Palestine PolytechnicUniversity

College of Engineering

Civil & Architectural Engineering Department

Surveying and Geomatics Engineering

Graduation Project

" Study of geoid heights in Palestine "

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In accordance with the recommendation of project supervisor and acceptance of all examining committee members, this project has been submitted to the Department of Civil and Architectural Engineering in the college of Engineering and Technology in partial fulfillment of requirements of the department for degree of Bachelor of Surveying and Geomatics Engineering.

Signature of Project Supervisor Signature of Department Chairman

Name…………………………… Name ……………………………..

Hebron – Palestine

2014

بسم ا□ الرحمن الرحيم

يرفع ا□ الذين آمنوا منكم

والذين أوتوا العلم درجات

صدق ا□ العظيم

المجادلة . الآية-١١

الإهداء

إلى من انار بعلمه ظلمات الجهل ... رسول الله صلى الله عليه وسلم

إلى ورثة الأنبياء بعلمهم ...**علماؤنا الأجلاء**

الى الذين رووا تراب فلسطين بدمائهم الطاهرة ... شهداؤنا **الابرار**

إلي كل من أضاء بعلمه عقل غيره

أو هدى بالجواب الصحيح حيرة سائليه

فأظهر بسماحته تواضع العلماء

وبرحابته سماحة العارفين

إلى من تعهداني بالتربية في الصغر ، وكانا لي نبراساً يضيء فكري بالنصح ، و التوجيه في الكبر

أمي ، وأبي

حفظهما الله

إلى من شملوني بالعطف ، وأمدوني بالعون ، وحفزوني للتقدم... إخوتي ، وأخواتي

رعاهم الله.

ثم إلى كل من علمني حرفاً أصبح سنا برقه يضيء الطريق أمامي

إلى من علمني النجاح والصبر

مشرفنا الدكتور غادي زكارنة

الى كل من ساعد وساهم في انجاز هذا العمل

إليكم جميعا نهدي هذا **العمل**

شكر وتقدير

ومن حق النعمة الذكر، وأقل جزاء للمعروف الشكر ...

... فبعد شـــكر المــولى عز وجل ، المتــفضل بجليل النعم ، وعظيم الجزاء

يجدر بي أن أتــقــدم ببالغ الامتــنـان ، وجزيـــل العرفان إلى كل من وجهني ، وعلمني ، وأخذ بيدي في سبيل إنجاز هذا المشروع .. وأخص بذلك مشرفي ، الأستـاذ الدكتـور:غادي زكارنة، الذي قوم ، وتابع ، وصوب ، بحسن إرشاده لي في كل مراحل المشروع ، والذي وجدت في توجيهاته حرص المعلم ، التي تــؤتي ثمارها الطيــبة بإذن الله ...

كما أتقدم بخالص الشكر والتقدير إلى الاستاذ احمد حرباوي ... الذي ساعدنا في العمل الميداني للمشروع

كما أحمل الشكر والعرفان إلى كل من أمـدني بالعلم ، والمعرفـة ، وأسدى ليَّ النصح ، والتوجيه ، وإلى ذلك الصرح العلمي الشامخ متمثلاً في جامعة بوليتكنك فلسطين ، وأخص بالذكر كلية الهندسة ، و دائرة الهندسة المدنية والمعمارية ، والقائمين عليها ...

كما أتوجه بالشكر إلى كل من ساندني بدعواته الصادقة ، أو تمنياته المخلصة ...

أشكرهم جميعاً وأتمنى من الله عز وجل أن يجعل ذلك في موازين حسناتهم

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CHAPTER ONE
INTRODUCTION

1-1 Background

Geiod : is the surface within or around the earth that is everywhere normal to the direction of gravity and coincides with mean sea level in the oceans .

Coordinates system :A system for specifying points using coordinates measured in some specified way. The simplest coordinate system consists of coordinate axes oriented perpendicularly to each other, known as Cartesian coordinates .

GNSS : GNSS (Global Navigation satellite system) is a satellite system that is used to pinpoint the geographic location of a user's receiver anywhere in the world.

1-2 Objective

This project aim to study the accuracy of local and international geoid systems that are available in the internet , in compared with land surveying and GNSS. based on this comparison we can determine the appropriateness of each systems to be applied in Palestine.

when comparing these systems ,we need defined height points based on height system being used in Palestine (orthometric height(H)) , we will measure the height of these points by using GNSS from ellipsoid ,and at the end we will use the Geographic information system (GIS) to achieve and analyze the results in Palestine, we also show the resulting errors throughout using these systems.

1-3 Time schedule

The time schedule shows the stages of developing in our work and the process of project growth that include Project determination, studying, collecting data, designing the entire system. Table 1-1 shows the first semester project growth. All tasks are referred to the theoretical background and the whole system analysis.

Table 0-1 Time Schedule for first semester

Tasks \ Weeks	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	**16**
discuss the project idea	→	→														
Project analysis and plane			→	→	→	→										
Training on using GNSS devices						→	→	→	→							
collecting data									→	→						
discuses the data with the supervisor and modify it											→	→	→	→	→	
Presentation																☆

1-4 Project Scope

This project consists of four chapters as follows:

Chapter one :is an introduction for the desecration of the project and its limitation.

Chapter two : is an Introduction to coordinates system and datum transformation and GNSS position calculation, and their relations.

Chapter three : Discusses the idea of GNSS , and how to calculate the position on the earth ,using different satellite navigation systems .

Chapter four : is an introduction to the gravity field of the earth and Global geoid / gravity models .

CHAPTER TWO
INTRODUCTION TO COORDINATE SYSTEMS

2-1 Introduction

Geodesy is the science of the measurement and mapping of the earth surface as a part science, the problems of geodesy are to determine the of geosciences and engineering external gravity field of the earth of the earth and other celestial bodies figure and the external observations of the surfaces of these as a function of time by internal and bodies[5].

2-2 Figure of the earth (ellipsoid)

The shape of the earth is ellipsoid because the distance from the center of the earth to the equator is larger than the distance from the center to the poles by about 23 km. to make an ellipsoid model of the earth, Rotate the ellipse about the shorter polar axis (semi-minor axis b) to form a solid surface, see figure (2-1) A datum is defined by choosing an ellipsoid and then a primary reference point[1].

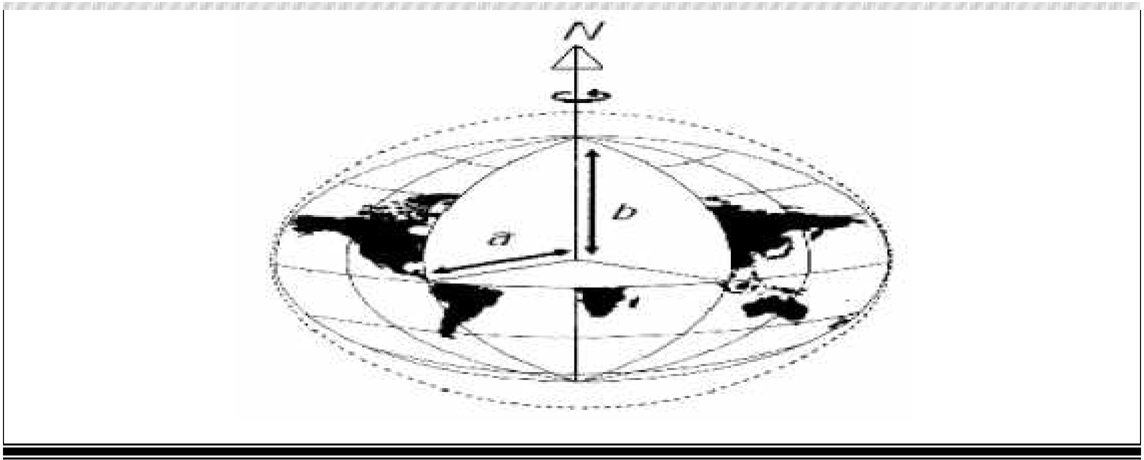


Fig 2-1 : Making ellipsoid [1]

The reference ellipsoid of the Palestine_1923_Grid, Palestine_1923_Israel_CS_Grid, and Palestine_1923_Palestine_Belt is the Clarke_1880_Benoit. The reference ellipsoid of the Israel_TM_Grid is the Geodetic Reference System 0f 1980 (GRS80).

