^3\,cos^2\theta\sin\theta}{L\left(1-\frac{1R^2}{2L^2}sin^2\theta\right)}+\frac{m_3R^4\cos 2\theta\sin\theta\cos\theta}{L^2\left(1-\frac{1R^2}{2L^2}sin^2\theta\right)}+\frac{m_2R^2\sin\theta\cos\theta l}{L}+\ m_2R^2\sin\theta\cos\theta\,\frac{l^2}{L^2}+$$
$$\left.\frac{m_2R^3lcos^2\theta\sin\theta}{LL^2\left(1-\frac{1R^2}{2L^2}sin^2\theta\right)}+\frac{m_2R^2\sin\theta\cos\theta l}{L}+\frac{m_2R^4\cos 2\theta\sin\theta\cos\theta l^2}{L^4\left(1-\frac{1R^2}{2L^2}sin^2\theta\right)}-m_2R^2\sin\theta\cos\theta+2m_1r^2\sin\theta\cos\theta\right)+f_eR\sin\theta+$$
$$\frac{f_eR^2\cos\theta\sin\theta}{L\left(1-\frac{1R^2}{2L^2}sin^2\theta\right)}\quad-\frac{m_2glR\cos\theta}{L\left(1-\frac{1R^2}{2L^2}sin^2\theta\right)}+m_2gR\cos\theta+m_1gr\cos\theta+\frac{\frac{1}{2}m_2glR^3\cos\theta sin^2\cos\theta}{L^3\left(1-\frac{1R^2}{2L^2}sin^2\theta\right)}-\tau$$

$$(4.31)$$

Approximated equation (4.31), then

$$\tau=(\mathrm{m}_2\mathrm{R}^2+\mathrm{I}_1+\mathrm{m}_1\mathrm{r}^2)\ddot{\theta} \qquad (4.32)$$

Chapter 5: Controller Design and simulation

In this chapter will design and applying different controllers in applications (pendulum, inverted pendulum and crank slider mechanism).

In the upcoming sections linear control theories and strategies will be tested and discussed, including:

- PD controller.
  A PD controller is described by the transfer function:
  K(s)=kp+kds
  A PD controller thus adds a single zero to the loop transfer function, the genal block diagram as shown in figure (5.1) [11].

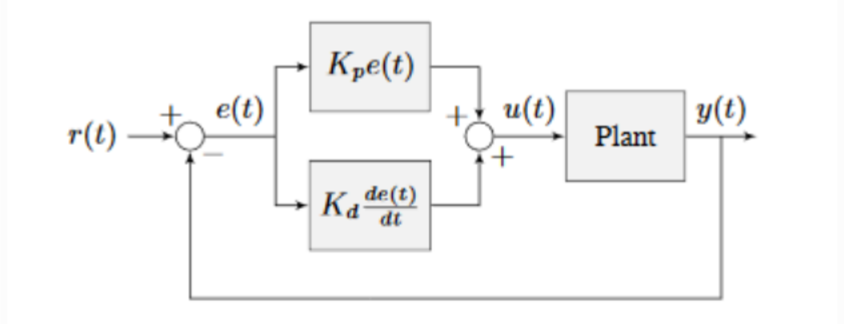


Figure 5.1: General block diagram for PD controller[11].

- PID controller

  PID is a proportional-integral-derivative controller which creates a control loop feedback mechanism. A PID controller calculates an "error" (e) value as the difference between a measured process variable and a desired set point. The desired closed loop dynamics is obtained by adjusting the three parameters KP (proportional gain), KI (integral gain) and KD (derivative gain), based on the linear system transfer function[11].

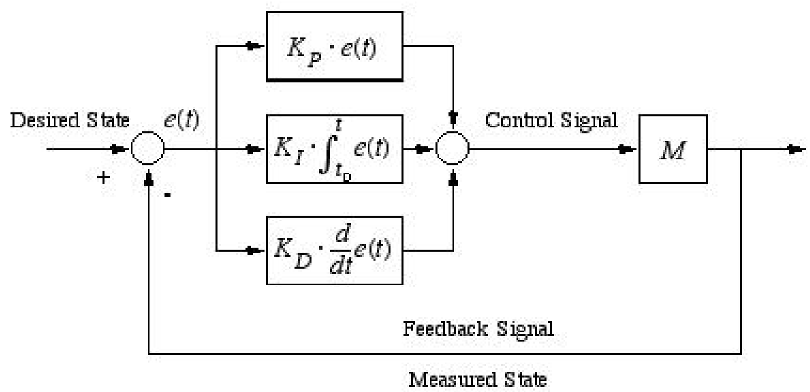


Figure 5.2: PID controller[11].

- State Feedback controllers.

In control engineering, a state space representation is a mathematical model of a physical system as a set of input, output and state variables related by first-order differential equations. The state space representation (also known as the "time-domain approach") provides a convenient and compact way to model and analyze systems with multiple inputs and outputs. Unlike the frequency domain approach, the use of the state space representation is not limited to systems with linear components and zero initial conditions. However, in state feedback method you can place the eigenvalues anywhere in the S-plane to get the desired response

In order to obtain the state-space representation for any system, you need to know system inputs, outputs, in addition to the states. The general linear time invariant state space model that is used throughout this chapter[11]:

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

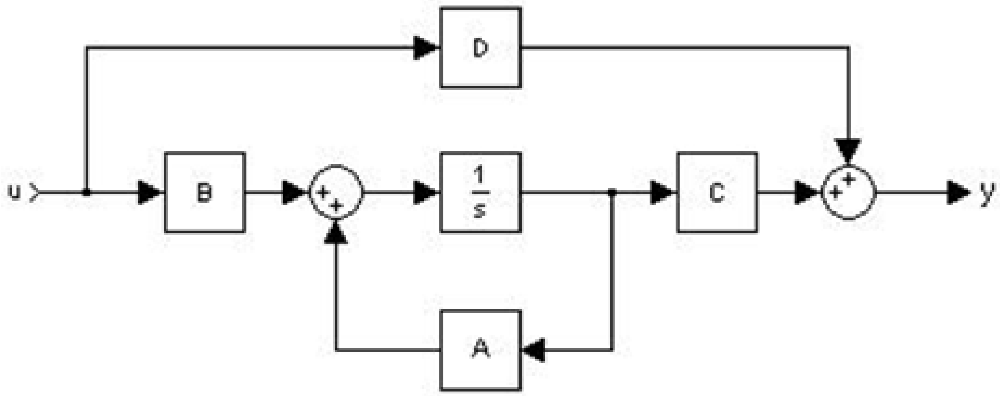


Figure 5.3: Block diagram representation of the state space equation[11].

- Extended System

it is desired to design a state feedback controller that able to track a desired reference input of the applications [11].

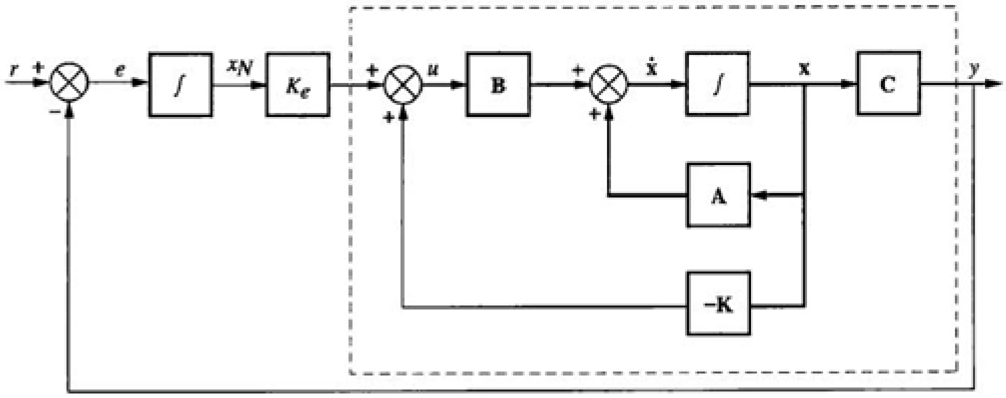


Figure 5.4: Integrator control for steady state error design[11].

5.1 Controllers Design and simulation for pendulum

In this section every linear control strategy used is discussed in details, after that a Simulink model built for the controller, and then shown the simulated results.

5.1.1 PD controller for pendulum

As equation (4.3) the transfer function

Where:
J=0.0017843 Kg.m^2
m=0.147 Kg
L=0.02 m
g=9.81 m/s^2

$$\frac{\Theta(s)}{T(s)} = \frac{Kds+Kp}{Js^2+mg*\frac{L}{2}} \tag{5.1}$$

By definition: the general form of the transfer function of a second system is

$$C(s) = \frac{wn^2}{s^2+2\zeta\omega ns+wn^2} \tag{5.2}$$

Then the characteristic equation

$$s^2 + 2\zeta\omega ns + \omega n^2 \tag{5.3}$$

The characteristic equation from closed loop for the transfer function

$$s^2 + \frac{Kds}{J} + \frac{mgL+Kp}{2J} = \ s^2 \ + 2\zeta\omega ns + \omega n^2 \qquad (5.4)$$

From equation (5.4)

Kp=$wn^2J - mg\frac{L}{2}$ (5.5)

Kd=$2\zeta\omega nJ$ (5.6)

As design requirement will do two experiments:

1. let of the system equals to the settling time 0.55 sec and the percent over shoot (%OS) to be 10%, from equations (5.5) and (5.6) The controller gains are:

   Kp=0.1126
   Kd=0.0256

- Simulink model for PD controller

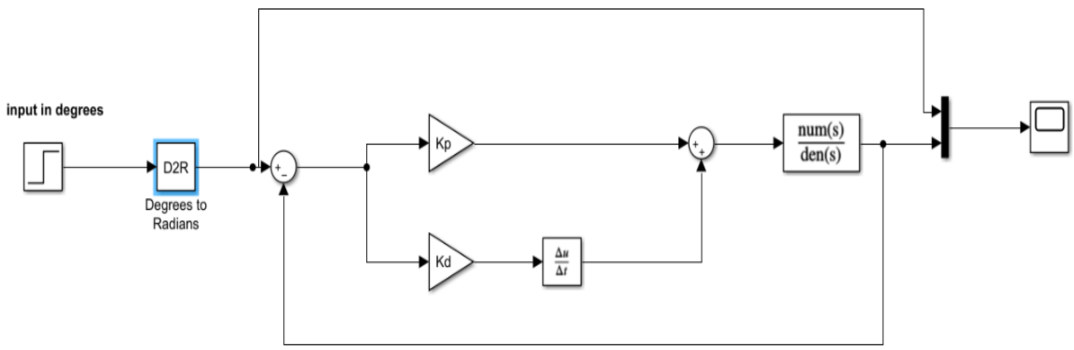


Figure 5.5: PD controller

- Simulation results

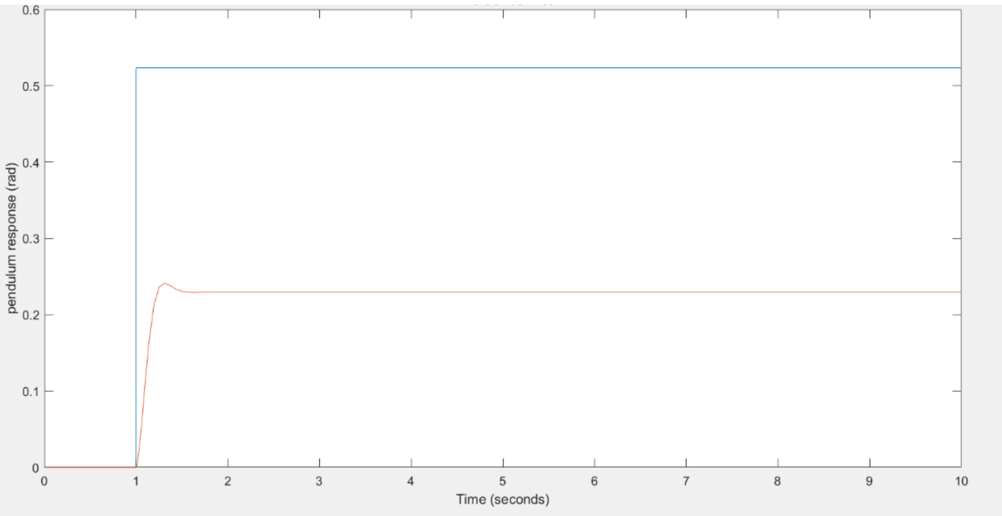


Figure 5.6: Response (PD) for first experiments

2. let of the system equals to the settling time 0.2 sec and critically damped, from equations (5.5) and (5.6) The controller gains are:

   Kp=0.5694
   Kd=0.0713

- Simulation results

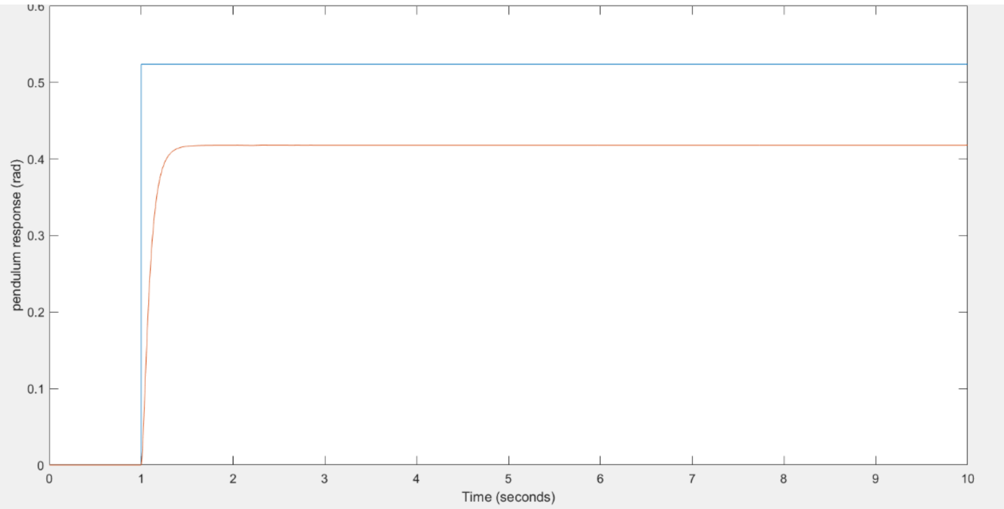


Figure 5.7: Response (PD) for second experiments

5.1.2 PID controller for pendulum

As design requirement let of the system equals to the settling time 0.06 sec and the percent over shoot (%OS) to be 10%. For designing the controller its preferred to use MATLAB SISOTOOL, because it's simple and easy function. So, the controller gains are:

Kp=4.52
Ki=23.1
Kd=0.217

- Simulink model for PID controller

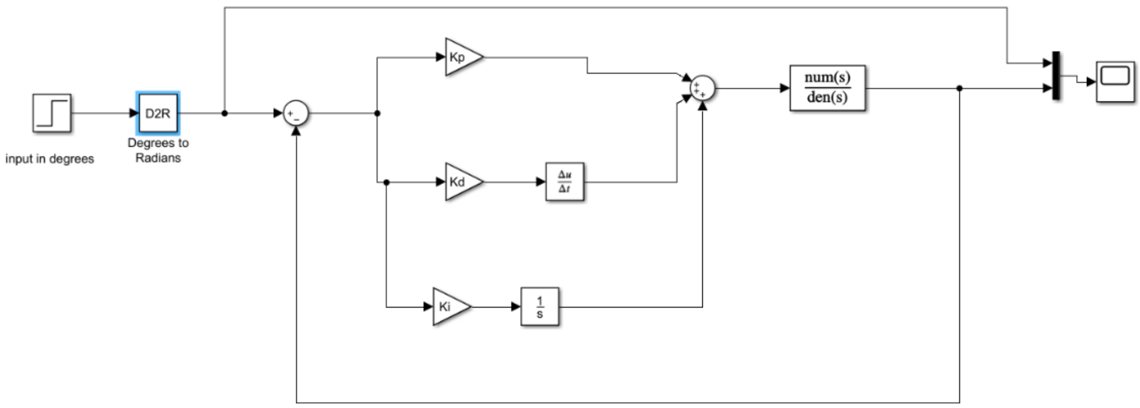


Figure 5.8: PID controller

- Simulation results

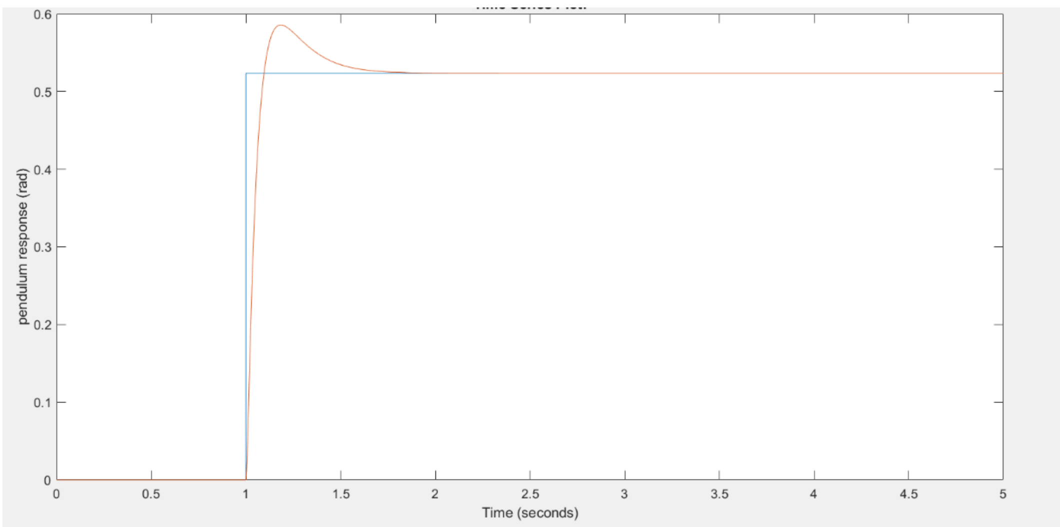


Figure 5.9: Response (PID) for first experiment

5.1.3 State feedback controller for pendulum

As equation (4.3) let

$x_1 = \theta \rightarrow \dot{x_1} = x_2$
$x_2 = \dot{\theta} \rightarrow \dot{x_2} = \ddot{\theta}$

the matrices are

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -mg\theta * \frac{L}{J2} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{T}{J} \end{bmatrix} u \tag{5.7}$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{5.8}$$

As design requirement, let of the system equals to the settling time 0.6 sec and the percent over shoot (%OS) to be 10%. the controller gains are:

K=[19.12   12]

- Simulink model for state feedback controller with disturbance

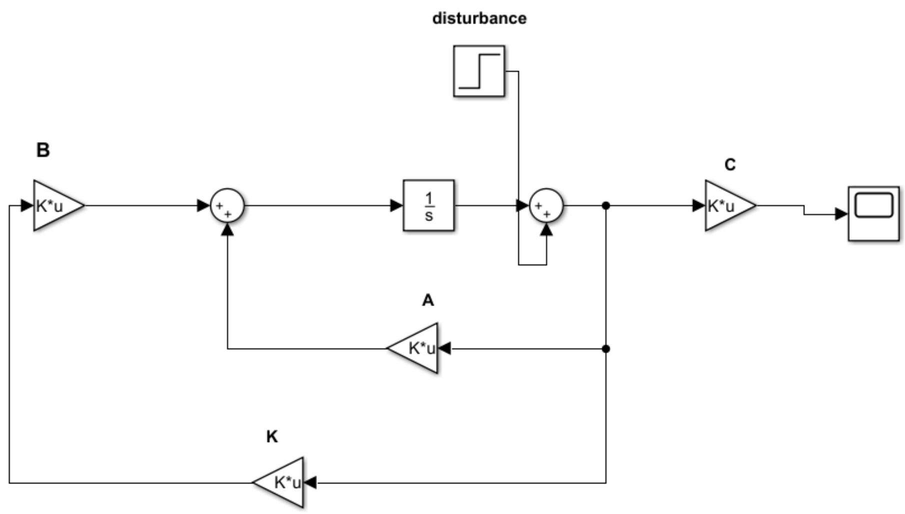


Figure 5.10: State feedback controller with disturbance

- Simulation results

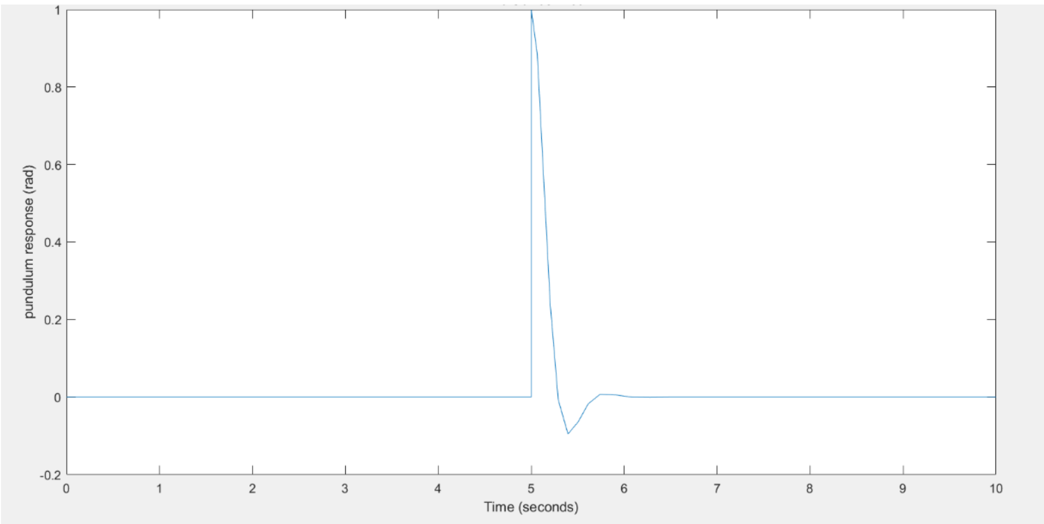


Figure 5.11: Response for state feedback controller with disturbance

5.1.4 Extended system for pendulum

The matrices for extended system are

$$A_e=\begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} \rightarrow \quad A_e=\begin{bmatrix} 0 & 1 & 0 \\ -mg\theta * \frac{L}{J2} & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

$$B_e=\begin{bmatrix} B \\ 0 \end{bmatrix} \quad \rightarrow \quad B_e=\begin{bmatrix} 0 \\ \frac{T}{J} \\ 0 \end{bmatrix}$$

As design requirement, let of the system equals to the settling time 1.66 sec and the percent over shoot (%OS) to be 10%. the controller gains are:
Ks=[223 65]
Ki=960

- Simulink model for Extended system

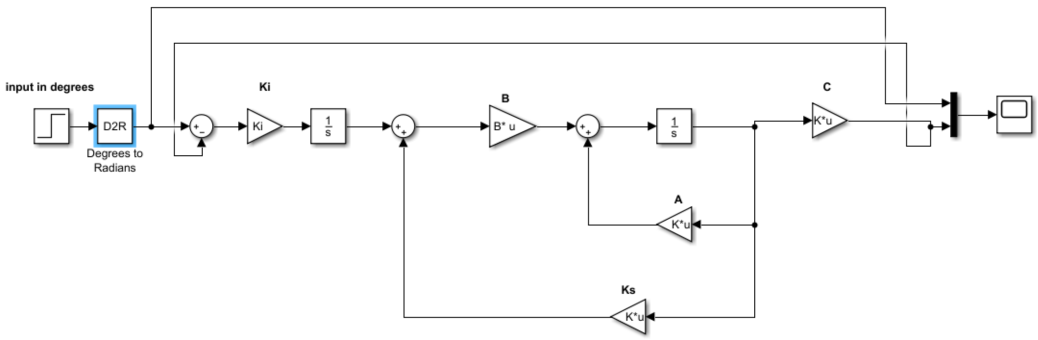


Figure 5.12: Extended system for pendulum

- Simulation results

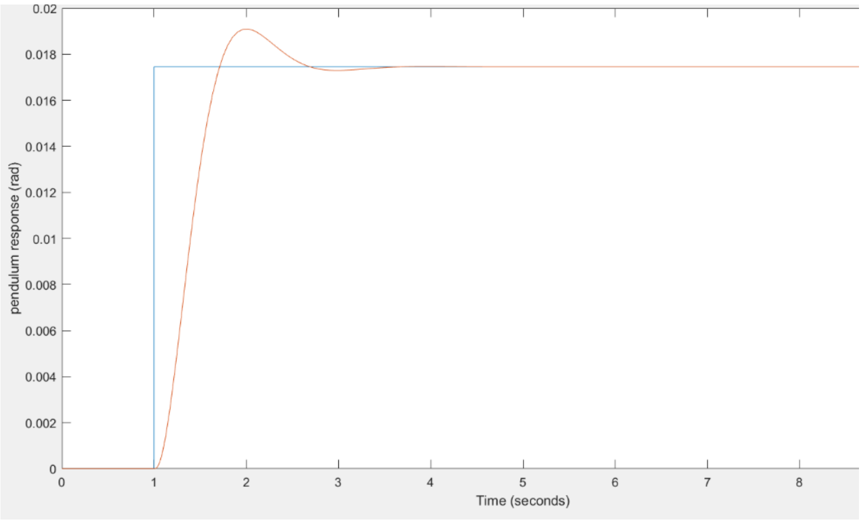


Figure 5.13: Response for extended system

5.2 Controllers Design and simulation for crank slider mechanism

In this section every linear control strategy used is discussed in details, after that a Simulink model built for the controller, and then shown the simulated results.