In ellipsoidal coordinates the earth is considered to be an ellipsoid with semi-major axis (the radius of the equatorial circle) and semi-minor axis[1] .

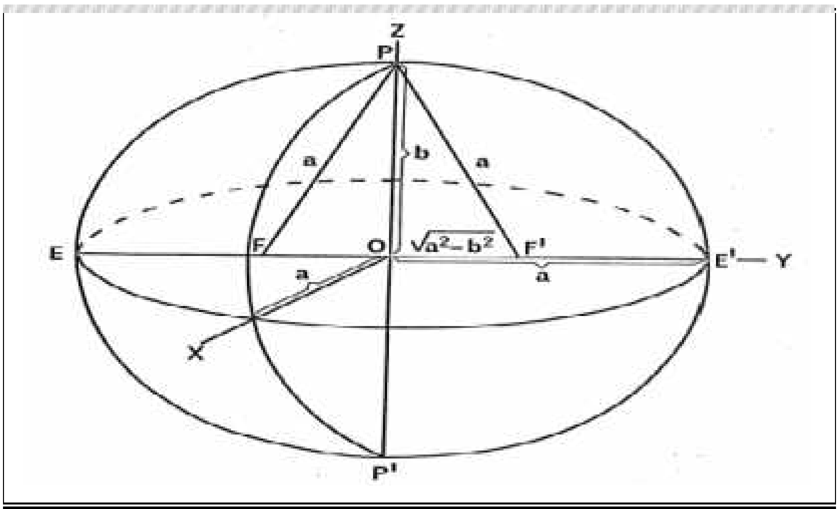


fig 2-2 : ellipsoidal parameters[11]

Other the basic derived parameters can be calculated using the basic axis:

$$f = \frac{a-b}{a} \quad (2\text{-}1)$$

$$e^2 = \frac{a^2-b^2}{a^2} = f(2-f) \quad (2\text{-}2)$$

$$c = \frac{a^2}{b} = \frac{a}{1-f} \quad (2\text{-}3)$$

$$n = \frac{a-b}{a+b} \quad (2\text{-}4)$$

$$W = (1 - e^2 \sin^2 Bi^R)^{1/2} \quad (2\text{-}5)$$

$$V = (1 + e^2 \cos^2 Bi^R)^{1/2} \quad (2\text{-}6)$$

$$N = \frac{a}{W} \quad (2\text{-}7)$$

$$M = \frac{c}{V^3} \quad (2\text{-}8)$$

Where:

f :The flattening of the ellipsoid.

e^2 :The first eccentricity squared.

c :The polar radius of curvature.

n : Second flattening.

W : First auxiliary quantity.

V : Second auxiliary quantity.

M :Radius of curvature in the meridian.

N :Radius of curvature in the prime vertical.

2-3 Coordinate system

In this chapter introduce the coordinate system and the mathematical figure of the earth is applied to the classical definition of the Geoid, defined as equipotential (level) surface of the earth gravitation field. In average this surface coincides with mean sea level (MSL).

A reference surface is chosen so that reductions are applied to this surface. At the beginning this surface was defined as a sphere, later it was defined as a rotational ellipsoid.The modern definition of the ellipsoid. the Geoid height values above the GRS80 ellipsoid[10].

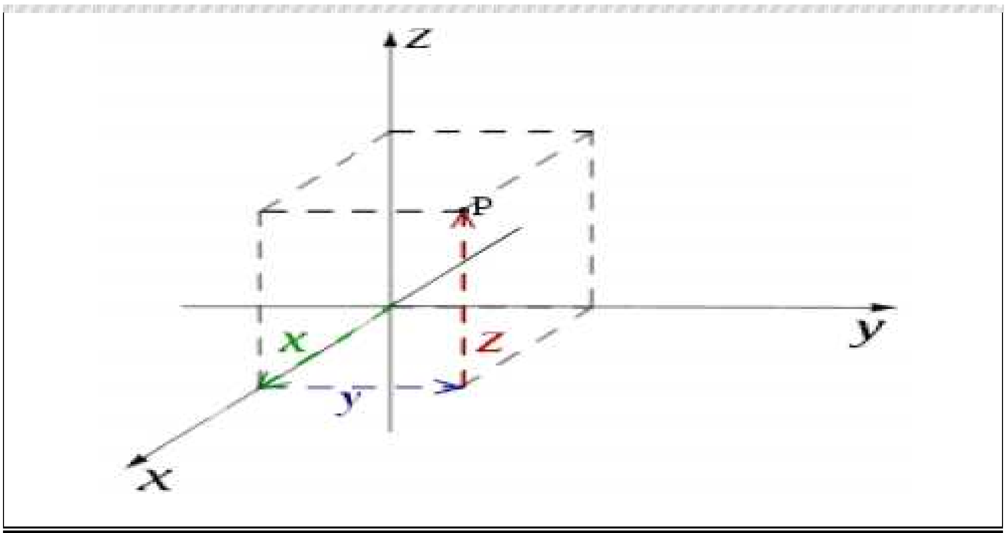


Fig 2-3 : Coordinate systems [1]

2-3-1 Geocentric coordinate system (X,Y,Z)

The Geocentric coordinate system is based on a normal (X,Y,Z) coordinate system with the origin at the center of Earth. This is the system that GPS uses internally for doing it calculations, but since this is very unpractical to work with as a human being (due to the lack of well-known concepts of east, north, up, down) it is rarely displayed to the user but converted to another coordinate system[6]

An alternative method of defining a 3D position on the surface of the Earth is by means of geocentric coordinates (X,Y,Z), also known as 3DCartesiancoordinates. The system has its origin at the mass-centre of the Earth with the X- axis and Y-axis in the plane of the equator. The X-axis passes through the meridian of Greenwich, and the Z-axis coincides with the Earth's axis of rotation. The three axes are mutually orthogonal and form a right-handed system. Geocentric coordinates can be used to define a position on the surface of the Earth (point P in figure(2-4))[6].

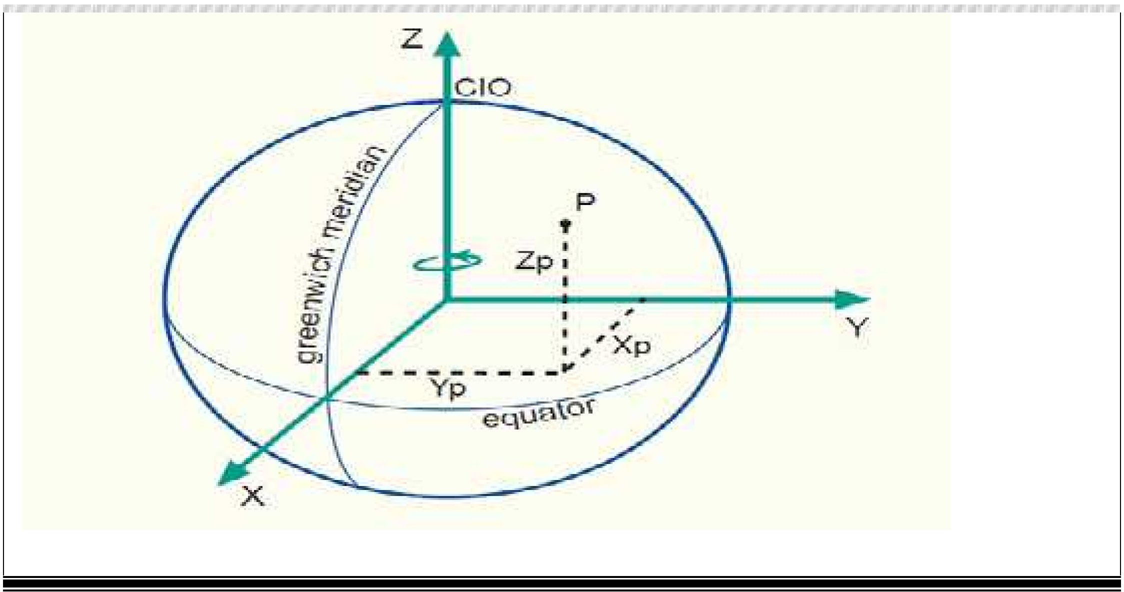


Fig 2-4 : An illustration of the geocentric coordinate system [6]

2-3-2 Geographic coordinate system (λ ,φ, h)

The Geographic coordinate system is probably the most well-known. It is based on angles relative to a prime meridian and Equator usually as Longitude and Latitude. Heights are usually given relative to either the mean sea level or the datum.

The most widely used global coordinate system consists of lines of geographic latitude (φ) and longitude (λ) Lines of equal latitude are called parallels and ellipsoidal height (h). They form circles on the surface of the ellipsoid. Lines of equal longitude are called meridians and they form ellipses on the ellipsoid. Both lines form the graticule when projected onto a map plane, using a specific map projection[6].

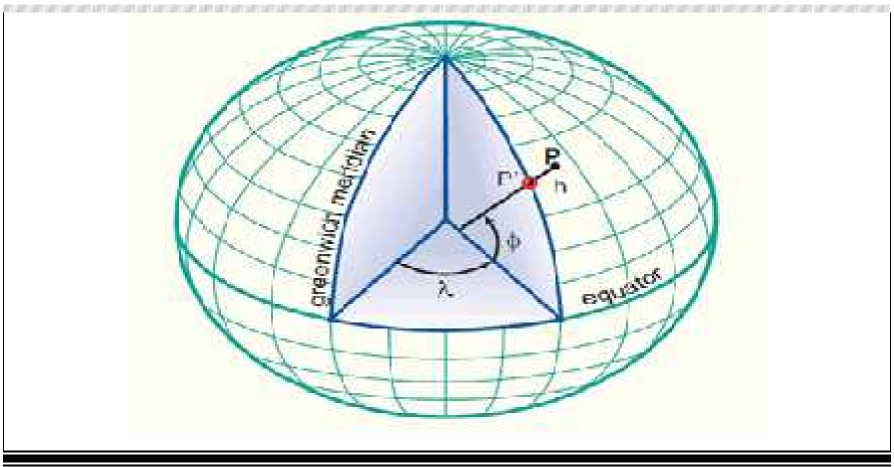


Fig 2-5 : The latitude (φ) and longitude (λ) angles and the ellipsoidal height (h) represent the 3D gegraphic coordinate system [6]

Latitude and longitude and ellipsoidal height represent the geographic coordinates λ ,φ of a point P with respect to the selected reference surface. They are also called geodetic coordinates or ellipsoidal coordinates when an ellipsoid is used to approximate the shape of the Earth. Geographic coordinates are always given in angular units, . The ellipsoidal height (h) of a point is the vertical distance of the point in question above the ellipsoid. It is measured in distance units along the ellipsoidal normal from the point to the ellipsoid surface. geographic coordinates can be used to define a position on the surface of the Earth (point P in figure (2-5)) [6].

The geocentric coordinates (X,Y,Z) can be calculated using the geographic coordinates (λ ,φ, h):

$$X = (N + h) \cos\varphi \cos\lambda \qquad (2\text{-}9)$$

$$Y = (N + h) \cos\varphi \sin\lambda \qquad (2\text{-}10)$$

$$Z = ((1 - e^2) N + h) \sin\varphi \qquad (2\text{-}11)$$

The inverse problem is solved in an iterative solution (Torge Methode):

$$\lambda = \tan^{-1}\frac{Y}{X} \qquad (2\text{-}12)$$

$$h = \frac{\sqrt{X^2+Y^2}}{\cos\varphi} - N \qquad (2\text{-}13)$$

$$\varphi = \tan^{-1} \frac{Z}{\sqrt{X^2+Y^2}} \left(1 - e^2 \frac{N}{N+h}\right)^{-1} \qquad (2\text{-}14)$$

As initial value to start the iterative solution

$$\varphi = \tan^{-1} \frac{Z}{\sqrt{X^2+Y^2}} \quad 1 - e^2 \quad ^{-1} \qquad (2\text{-}15)$$

For the geodetic (or geographic) latitude, there are two other type of latitudes. These are the astronomic latitude (**φ**) and the geocentric latitude (φ'). The astronomic latitude (**φ**) (figure 2-6) is the angle between the equatorial plane and the normal to the Geoid . It differs from the geodetic (or geographic) latitude only slightly, due to the slight deviations of the Geoid from the reference ellipsoid ,These deviation are called the deflection of vertical. The astronomic latitude (**φ**) is the latitude which results directly from observations of the stars, uncorrected for vertical deflection, and applies only to positions on the Earth's surface. Astronomic observations were used to establish local horizontal (or geodetic) datum's in building the older geodetic networks[10].

The geocentric latitude (φ') is the angle between the equatorial plane and a line from the center of the ellipsoid (used to represent the Earth). This value usually differs from the geodetic latitude, unless the Earth is represented as a perfect sphere. Both geocentric and geodetic latitudes (φ) refer to the reference ellipsoid and not the Geoid[10].

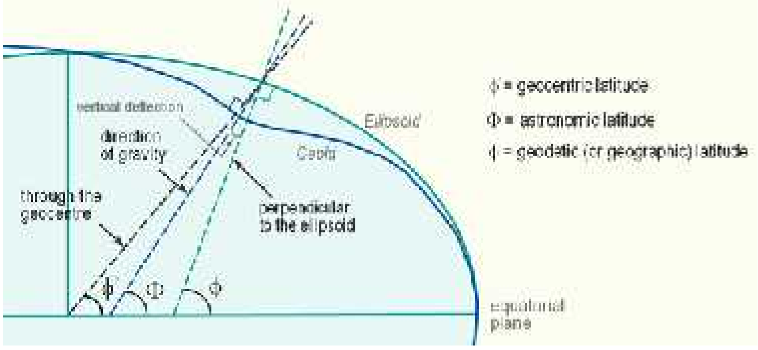


Fig 2-6 Three different latitudes: the geodetic (or geographic) latitude (φ), the astronomic latitude (φ) and the geocentric latitude (φ') [10]

2-3-3 Topocentric coordinate systems (local coordinate system)

Definition: point of origin with know geographic coordinate P_0 (λ,φ, h)or (X,Y, Z). The x-direction is defined to the north by the horizon, The y-direction is to the east, And the z-direction is perpendicular to the xy-plane to above in the zenith direction. The position of the point is defined by the slope (s) distance, Azimuth (zi), and zenith angle or (x, y, z) local coordinates with respect to the point P[13].