5.2.1 PD controller for crank slider mechanism

We can design controller from equation (4.34)

Where:

J_1=7.7× 10^{-5} Kg.m^2

m_1=0.055 Kg

m_2=0.1156 Kg

r=2.75× 10^{-2} m

g=9.81 m/s^2

$$\frac{\Theta(s)}{T(s)} = \frac{Kds+Kp}{(m_2R^2+J_1+m_1r^2)s^2} \quad (5.9)$$

The characteristic equation from closed loop for the transfer function

$$s^2 + \frac{Kds}{m_2R^2+J_1+m_1r^2} + \frac{Kp}{m_2R^2+J_1+m_1r^2} \quad (5.10)$$

By definition: the general form of the transfer function of a second system is

$$C(s) = \frac{wn^2}{s^2 + 2\zeta\omega ns + wn^2} \quad (5.11)$$

Then the characteristic equation

$$s^2 + 2\zeta\omega ns + \omega n^2 \quad (5.12)$$

From equations (5.2) and (5.4)

Kp=$wn^2(m_2R^2 + J_1 + m_1r^2)$ (5.13)

Kd=$2\zeta\omega n(m_2R^2 + J_1 + m_1r^2)$ (5.14)

As design requirement let of the system equals to the wn 4 rad/s and
the damping ratio to be 0.6, from equations (5.13) and (5.14) The controller gains are:

Kp=0.0075
Kd=0.0023

- Simulink model for PD controller

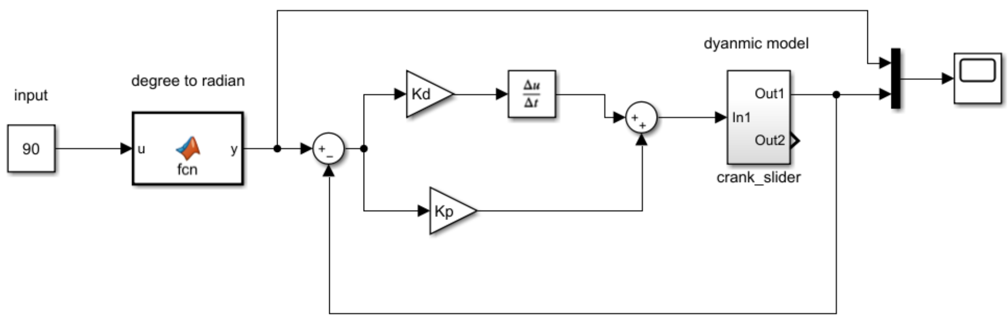


Figure 5.14: PD controller for crank mechanism

- Simulation result

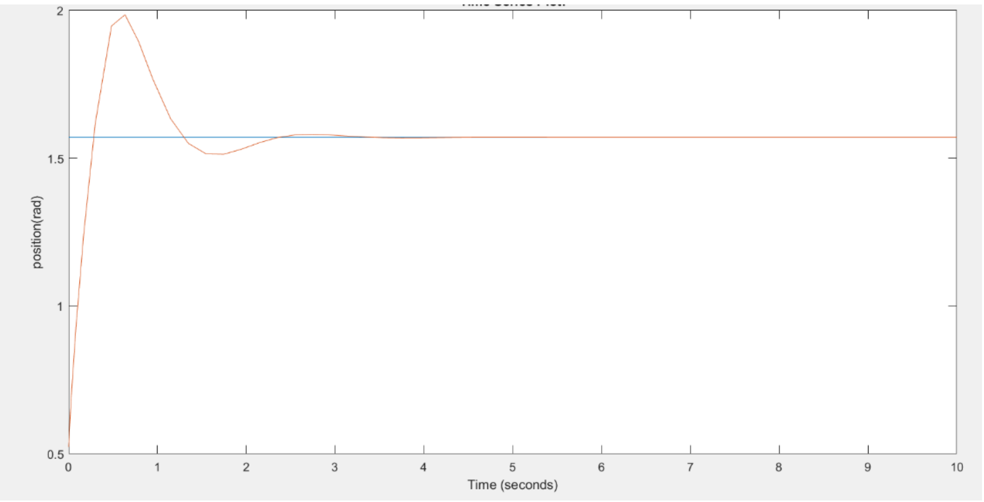


Figure 5.15: Response for PD controller

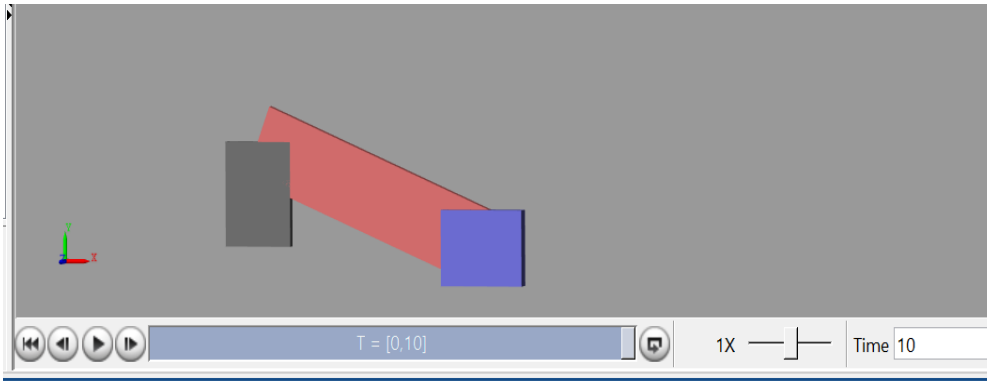


Figure 5.16: the crank slider after run PD controller

5.2.2 PID controller for crank slider mechanism

As design requirement let of the system equals to wn 4 rad/sec and the damming ratio to be 0.7. For designing the controller its preferred to use MATLAB SISOTOOL, because it’s simple and easy function. So, the controller gains are:

Kp=0.00815
Ki=0.00168
Kd=0.00273

- Simulink model for PID controller

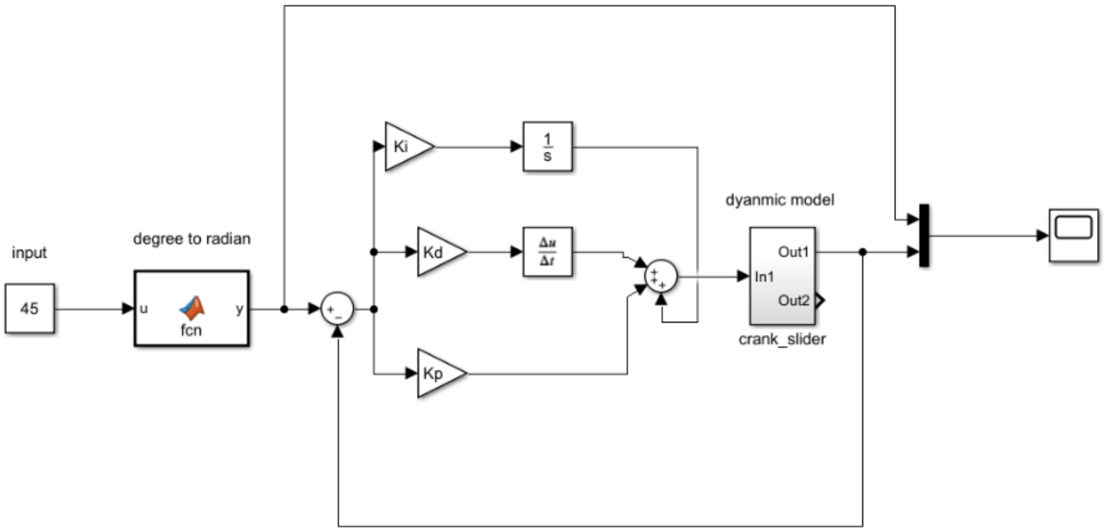


Figure 5.17: PID controller for crank slider

- Simulation result

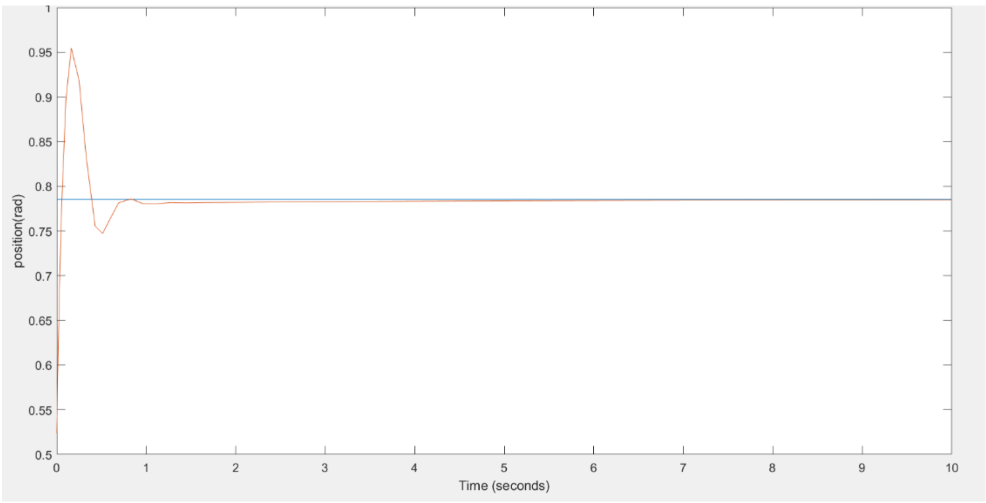


Figure 5.18: Response for PID controller

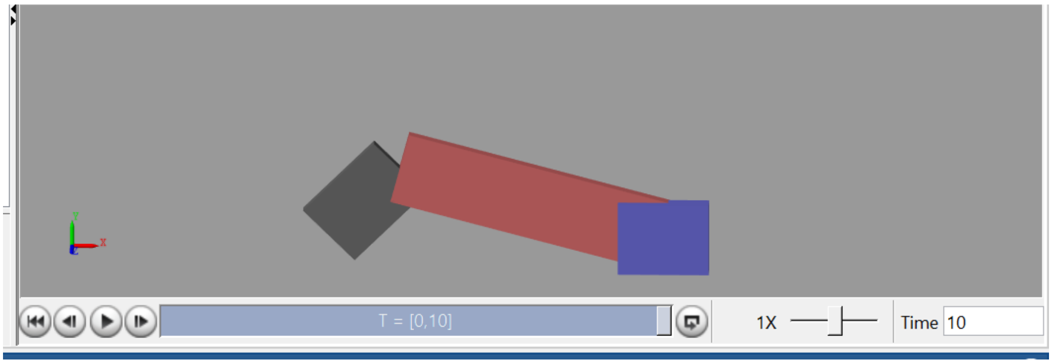


Figure 5.19: The crank slider after run PID controller

5.2.3 State feedback controller for crank slider mechanism

As design requirement, let of the system equals to the wn 40 rad/sec and damping ratio 0.6. the controller gains are:

K= [0.7541 0.022]

- Simulink model for state space controller

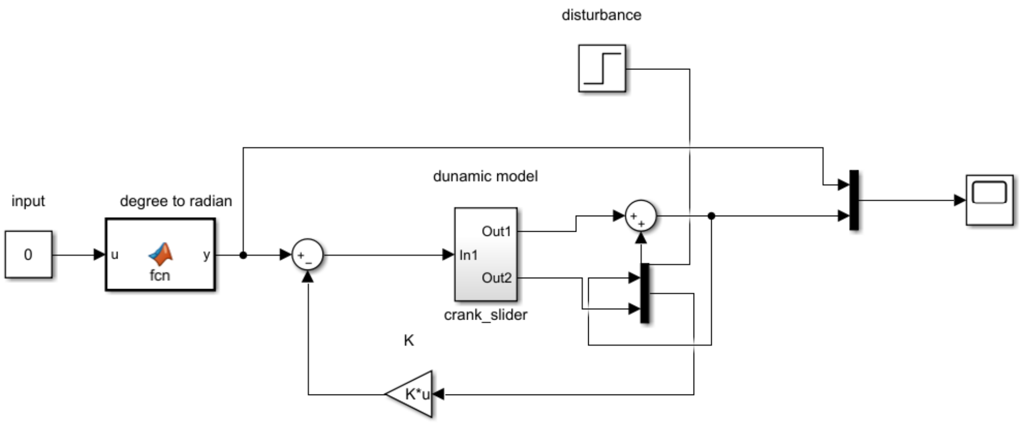


Figure 5.20: state space controller

- Simulation result

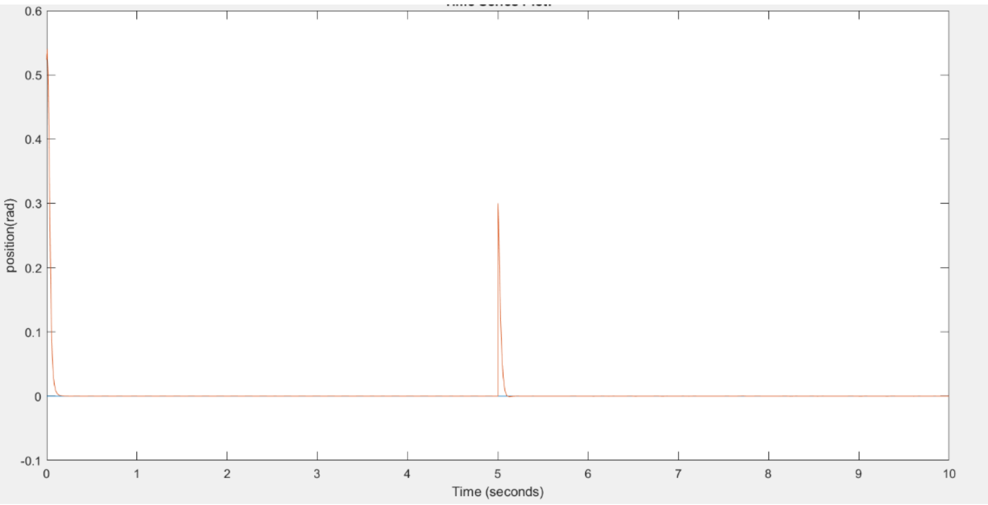


Figure 5.21: Response for state space with disturbance

5.2.4 Extended system for crank slider mechanism

As design requirement, let of the system equals to the wn 40 rad/sec and damping ratio 0.6. the controller gains are:

Ks=[2.112 0.0509]

Ki=45.25

- Simulink model for extended system controller

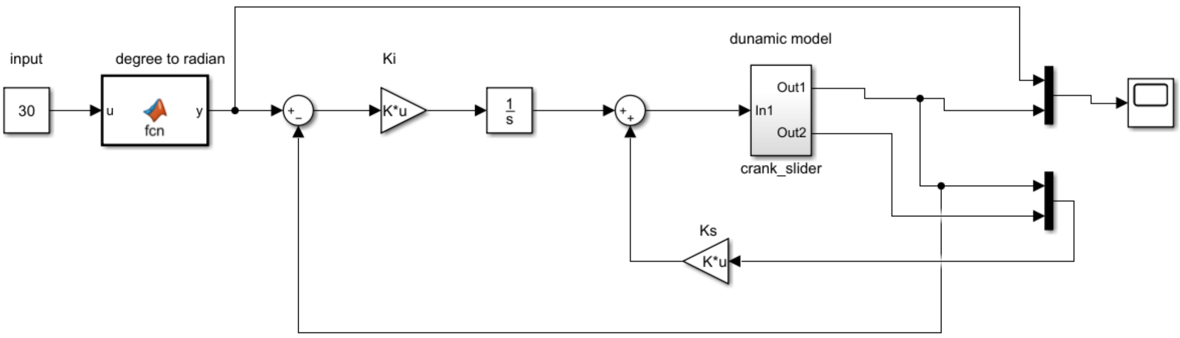


Figure 5.22: Extended system controller

- Simulation result

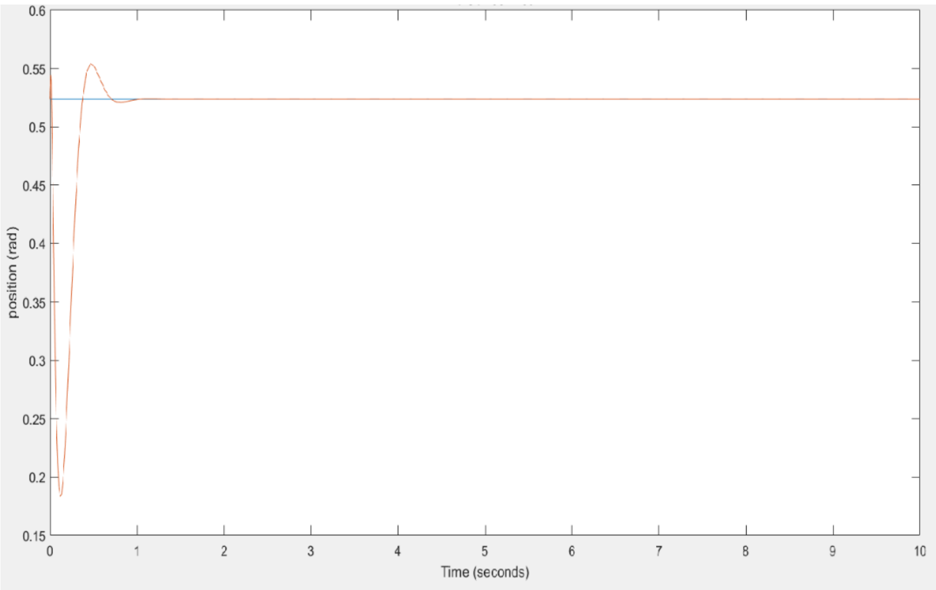


Figure 5.23: Response extended system controller

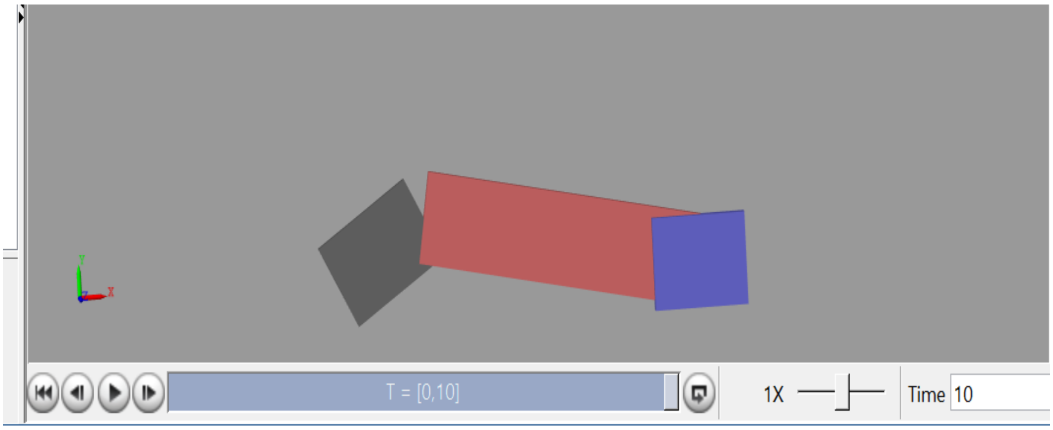


Figure 5.24: The crank slider after run extended system controller

Chapter 6: Selection of electrical components and processing unit

This chapter reviews the sensor, controller selection and technologies for each one. The interface between human and Exoskeleton and how the signal exchanges in order to drive an actuator, provide feedback for human motor control.

6.1 Selecting Actuating Subsystem

In Exoskeleton the most important component is the actuators, Based on the conceptual design that is presented in chapter 2, The apparatus has one actuator and selected servo motor for many reasons, the motor available in university, the motor have built in encoder and used encoder to know position of applications and the motor has driver.

The advantages for servo driver

- Can interface with DAQ
- The ability to generate enough torque independently of the speed of the motor
- The ability to control the speed independently of the torque value
- Simple and precise control.

The parameter for our motor:
Model: ECMA-C10604RS
Output: 3000 r/min, 1.27 Nm., 0.4 KW

Figure 6.1: Servo motor[12].

6.2 Selecting interface Subsystem

The needing to use this subsystem is to convert the control signal that coming from the matlab to analog signal and send it to the servo driver, and transmit the reading of the counter to the matlab.

We faced many problems to interface between servo driver and matlab software, in the following review the hardware was used and the problem for each one.

- NI myDAQ

This hardware can supply analog command using matlab software from (-10 – 10) voltage and it make support this hardware as encoder input block, figure (6.2) shows NI myDAQ hardware. suitable for our servo driver, but the problem for this hardware can't enter the number of counting for encoder to matlab software.

Figure 6.2: NI myDAQ[13].

- Raspberry pi 3b
  The advantage for Raspberry pi is easy to connect with other external modules by different communication protocols as (SPI,I2C,…etc), this protocols supported form matlab software, the problem fot this hardware doesn't has analog output and hardware counter for encoder so we need to external modules for analog output and hardware counter, and these modules don't exists in the local market, figure (6.3) shows Raspberry pi 3b hardware.

Figure 6.3: Raspberry pi 3b[14].

- DAQ PCI 6221
  Based on the above we selected the DAQ PCI 6221 because we don't have another choice for lake of time, it available in university and it has a package included in matlab Simulink and easy to use it, figure (6.4) shows DAQ PCI 6221 hardware.

Figure 6.4: DAQ PCI 6221

6.3 Selecting sensor

we need sensor to measurement angle of applications, and the most sensor common and used for this job is optical encoder, luckily our servo motor has built in 20-bit incremental encoder, and this encoder characterized by high accuracy.

6.4 Selecting other electorships

- Quad Eia-422/423-line receiver with three-state outputs

  The output encoder signal from servo driver is deferential signal, selected this IC to convert differential signal to pulse signal pin connection for IC as shown in figure.

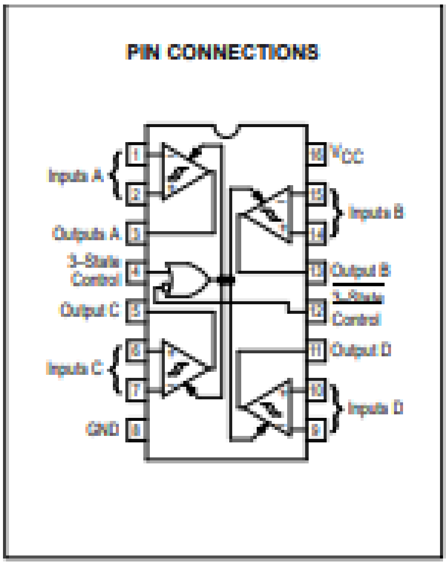


Figure 6.5: Quad Eia-422/423 converter [15].

Chapter 7: Experimental results

This chapter reviews the experimental results by using DAQ PCI-6221 with matlab software (real-time desktop), and applied different controllers on pendulum and crank slider mechanism.

7.1 Experimental results for pendulum

7.1.1 PD controller for pendulum

Applied the gains controller as chapter 5, and the result as shown below.

- Experimental model for PD controller

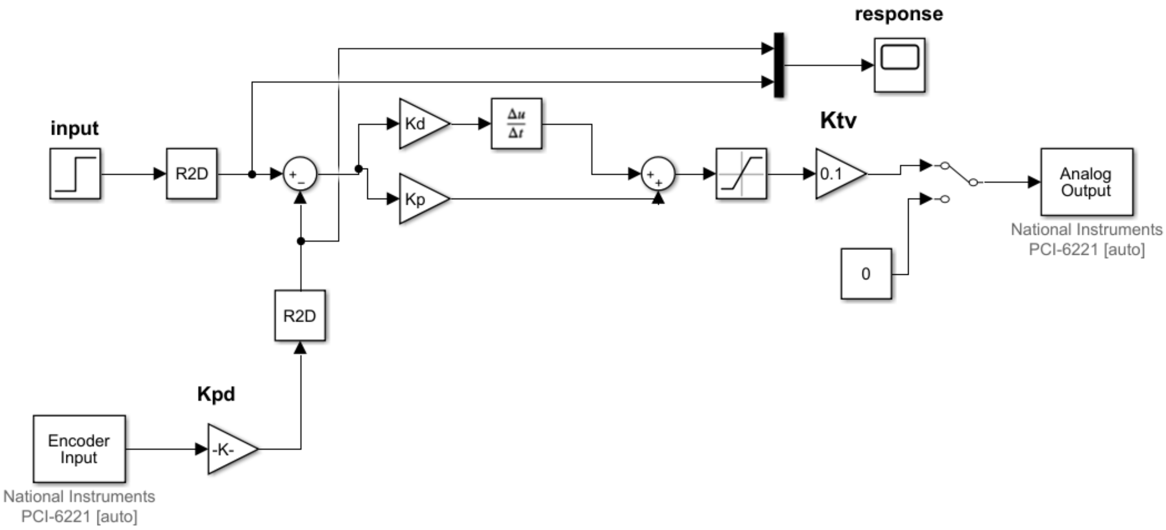


Figure 7.1: Experimental model for PD controller

- Experimental result

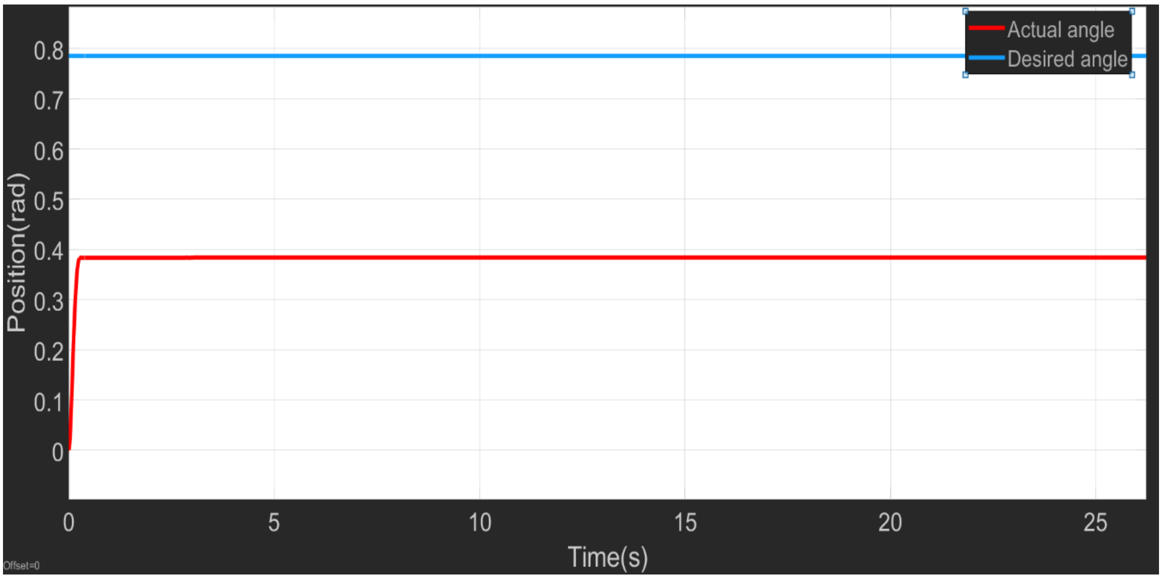


Figure 7.2: Experimental Result for PD controller (45 degree)