Where:

$x = s.\cos Az.\sin zi$

$y = s.\sin Az.\sin zi$

$z = s.\cos zi$ (2-16)

To convert from Topocentric to geocentric coordinate the following equation can be applied in matrix form :

$X = \begin{matrix} X \\ Y \\ Z \end{matrix} \quad , \quad x = \begin{matrix} x \\ y \\ z \end{matrix}$

$\Delta X = X - X_{P0}$ (2-17)

$x = A^{-1}\Delta X = A^{T}\Delta X$ (2-18)

$\Delta X = A_X$

$$\begin{matrix} \Delta X \\ \Delta Y \\ \Delta Z \end{matrix} = \begin{matrix} -\sin\varphi\cos\lambda & -\sin\lambda & \cos\varphi\cos\lambda \\ -\sin\varphi\cos\lambda & \cos\lambda & \cos\varphi\cos\lambda \\ \cos\varphi & 0 & \sin\varphi \end{matrix} \quad (2\text{-}19)$$

$X = X_{P0} + \Delta X$ (2-20)

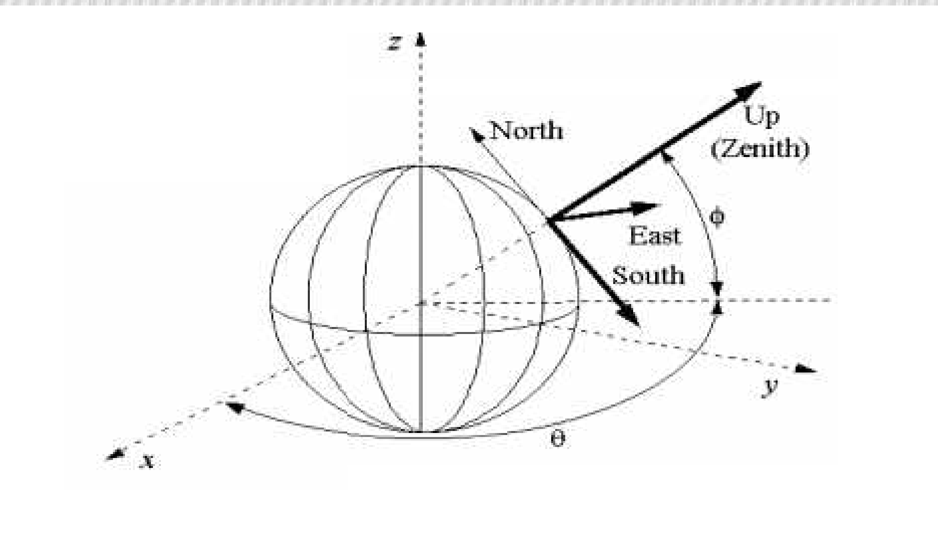


Fig 2-7 : Topocentric coordinate systems (local coordinate systems)[13].

2-4 Datum transformations

The coordinates of all locations on the earth are defined referring to a datum. While a spheroid nearly represents the shape of the earth, a datum defines the position of a spheroid relative to the center of the earth. A point on the surface of the earth is matched to a particular position on the surface of the ellipsoid. This point is known as the origin point of the coordinates system on the datum. The coordinates of the origin point of coordinates system are fixed, and all other points are calculated referring to it. The coordinate system origin of a local datum is not at the center of the earth. The center of the spheroid of a local datum is offset from the earth's center, depending on a global datum like WGS84[3].

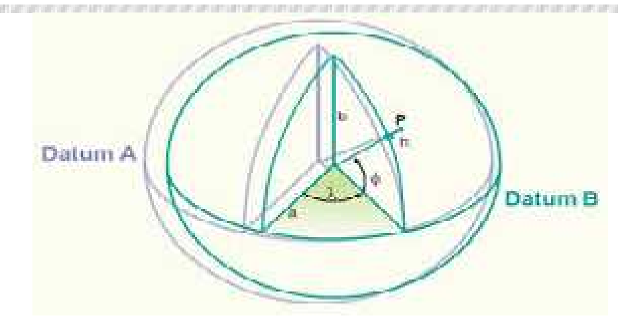


Fig 2-8 : Datum transformations .[3]

A datum provides a frame or reference for measuring locations on the surface of the earth. It defines the origin and orientation of latitude and longitude lines. Whenever change the datum, or more correctly, the geographic coordinate system, the coordinate values of a point will change[3].

There are three common methods of making these transformations from one datum to on another. These methods are 3D similarity, Helmert and Moldensky method the basic parameters for datum transformation[13].

2-4-1 3D Similarity 7-Parameter

This method is more complex and accurate datum transformation**.** The 3D similarity 7-parameter has seven parameter transformations that include three translation parameters, three rotation parameters and a scale parameter[13].

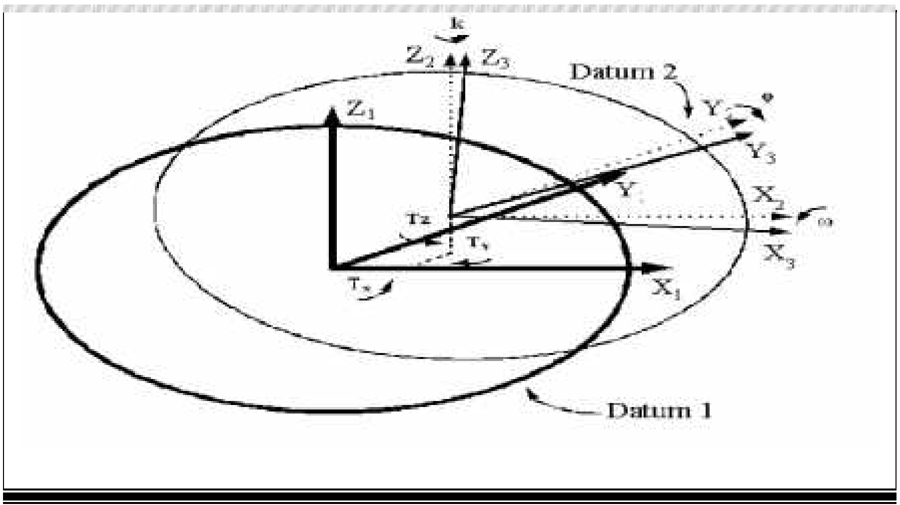


Figure 2-9 : Parameter Transformation[13]

Parameters are: S, ω, φ, k, T_x, T_y ,and T_z the equations for the 3D similarity 7-parameter transformation are:

$$X = S(m11x + m21y + m31z) + T_x \tag{2-21}$$

$$Y = S(m12x + m22y + m32z) + T_y \tag{2-22}$$

$$Z = S(m13x + m23y + m33z) + T_z \tag{2-23}$$

In matrix form:

$$\begin{matrix} X \\ Y \\ Z \end{matrix} = \begin{matrix} m11 & m21 & m31 \\ m12 & m22 & m32 \\ m13 & m23y & m33 \end{matrix} \begin{matrix} x \\ y \\ z \end{matrix} + \begin{matrix} Tx \\ Ty \\ Tz \end{matrix} \quad (2\text{-}24)$$

X = S .M . x +Tx (2-25)

Y = S .M . y +Ty (2-26)

Z = S .M . z +Tz (2-27)

Where;

X, Y, Z: point coordinates in the target system

x, y, z: point coordinates in the source

S: scale

M: rotation matrix

X, y, z: coordinate system in second datum

T: translation matrix

$m11 = cos\varphi \cos k$ (2-28)

$m12 = \sin w\, sin\varphi \cos k + \cos w \ \sin k$ (2-29)

$m13 = -\cos w\, sin\varphi \cos k + \cos w \ \sin k$ (2-30)

$m21 = -cos\varphi \sin k$ (2-31)

$m22 = -s\,in\, w\, sin\varphi \sin k + \cos w \ \sin k$ (2-32)

$m23 = \cos w\, sin\varphi \cos k + \cos w \ \cos k$ (2-33)

$m31 = \sin\varphi$ (2-34)

$m32 = -\sin w\, cos\varphi$ (2-35)

$m33 = \cos w\, cos\varphi$ (2-36)

2-4-2 Helmert transformations

The Helmert transformations parameters are three linear shifts (Tx, Ty, Tz), three angular rotations around each axis ($r_x,\ r_y, r_z$), and scale factor(S), The rotation values are given in decimal seconds[13].

$$\begin{matrix} X \\ Y \\ Z \end{matrix} (new) = \begin{matrix} Tx \\ Ty \\ Tz \end{matrix} .S \begin{matrix} 1 & r_z & -r_y \\ -r_z & 1 & r_x \\ r_y & -r_x & 1 \end{matrix} . \begin{matrix} X \\ Y \\ Z \end{matrix} (original) \quad (2\text{-}37)$$

$X = S\,(X + r_z.Y - Z.\,r_y) + Tx$

$Y = S(-r_z\, X + .Y + Z.\,r_x) + Ty$

$Z = S(r_y.X - r_x.Y + Z) + Tz$ (2-38)

Where;

X, Y, Z: point coordinates in the target system

x, y, z: point coordinates in the source

S : scale factor

r_x, r_y, r_z : angular rotations

Tx, Ty, Tz : linear shifts

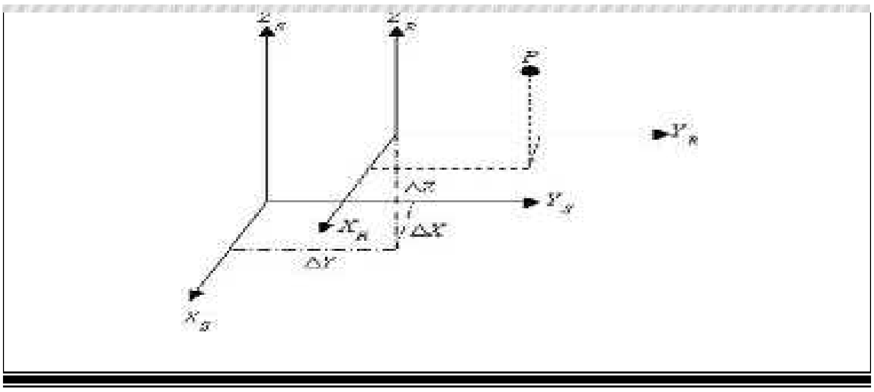


Fig 2-10 : Coordinate transformation between different datums [13].

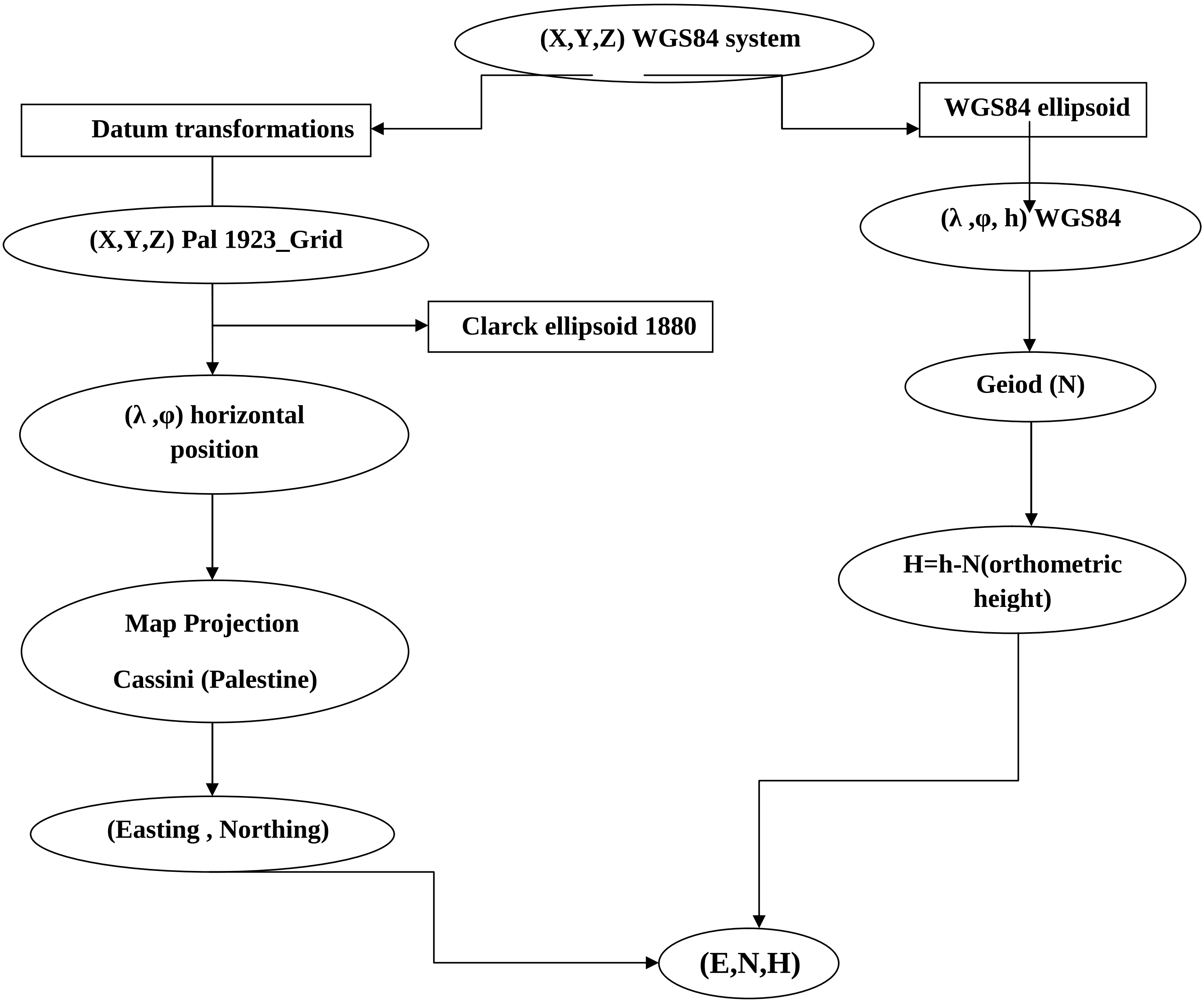
2-5 GNSS Position Calculation

The global navigation satellite system (GNSS) positioning for receiver's position is derived through the calculation steps, or algorithm, given below. In essence, a GNSS receiver measures the transmitting time of GNSS signals emitted from four or more GNSS satellites and these measurements are used to obtain its position (spatial coordinates) and reception time[3].