7.1.2 PID controller for pendulum

The controllers gains as chapter 5, and the result as shown below.

.

- Experimental model for PID controller

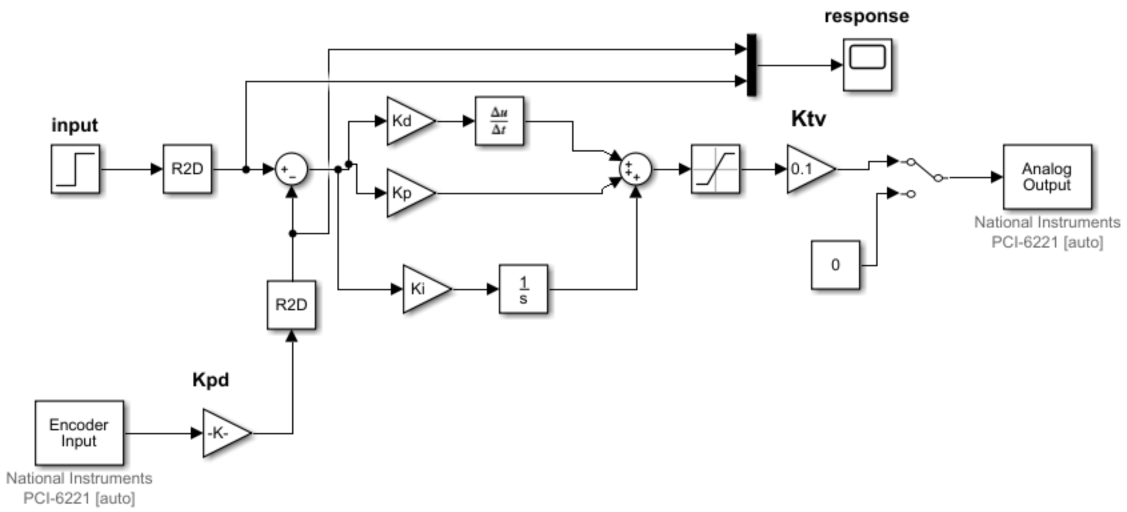


Figure 7.3: Experimental model for PID controller

- Experimental result

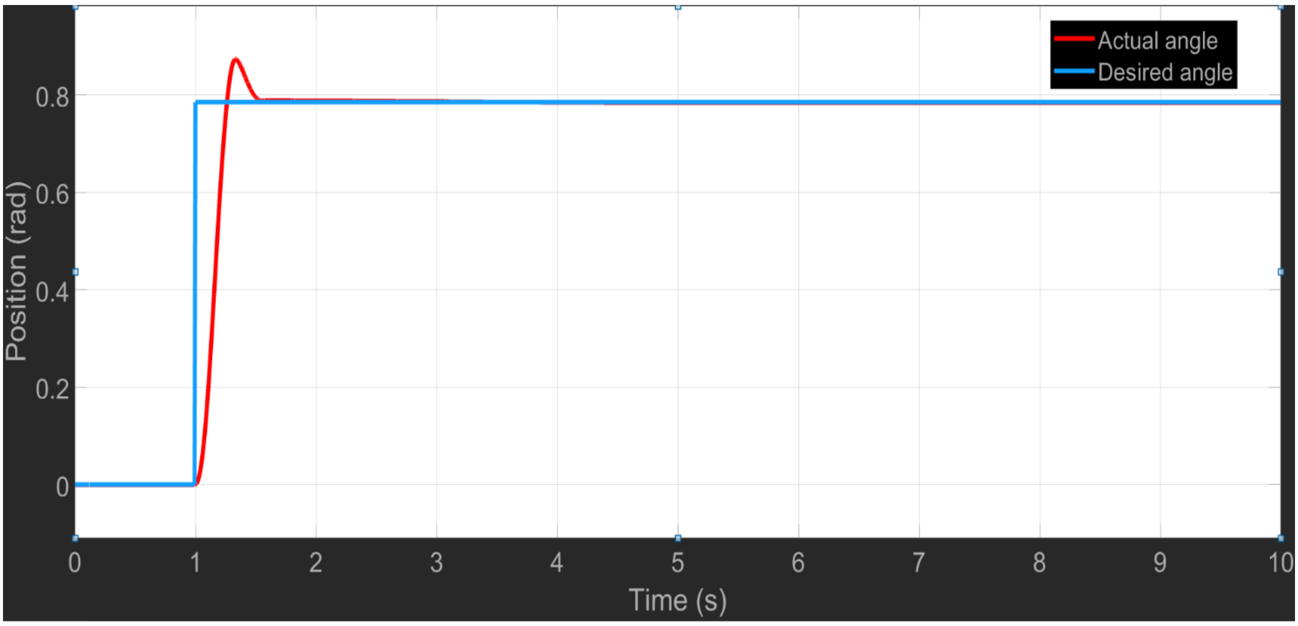


Figure 7.4: Experimental Result for PID controller (45 degree)

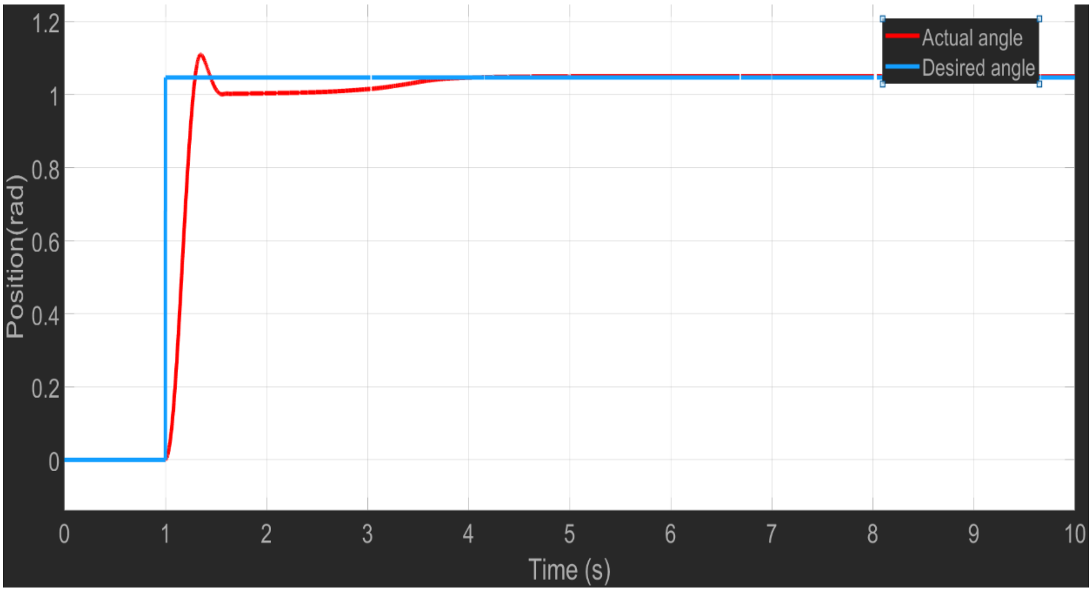


Figure 7.5: Experimental Result for PID controller (60 degree)

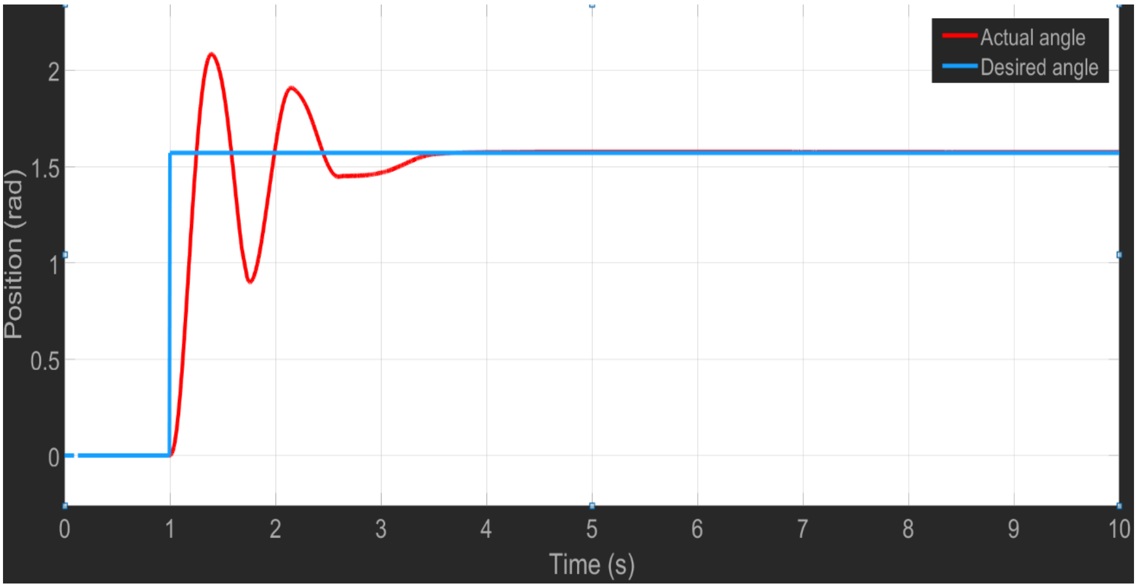


Figure 7.6: Experimental Result for PID controller (90 degree)

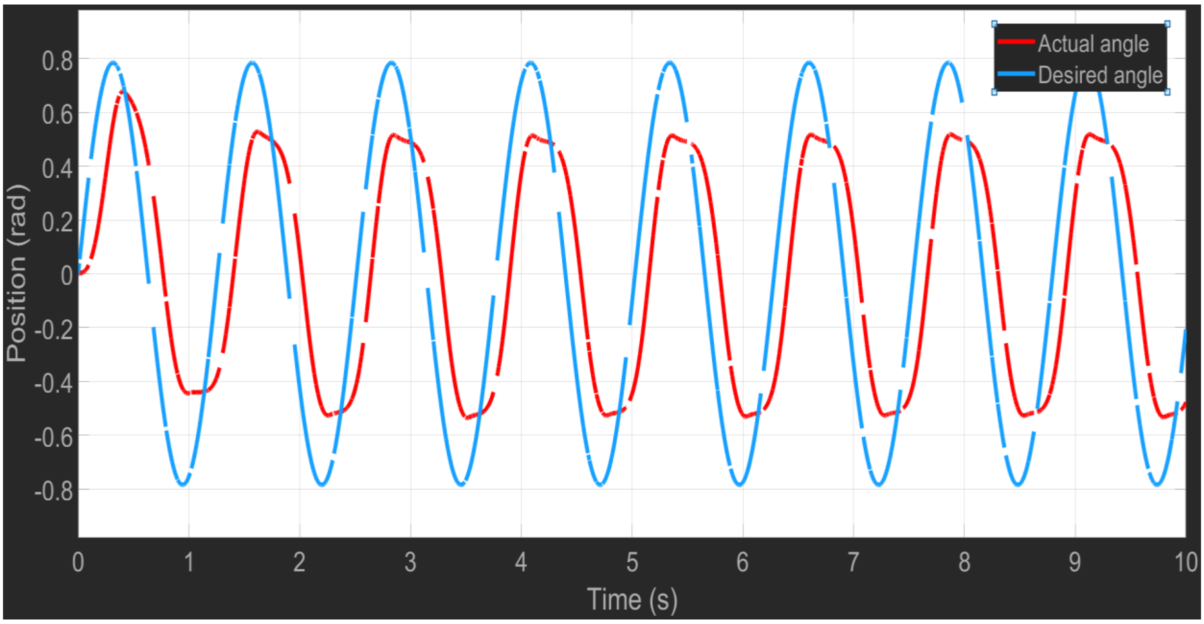


Figure 7.7: Experimental Result for PID controller (-45 - 45)

7.1.3 State feedback controller for pendulum

The controllers gains as chapter 5, and the result as shown below.