GPS: WGS 84 (origin + orientation)

In height : GPS (ellipsoid at height reference) , Reference: ellipsoid .

Leveling : geoid (orthometric height reference).


(X,Y,Z) WGS84 system
Datum transformations
WGS84 ellipsoid
(X,Y,Z) Pal 1923_Grid
(λ ,φ, h) WGS84
Clarck ellipsoid 1880
Geiod (N)
(λ ,φ) horizontal position
H=h-N(orthometric height)
Map Projection
Cassini (Palestine)
(Easting , Northing)
(E,N,H)

CHAPTER THREE

GNSS

Introduction

Originally designed for military and intelligence applications at the height of the Cold War in the 1960s, with inspiration coming from the launch of the Soviet spacecraft Sputnik in 1957, the global positioning system (GPS) - is a network of satellites that orbit the earth at fixed points above the planet and beam down signals to anyone on earth with a GPS receiver. These signals carry a time code and geographical data point that allows the user to pinpoint their exact position, speed and time anywhere on the planet.[7]

Transit was the first satellite system launched by the USA and tested by the US Navy in 1960. Just five satellites orbiting the earth allowed ships to fix their position on the seas once every hour. In 1967 Transit was succeeded by the Timation satellite, which demonstrated that highly accurate atomic clocks could be operated in space. GPS developed quickly for military purposes thereafter with a total of 11 "Block" satellites being launched between 1978 and 1985.

However, it wasn't until the USSR shot down a Korean passenger jet- flight 007 - in 1983 that the Reagan Administration in the US had the incentive to open up GPS for civilian applications so that aircraft, shipping, and transport the world over could fix their positions and avoid straying into restricted foreign territory. [7]

3-2 Idea of GNSS

A GNSS provides autonomous geo-spatial positioning with global coverage. The most well-known and most utilized GNSS system is the United States NAVSTAR Global Positioning system (GPS).

which consists of a constellation of 24 or more medium Earth orbit satellites in six different orbital planes and a worldwide ground control/monitoring network of stations .

This network monitors the health and status of the satellites and also uploads navigation and other data to the satellites. Each satellite broadcasts ranging codes and navigation data on two frequencies (L1 @ 1575.42MHz and L2 @ 1227.6 MHz) using a technique called Code Division Multiple Access (CDMA), i.e., each satellite uses different ranging codes that have low cross-correlation properties with respect to one another.

The navigation data enables the receiver to determine the location of the satellite at the time of signal transmission. The satellite uses an atomic clock to maintain synchronization with all the satellites in the constellation.

The receiver compares the time of broadcast encoded in the transmission with the time of reception measured by an internal clock and estimated by correlating the spread spectrum code of the satellite with a known replica. Since the orbital positions of the satellites (ephemeris data) as well as the time of transmission of the signals can be extracted from the navigation message contained in the signals transmitted from the satellites to the receiver, the ranges between the satellites and the receiver can be calculated.

If three satellites are used, then the three ranges will define three spheres centered at the satellite positions and intersecting at the receiver position. In reality, a fourth satellite is required because the time offset in the GPS receiver clock, with respect to GPS system time, is also unknown (to minimize the size, cost and complexity of the receiver, a crystal clock is usually employed in navigation receivers). Thus, instead of true ranges, pseudo ranges.

Fig 3-1: idea of GNSS [2]

3-3 GNSS Augmentation Systems

In order to enhance standalone GNSS performance, there exists a number of augmentation systems and a user equipment can be configured to make use of inertial sensors for added robustness in the presence of jamming or to aid vehicle navigation when the satellite signals are blocked in “urban canyons”.

GNSS Augmentation involves using external information, often integrated into the calculation process, to improve the accuracy, availability, or reliability of the satellite navigation signal. There are many such systems in place and they are generally named or described based on how the GNSS sensor receives the information. Some systems transmit additional information about sources of error (such as clock drift, ephemeris, or ionospheric delay), others provide direct measurements of how much the signal was off in the past, while a third group provides additional navigational or vehicle information to be integrated in the calculation process. Examples of augmentation systems include the US’s Wide Area Augmentation System (WAAS), the European Geostationary Navigation Overlay Service (EGNOS), Japan’s MTSAT Satellite-based Augmentation System (MSAS); India’s Regional Navigation Satellite System (IRNSS), Russia’s Wide-area SDCM (System of Differential Corrections and Monitoring); as well as Differential GPS, and Inertial Navigation Systems. [2]

٣-4 GPS segments

The Global Positioning System (GPS) provides users with positioning, navigation, and timing (PNT) services. This system consists of three segments: the space segment, the control segment, and the user segment.[9]

٣-4-١the space segment

The GPS space segment consists of a constellation of satellites transmitting radio signals to users. The Air Force manages the constellation to ensure the availability of at least 24 GPS satellites, 95% of the time. For the past several years, the Air Force has been flying 31 operational GPS satellites, plus 3-4 decommissioned satellites ("residuals") that can be reactivated if needed.[9]

The GPS system has satellites in different generation calls bocks the first one is called Block I , block I experimental in nature with a focus on military applications and this block consists of 11 prototype satellites, and were launched between 1978 to 1985. And the second block is Block IIA, Block IIA is an upgraded version of the GPS Block II satellites launched in 1989-1990. The "II" refers to the second generation of GPS satellites, although Block II was actually the first series of operational GPS satellites. The "A" stands for advanced.[9]

Another one called Block IIR, The IIR series were produced to replace the II/IIA series as the II/IIA satellites gradually degraded or exceeded their intended design life. The "R" in Block IIR stands for replenishment.

Developed by Lockheed Martin, the production consisted of a total of 13 satellites: SVN-41 through SVN-47, SVN-51, SVN-54, SVN-56, and SVN-59 though SVN-61. The first successful launch occurred in July 1997, and the last in November 2004.[d] Satellites have the capability to autonomously navigate (AUTONAV) themselves and generate their own 50Hz navigation message data[13].

Then we have Block IIR (M) The IIR(M) series of satellites are an upgraded version of the IIR series, completing the backbone of today's GPS constellation. The "M" in IIR(M) stands for modernized, referring to the new civil and military GPS signals added with this generation of spacecraft. Developed by Lockheed Martin, there are eight IIR(M) satellites: SVN-48, SVN-49, SVN-50, SVN-52, SVN-53, SVN-55, SVN-57, and SVN-58.[9]

The last block called Block IIF the IIF series expand on the capabilities of the IIR(M) series with the addition of a third civil signal in a frequency protected for safety-of-life transportation. The "F" in IIF stands for *follow-on*. Compared to previous generations, GPS IIF satellites have a longer life expectancy and a higher accuracy requirement. Each spacecraft uses a mix of rubidium and cesium atomic clocks to keep time within 8 billionths of a second per day. The IIF series will improve the accuracy, signal strength, and quality of GPS. Developed by Boeing, the IIF series includes a total of 12 satellites: SVN-62 through SVN-73. [9]

3-4-2Control Segment

The control segments is a group of ground stations that monitor and operate the GPS satellites. There are monitoring stations spaced around the globe and one master control station located in Colorado springs, Colorado each station sends information to the conrol station which then updates and corrects the navigational message of the satellites. There are actually five major monitoring systems. [8]

٣-٤-٣ User Segment

consist The radio signal takes to travel from a GPS satellite until it arrives at the GPS of GPS receiver units with capability to obtain real time positioning GPS receivers hand-held radio-receivers/computers which measure the time that antenna. Using the travel time multiplied by the speed of light provides a calculation of range to each satellite in view. From this and additional information on the satellites orbit and velocity, the internal GPS receiver software calculates its position through a process of[13]

3-٥ GPS Positioning Modes

Positioning with GPS can be performed by either of two ways: point positioning or relative positioning. GPS point positioning employs one GPS receiver that measures the code pseudoranges to determine the users position instantaneously, as long as four or more satellites are visible at the receiver.