- Experimental model for state-space controller

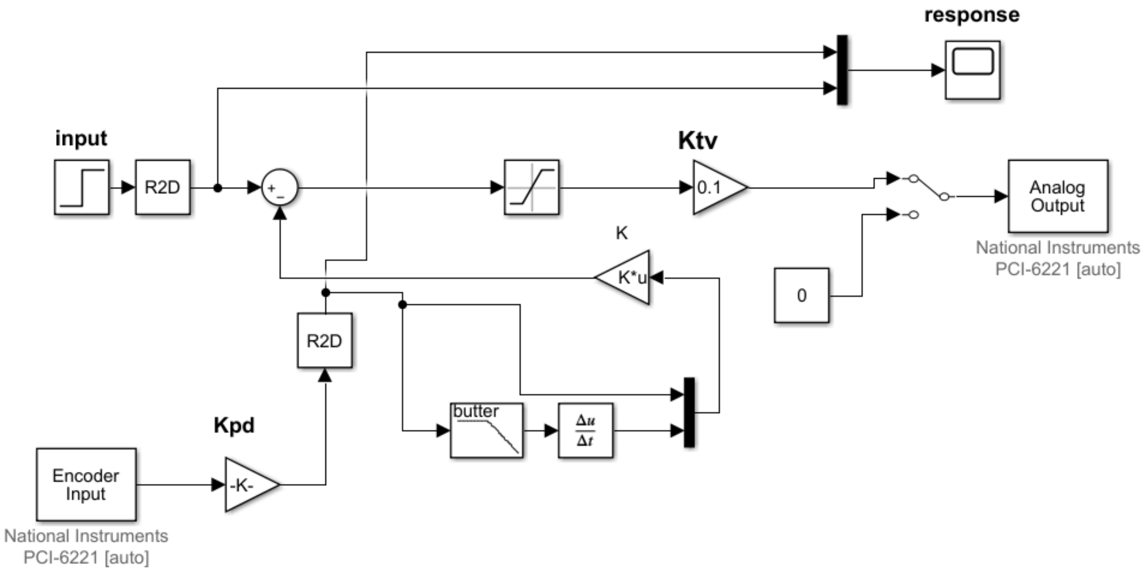


Figure 7.8: Experimental model for State space controller

- Experimental result

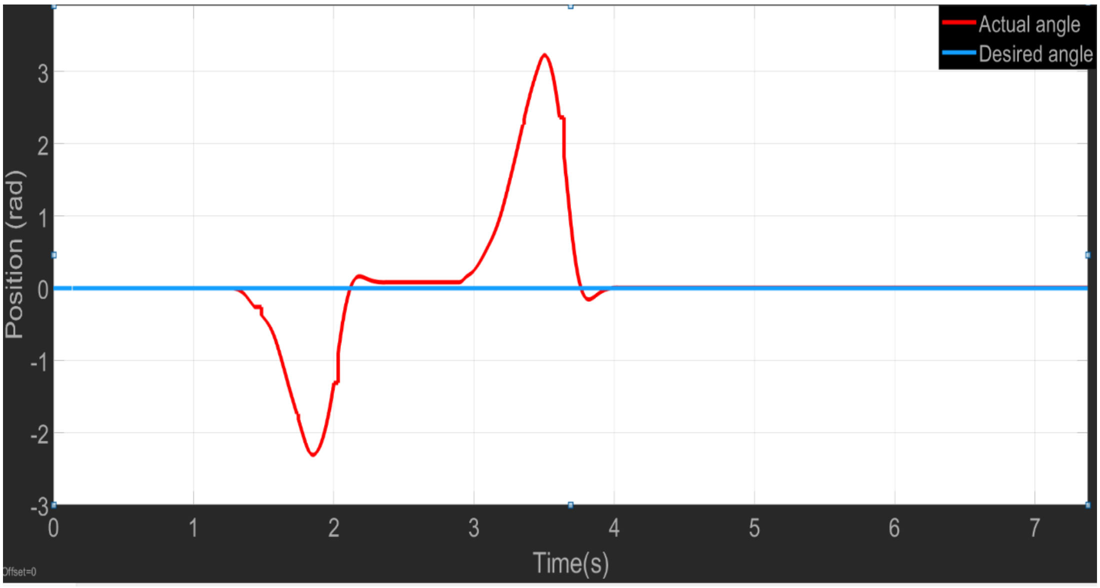


Figure 7.9: Experimental Result for state-space with disturbance

7.1.4 Extended system for pendulum

The controllers gain as chapter 5, and the result as shown below

- Experimental model for extended system controller

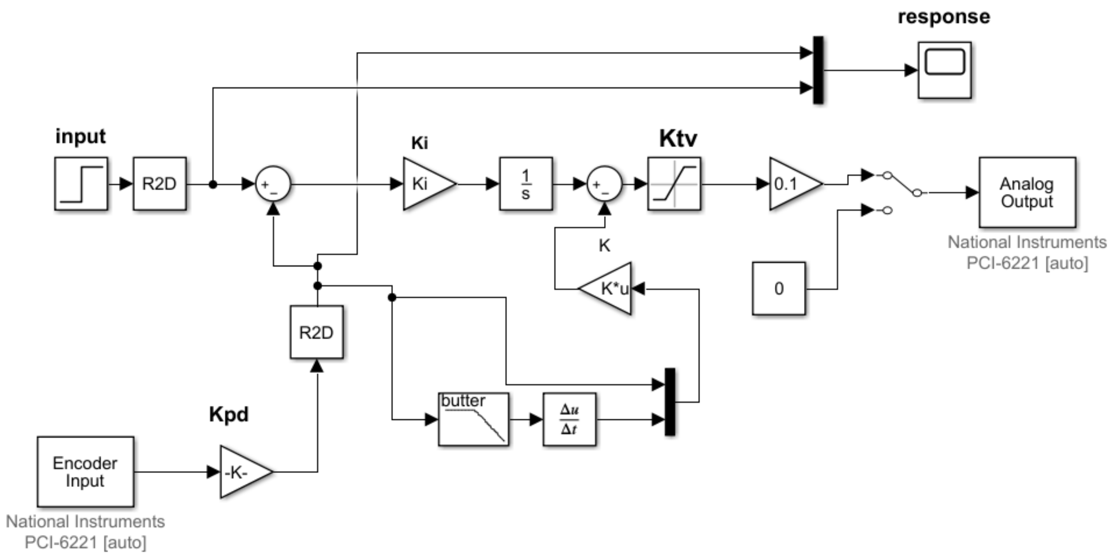


Figure 7.10: Experimental model for extended system controller

- Experimental result

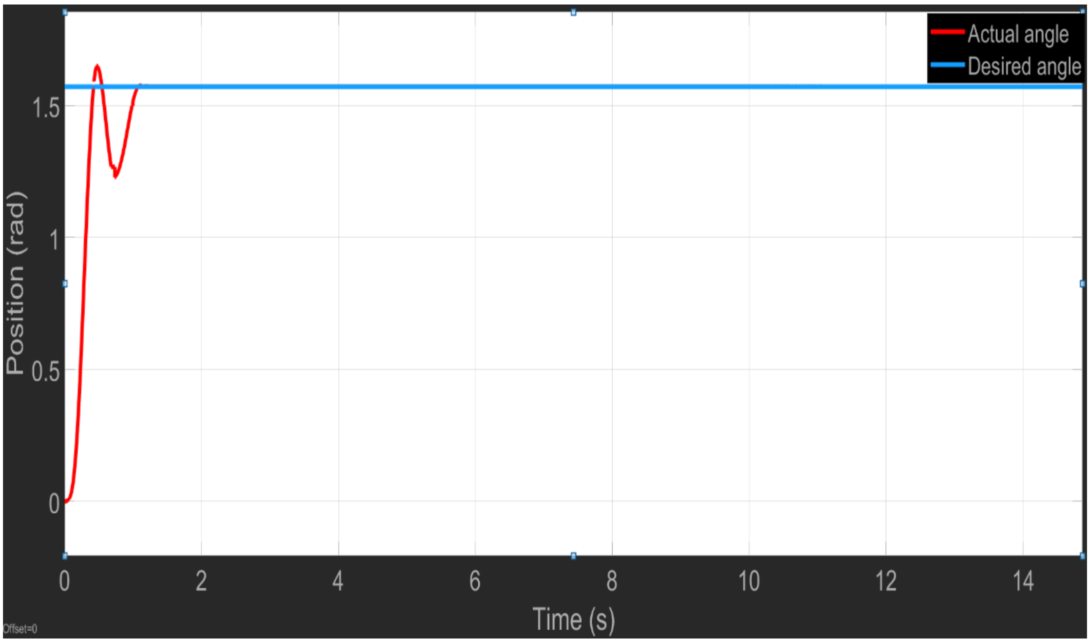


Figure 7.11: Experimental Result for Extended system (90 degree)

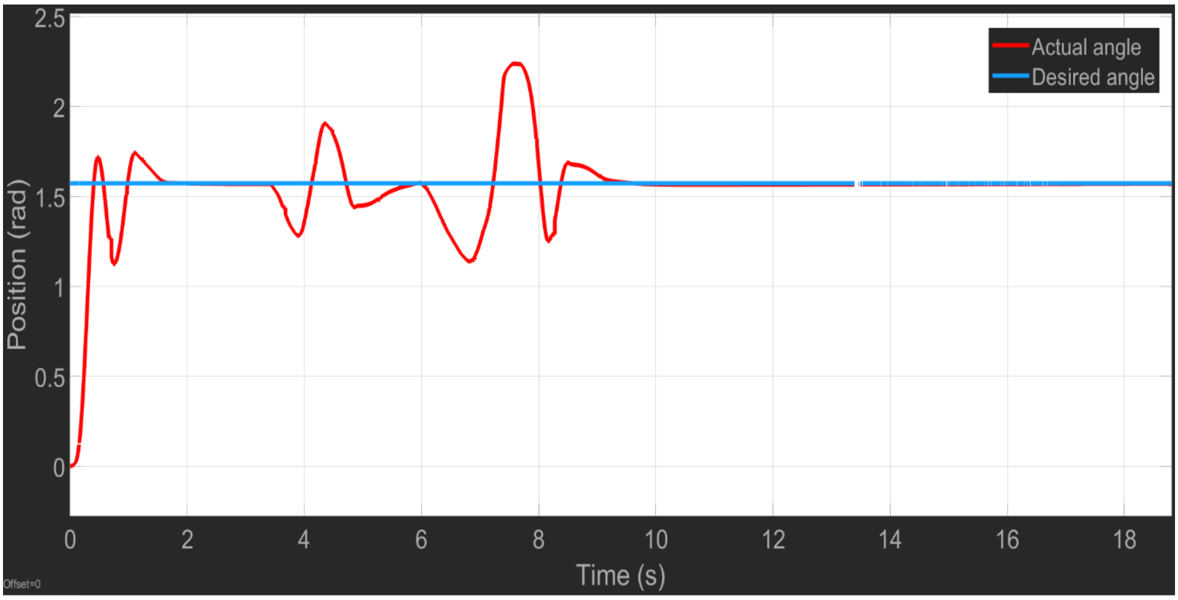


Figure 7.12: Experimental Result for Extended system (90 degree) with disturbance

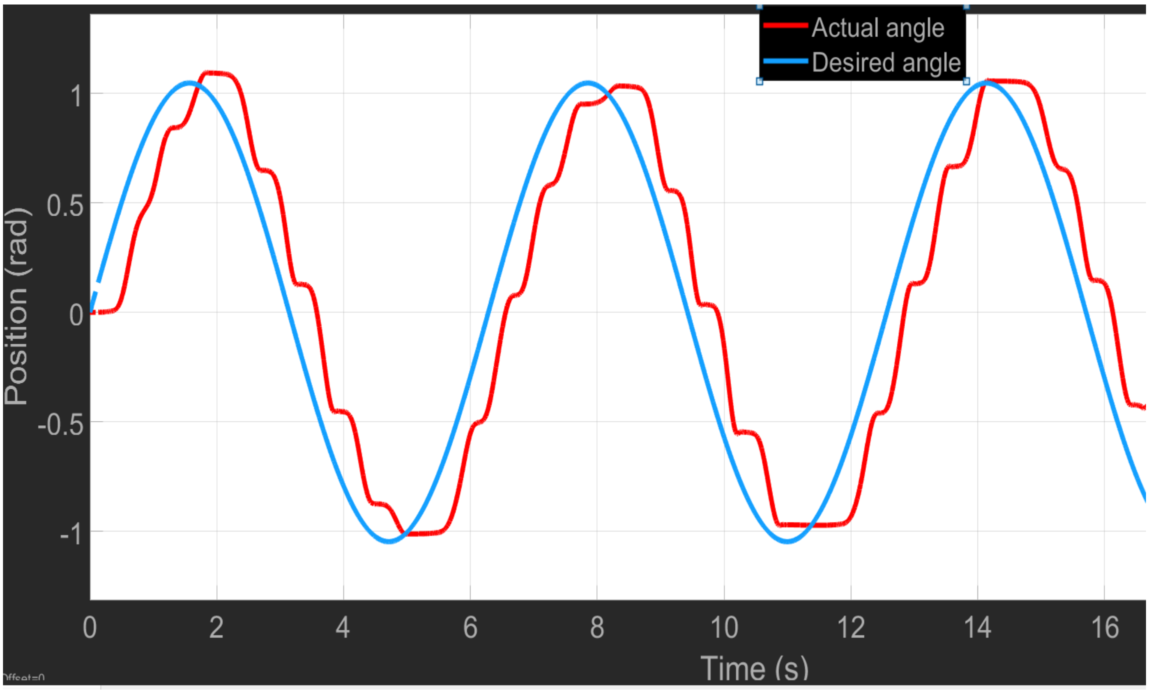


Figure 7.13: Experimental Result for PID controller (-60 - 60)

Chapter 8: Conclusion and Suggestion

8.1 Conclusion

In this graduation project we designed and built apparatus for control motion and applied different controllers in the applications.

We applied different controllers on pendulum (PD,PID, state space and extended system) and we found the most controllers robust are the state space and extended system, and when apply PD controller we found an error between actual and desired angle because the PD controller improve the transient response opposite other controllers, the controller of pendulum just work about small angle other ways the system be unstable because the mathematical model is nonlinear and we linearized it about small angle.

Other mechanism is the crank slider and when applied the different controllers on the mechanism doesn't work, and we think the controller doesn't work because the dynamic impedance focuses in the slider, where when compare the slider with crank and connecting rod too heavy and this point didn't take it into consideration when designed the mechanism.

8.2 Suggestion

The advantage for our project is can applied different applications in the same motor, and can add the other applications. Can use the PCI to USB adapter to make the apparatus portable, and redesign the crank slider and can applied other controllers on the applications.

Chapter 9: Manual

Operate PCI 6221 with Matlab

- Insert the card in computer

Complete the following steps to install a PCI or PCI Express DAQ board:

1. Power off and unplug your computer.
2. Remove the computer cover and/or the expansion slot cover.
3. Touch any metal part of the computer to discharge any static electricity.

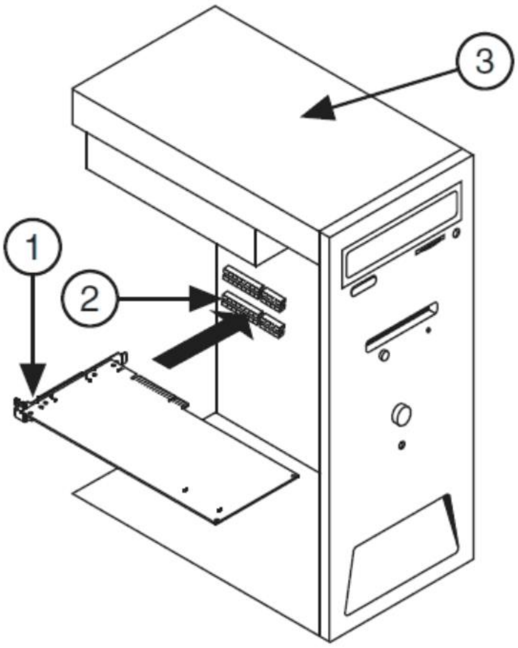


Figure 8.1: Installing a PCI/PCI Express Board

- Install the driver for this DAQ

you can install NI-DAQmx driver software. You must install NI-DAQmx driver software before installing any new NI hardware devices so Windows can detect them. You have three options for installation:

1. NI Device Drivers DVD: If you selected to install device drivers from the LabVIEW Platform DVD, you will be prompted to insert the NI Device Drivers DVD before continuing.
2. NI-DAQmx DVD: All NI DAQ devices include a DVD with the drivers you need to use the device.
3. Online Download: You can always find the latest NI-DAQmx driver software on the Drivers and Updates page.

   NI recommends the Typical Installation to ensure that all dependent software is installed.

   Note: After install the driver plz restart the computer

- Operate matalb software and install the Kernel Using MATLAB in these steps

1. In the MATLAB® Command Window, type:

```
sldrtkernel -install
```

   The MATLAB Command Window displays one of these messages:

   You are going to install the Simulink Desktop Real-Time kernel.
   Do you want to proceed? [y]:
   or:

   There is a different version of the Simulink Desktop Real-Time kernel installed.
   Do you want to update to the current version? [y]:
2. Type y to continue installing the kernel, or n to cancel installation without changing the installation.

   If you type y, the MATLAB environment installs the kernel and displays the message:

   The Simulink Desktop Real-Time kernel has been successfully installed.
3. If a message appears asking you to restart your computer, do so before attempting to use the kernel, or your Simulink Desktop Real-Time model does not run.
4. After installing the kernel, check the installation by typing:

```
rtwho
```

   The MATLAB Command Window displays a message that shows the kernel version number, followed by timer, driver, and other information.

- Hardware connection

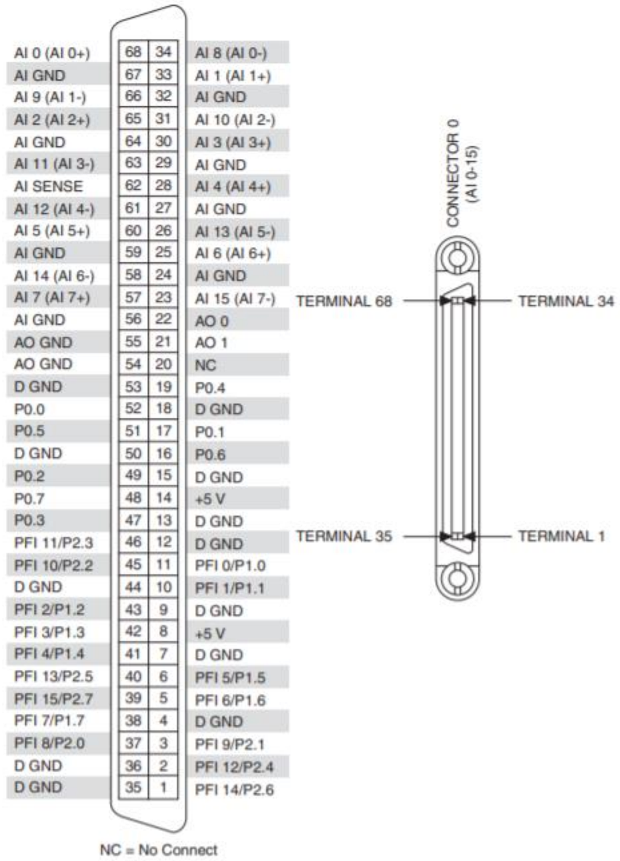


Figure 8.2: PCI/PXI-6221 Pinout

Encoder channel

Connect the A, B and Z channels as below

CTR 0 A	37 (PFI 8)
CTR 0 Z	3 (PFI 9)
CTR 0 B	45 (PFI 10)

Connect the analog input for driver to pins 22(AO 0) and 55 (GND AO)

- Select the DAQ in matlab Simulink

Chose the library Simulink real-time

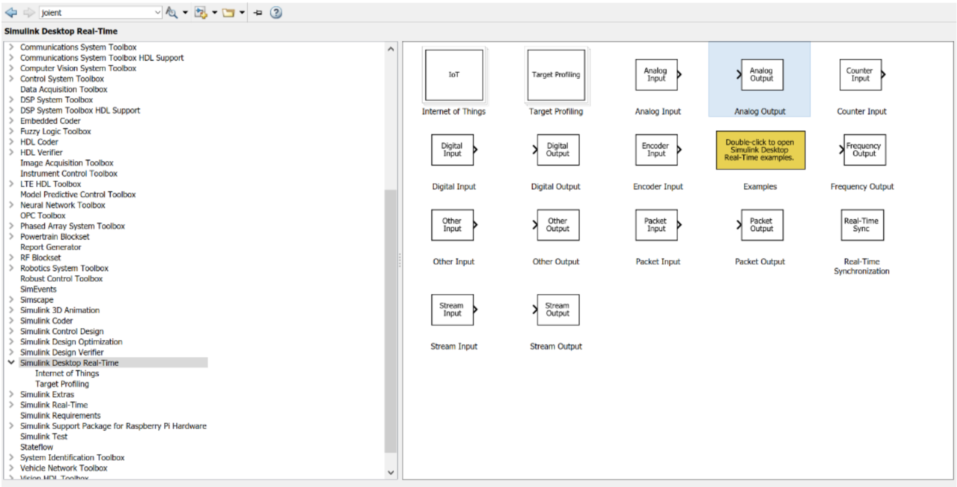


Figure 8.3: Simulink desktop real-time

Insert the block then click to Install new board → National Instruments (2) → PCI-6221