The expected horizontal positioning accuracy from the civilian C/A-code receivers has gone down from about 100m when selective availability was on, to about 22m in the absence of selective availability. GPS point positioning is used mainly when a relatively low accuracy is required. This includes recreation applications and low-accuracy navigation ,GPS relative positioning, however, employs two GPS receivers simultaneously tracking the same satellites.[1]

If both receivers track at least four common satellites, a positioning accuracy level of the order of a sub centimeter to a few meters can be obtained. Carrier-phase or/and pseudoranges measurements can be used in GPS relative positioning, depending on the accuracy requirements. The former provides the highest possible accuracy. GPS relative positioning can be made in either real-time or post mission modes. GPS relative positioning is used for high-accuracy applications such as surveying and mapping, GIS, and precise navigation.[1]

3-٥-1 single point positioning

GPS point positioning, also known as standalone or autonomous positioning, involves only one GPS receiver. That is, one GPS receiver simultaneously tracks four or more GPS satellites to determine its own coordinates with respect to the center of the Earth (Figure 3-2). Almost all of the GPS receivers currently available on the market are capable of displaying their point positioning coordinates.

To determine the receivers point position at any time, the satellite coordinates as well as a minimum of four ranges to four satellites are required. The receiver gets the satellite coordinates through the navigation message, while the ranges are obtained from either the C/A-code or the P(Y)-code, depending on the receiver type (civilian or military). As mentioned before, the measured pseudoranges are contaminated by both the satellite and receiver clock synchronization errors.[1]

Correcting the satellite clock errors may be done by applying the satellite clock correction in the navigation message; the receiver clock error is treated as an additional unknown parameter in the estimation process. This brings the total number of unknown parameters to four: three for the receiver coordinates and one for the receiver clock error. This is the reason why at least four satellites are needed.

It should be pointed out that if more than four satellites are tracked, the so-called least-squares estimation or Kalman filtering technique is applied. As the satellite coordinates are given in the WGS84 system, the obtained receiver coordinates will be in the WGS84 system as well. However, most GPS receivers provide the transformation parameters between WGS84 and many local datums used around the world.[1]

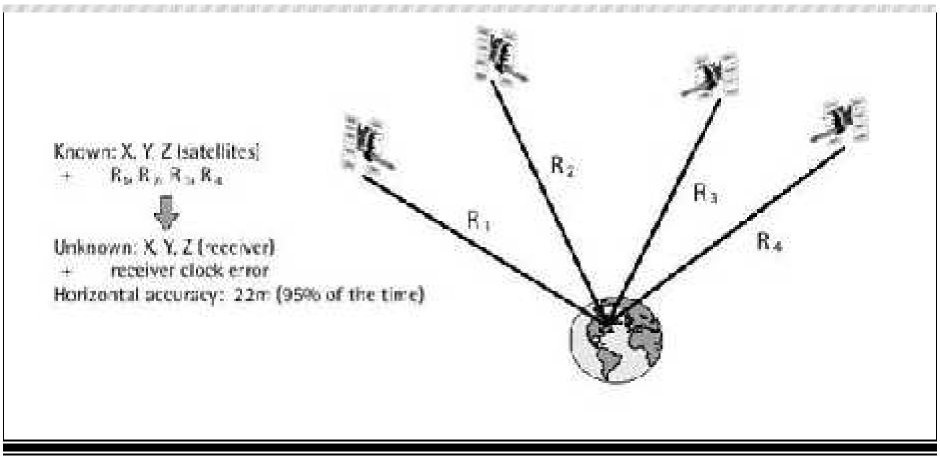


Fig3-2 (principle of GPS point position) [9]

Assume that the distance measured is accurate and under this condition at least four satellites are sufficient. In Figure 3-2 , there are four known satellites at locations (x_1, y_1, z_1) , or (x_2, y_2,z_2) and or (x_3, y_3 ,z_3) , and an unknown point at R_u or (X_u ,Y_u ,Z_u). If the distances between the three known points to the unknown point can be measured as p1 ,p2 and p3 these distances can be written as:

R=CΔt

$$R1=\sqrt{(x1-xu)^2+(y1-yu)^2+(z1-zu)^2}+C\,dt$$

$$R2=\sqrt{(x2-xu)^2+(y2-yu)^2+(z2-zu)^2}+C\,dt$$

$$R3=\sqrt{(x3-xu)^2+(y3-yu)^2+(z3-zu)^2}+C\,dt$$

$$R4=\sqrt{(x4-xu)^2+(y4-yu)^2+(z4-zu)^2}+C\,dt \qquad (3\text{-}1)$$

Where: Δt : receiver clock error

C: light velocity

X_u ,Y_u , Z_u :receiver position in GNSS system

X_i ,Y_i , Z_i : satellites position

Because there are three unknowns and three equations, the values of X_u,Y_u,Z_u can be determined from these equations. Theoretically, there should be two sets of solutions as they are second-order equations. Since these equations are nonlinear, they are difficult to solve directly. However, they can be solved relatively easily with linearization and iterative approach . In GPS operation, the positions of the satellites are given. This information can be obtained from the data transmitted from the satellites.[1]

The distances from the user (the unknown position) to the satellites must be measured simultaneously at a certain time instance. Each satellite transmits a signal with a time reference associated with it. By measuring the time of the signal traveling from the satellite to the user the distance between the user and the satellite can be found. The distance measurement is discussed in the next section.

3-٥-2GPS relative positioning

GPS relative positioning, also called differential positioning, employs two GPS receivers simultaneously tracking the same satellites to determine their relative coordinates , Of the two receivers, one is selected as a reference, or base, which remains stationary at a site with precisely known coordinates. The other receiver, known as the rover or remote receiver, has its coordinates unknown. The rover receiver may or may not be stationary, depending on the type of the GPS operation.

A minimum of four common satellites is required for relative positioning. However, tracking more than four common satellites simultaneously would improve the precision of the GPS position solution. Carrier phase and/or pseudorange measurements can be used in relative positioning. A variety of positioning techniques are used to provide a post-processing (post-mission) or real-time solution.[1]

GPS relative positioning provides a higher accuracy than that of autonomous positioning. Depending on whether the carrier-phase or the pseudorange measurements are used in relative positioning, an accuracy level of a sub centimeter to a few meters can be obtained. This is mainly because the measurements of two (or more) receivers simultaneously tracking a particular satellite contain more or less the

same errors and biases. The shorter the distance between the two receivers, the more similar the errors.[1]

Therefore, if we take the difference between the measurements of the two receivers (hence the name “differential positioning”), the similar errors will be removed or reduced.[1]

3-٦ Biases and errors

Generally, the biases that influence the GPS measurements fall into three categories: satellite biases, station biases, and observation dependent biases.

3-6-1 Satellite biases consist of biases in the satellite ephemeris (e.g., the satellite is not where the GPS broadcast data message or other orbital information tell us it is), and biases in models for the satellite clocks supplied in the broadcast message (e.g., the satellite clocks, even with the broadcast message models, are not perfectly synchronized to GPS time). These biases are thought to be uncorrelated between satellites. They affect both code and carrier beat phase measurements equally, and they depend on the number and the location of the tracking stations providing data for orbital determination, the orbital force model used, and the satellite geometry.