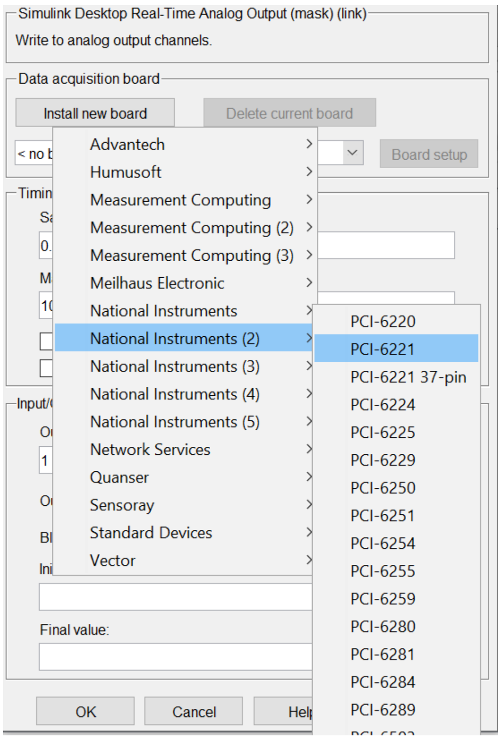


Figure 8.4: Chose the DAQ from block

Check the configuration block for analog output as shows in figure

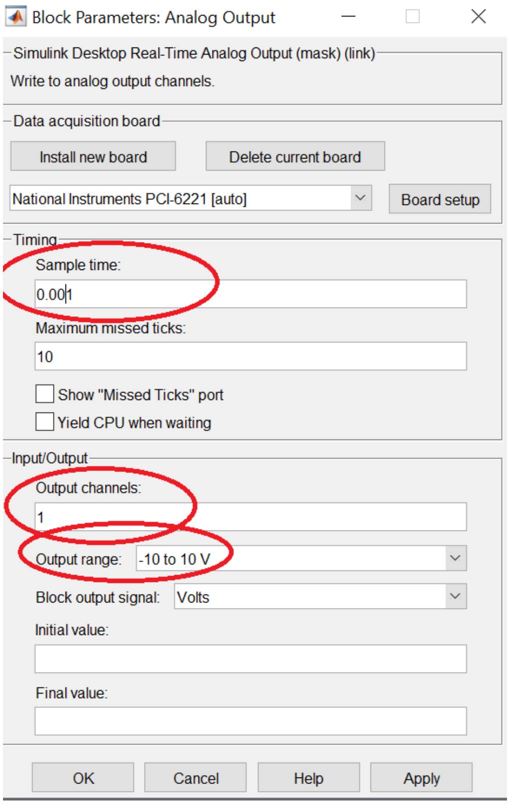


Figure 8.5: Configuration for analog output block

Check the configuration block for encoder input as shows in figure

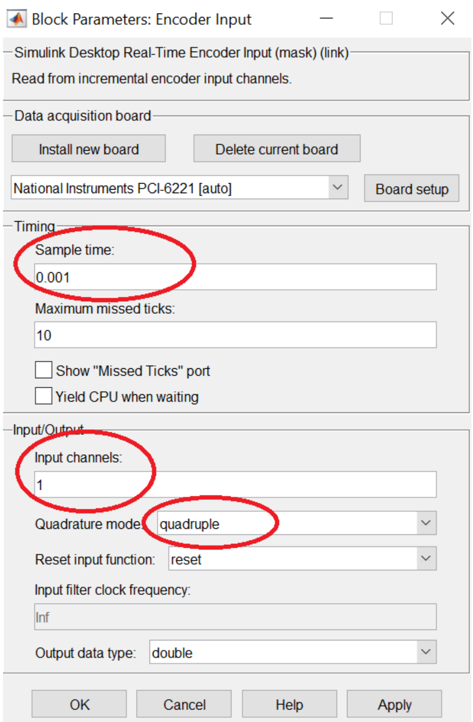
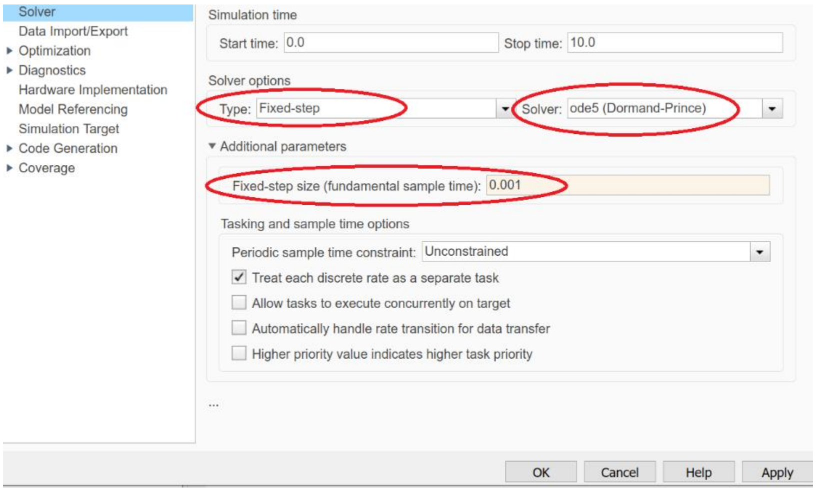


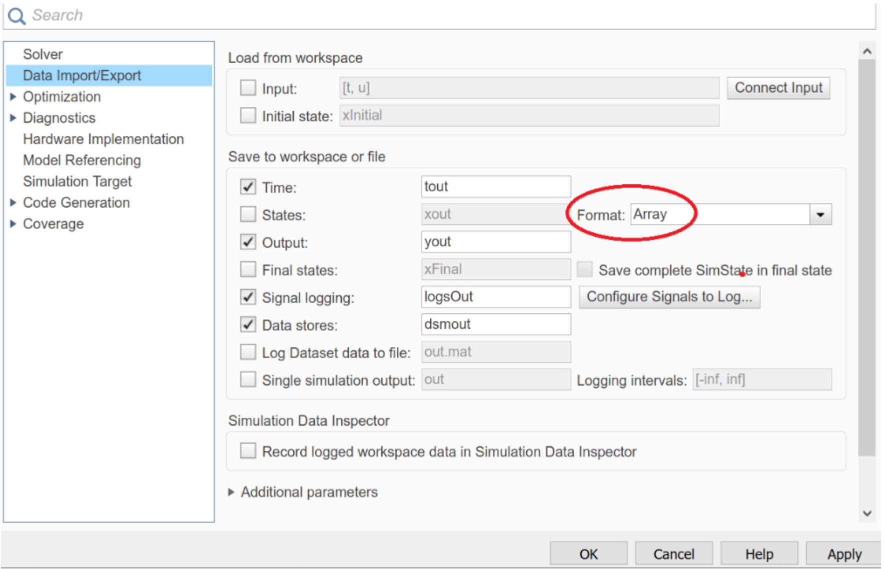
Figure 8.6: Configuration for encoder block

MATLAB Simulink configuration for real time applications

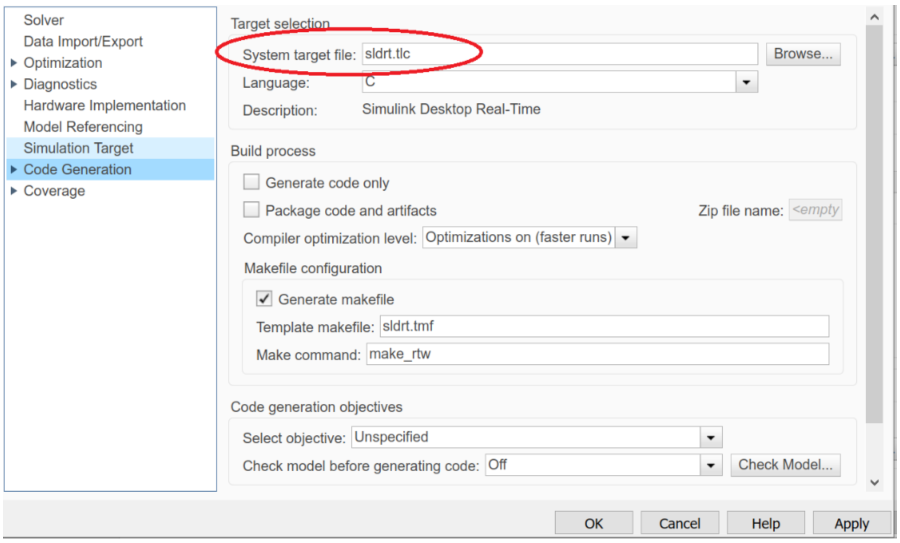
1.



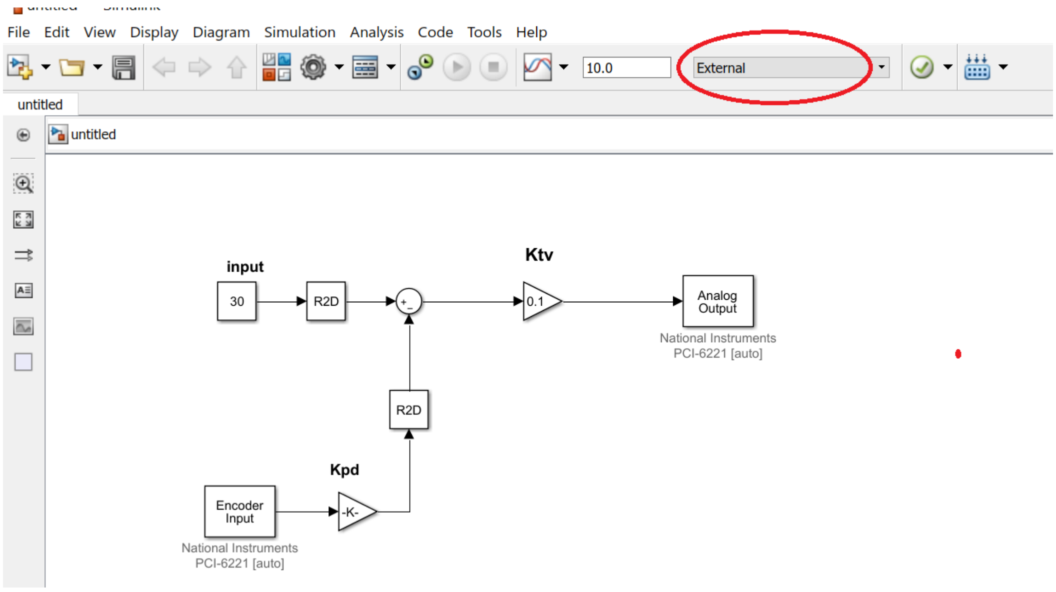
2.



3.



Check the apparatus using this Simulink model and put the Simulink in external mode



Where
Convert the torque to voltage (Ktv) =0.1
Convert the pulses to degree (Kpd) = 0.036

Note: remove the application when test the apparatus if the apparatus doesn't work check the hard ware connection and previous steps

Appendix A

Appendix A

Controller design (m-file)

A.1 PD controller for pendulum

```
clc
clear all
%%controller for pendulum
%definition of basic parameter
Ts=0.001                    %sample time
j_equvelant= 0.001784;      %inertia for pendulum
m=0.1471;                   %mass for pendulum
g=9.81                      %accelration of gravity
L=0.20                      %Length of pendulum
%controller gain
zeta=0.6;
wn=10;
Kp=wn*wn*j_equvelant+m*g*0.10
Kp=wn*wn*j_equvelant
```

A.2 PID controller for pendulum

```
%% pid controller
%%controller for pendulum
%definition of basic parameter
Ts=0.001                    %sample time
j_equvelant= 0.001784;      %inertia for pendulum
m=0.1471;                   %mass for pendulum
g=9.81                      %accelration of gravity
L=0.20                      %Length of pendulum;
num=[1]
dem=[j_equvelant  0 m*g*L/2 ]
TF=tf(num,dem)
% Controller gains (using sisotool function)
%at setlling time 0.06 and overshoot 10%
Kp=4.52
Ki=23.1
Kd=0.217
 %this gains from sisotools at wn=561 rad/s and zeta=0.5641
```

A.3 state space controller and extend system for pendulum

```
%state space and extended system
%definition of basic parameter
Ts=0.001                                %sample time
j_equvelant= 0.001784;                  %inertia for pendulum
m=0.1471;                               %mass for pendulum(Kg)
g=9.81                                  %accelration of gravity
L=0.20                                  %Length of pendulum;
A=[0 1;m*g*L/(2*j_equvelant) 0];        %system matrix
B=[0 1/j_equvelant]';                   %input matrix
C=[1 0]                                 %output matrix
R=ctrb(A,B) ,q=rank(R)                  %check controlability for state space
zeta=0.9
wn=8
p1=-zeta*wn-i*wn*sqrt(1-zeta^2)
p2=conj(p1)
p=[p1 p2]                              %desired pole for state space
K=place(A,B,p)                         %gain for state space
Ae=[A,zeros(2,1);-C 0]
Be=[B;0]
R1=ctrb(Ae,Be) ,q1=rank(R1)            %check controlability for extended system
pe=[p1 p2 -300]                        %desired pole for extended system
Ke=place(Ae,Be,pe)                     % gains for extended system
Ks=Ke(1:2) , Ki=-Ke(3)
```

A.4 PD controller for crank slider mechanism

```
%% PD controller for crank slider mechanism
%definition of basic parameter
Ts=0.001                         %sample time
g=9.81                           %accelration of gravity
m_1=0.05508;                     %mass for crank
m_2=0.1166;                      %mass for connecting rod
I=7.7e-5;                        %inertia for crank
R=5.5e-2;                        %lenrth of crank
r=R/2;
wn=4;
z=0.6;
O=m_2*R^2+I+m_1*r^2;            %equivilant inertia for crank slider
Kp=wn^2*O
Kd=2*z*wn*O
```

A.5 PID controller for crank slider mechanism

```
%% PID controller for crank slider mechanism
%definition of basic parameter
Ts=0.001                        %sample time
g=9.81                          %accelration of gravity
m_1=0.05508;                    %mass for crank
m_2=0.1166;                     %mass for connecting rod
I=7.7e-5;                       %inertia for crank
R=5.5e-2;                       %lenrth of crank
O=m_2*R^2+I+m_1*r^2;
num=[1]
dem=[O  0 0 ]
TF=tf(num,dem)
%Controller gains using (sisotool function)
Kp = 0.00815, Ki = 0.00168, Kd = 0.00273%at 0.7 and 4 rad/s
```

A.6 State space controller and extended system for crank slider mechanism

```
%% state space controller and extended system for crank slider mechanism
%definition of basic parameter
Ts=0.001                        %sample time
g=9.81                          %accelration of gravity
m_1=0.05508;                    %mass for crank
m_2=0.1166;                     %mass for connecting rod
I=7.7e-5;                       %inertia for crank
R=5.5e-2;                       %lenrth of crank
O=m_2*R^2+I+m_1*r^2;            %equivilant inertia for crank slider
A=[0 1;0 0];                    %system matrix
B=[0;1/O];                      %input matrix
C=[1 0];                        %output matrix
R=ctrb(A,B) ,q=rank(R)          %check controlability for state space
zeta=0.6 ,wn=40
p1=-zeta*wn-i*wn*sqrt(1-zeta^2)
p2=conj(p1)
p=[p1 p2]
K=place(A,B,p)                  %gains for state space
Ae=[A,zeros(2,1);-C 0]
Be=[B;0]
R1=ctrb(Ae,Be) , q1=rank(R1)
pe=[p1 p2  -60]
Ke=place(Ae,Be,pe)              %gains for extended system
Ks=Ke(1:2) ,Ki=-Ke(3)
```

A.7 MATLAB function code instead of z channel in encoder

```
function y = fcn(u)
k=fix(n/10000)
    y=u-10000*k
else
    y=u


;
```

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