3-6-2 Station biases usually consist of receiver clock biases and, for non-positioning types of GPS applications, such as time transfer and orbital tracking, of biases induced by uncertainties in the coordinates of the stations.

3-6-3 Observation dependent biases include those associated with the signal.[4]
propagation and other biases dependent on the observation type, such as, for instance, ambiguity biases inherent in the carrier beat phase observables. The effect of biases is removed, or at least suppressed, by an attempt to model them. They are assumed to have functional relations with a variety of arguments such as time, position, temperature, etc.

Beside biases, the accuracy of positions and/or time obtained by GPS is dependent on two general influences: the geometric strength of the satellite configuration being observed, and the errors affecting the measurements themselves plus the remnant from the biases after the main effects have been modeled out. The former have already been discussed in Chapter 5, where it was shown that the measurement errors propagate into the position proportionally to the various Dilution of Precision factors. Errors from each of the sources will have complicated spectral characteristics and other properties, and there will be correlations between some of these errors. However, at this stage in GPS development, the error models are usually limited to the simple approach of predicting typical standard deviations of uncorrelated equivalent range errors from each error source.[4]

3-٧ GNSS positioning methods

3-٧-1 Static surveying

This was the first method to be developed for GPS surveying. It can be used for measuring long baselines (usually 20km (16 miles) and over).

One receiver is placed on a point whose coordinates are known accurately in WGS84. This is known as the Reference Receiver. The other receiver is placed on the other end of the baseline and is known as the Rover.

Data is then recorded at both stations simultaneously. It is important that data is being recorded at the same rate at each station. The data collection rate may be typically set to 15, 30 or 60 seconds.[1]

The receivers have to collect data for a certain length of time. This time is influenced by the length of the line and the number of satellites observed and the satellite geometry, As a rule of thumb, the observation time is a minimum of 1 hour for a 20km line with 5 satellites and a prevailing GDOP of 8. Longer lines require longer observation times. Once enough data has been collected, the receivers can be switched off. The Rover can then be moved to the next baseline and measurement can once again commence . It is very important to introduce redundancy into the network that is being measured. This involves measuring points at least twice and creates safety checks against problems that would otherwise go undetected.

A great increase in productivity can be realized with the addition of an extra Rover receiver. Good coordination is required between the survey crews in order to maximize the potential of having three receivers.[1]

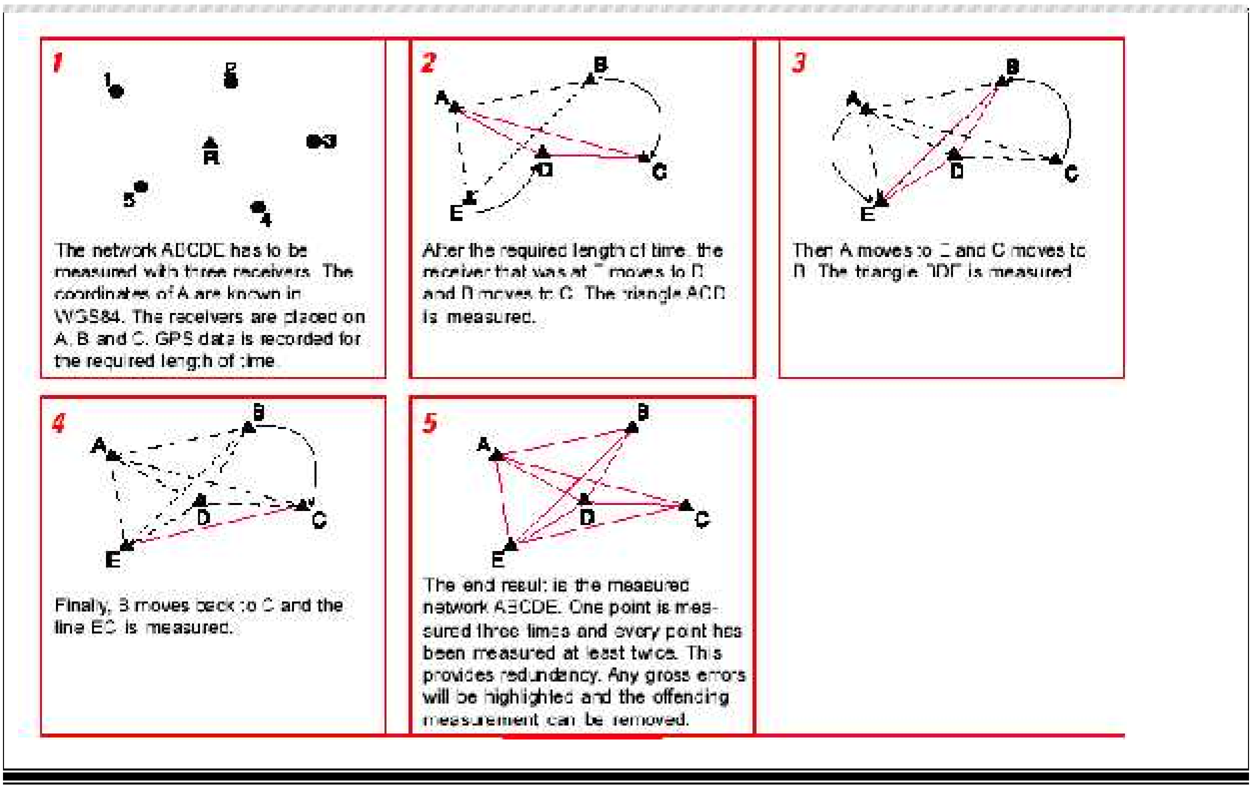


Fig 3-3 : method of static surveying[11] .

Static observations are required for all baselines greater than 20 kilometers in length. Static observations may be required for lines less than 20 kilometers depending on particular project requirements.

A minimum of five satellites shall be observed simultaneously for a minimum of 30 minutes, plus one minute per kilometer of base line length per session. Remember, sessions that are a bit longer than this minimum will provide worthwhile redundancy that could make data processing more robust and improve project results and Data sampling shall have an epoch time interval of 15 seconds or less , Typical achieved accuracy: sub-centimeter level (5 mm + 1 ppm).[1]

3-٧-2 Fast (Rapid) Static

In Rapid Static surveys, a Reference Point is chosen and one or more Rovers operate with respect to it. Typically, Rapid Static is used for densifying existing networks, establishing control etc. When starting work in an area where no GPS surveying has previously taken place, the first task is to observe a number of points, whose coordinates are accurately known in the local system. This will enable a transformation to be calculated and all hence, points measured with GPS in that area can be easily converted into the local system.[1]

The Reference Receiver is usually set up at a known point and can be included in the calculations of the transformation parameters. If no known point is available, it can be set up anywhere within the network. The Rover receiver(s) then visit each of the known points. The length of time that the Rovers must observe for at each point is related to the baseline length from the Reference and the GDOP.The data is recorded and post-processed back at the office. Checks should then be carried out to ensure that no gross errors exist in the measurements. This can done by measuring the points again at a different time of the day.

When working with two or more Rover receivers, an alternative is to ensure that all rovers operate at each occupied point simultaneously. Thus allows data from each station to be used as either Reference or Rover during post processing and is the most efficient way to work, but also the most difficult to synchronise, Another way to build in redundancy is to set up two reference stations, and use one rover to occupy the points as shown in the lower example on the next page.[1]

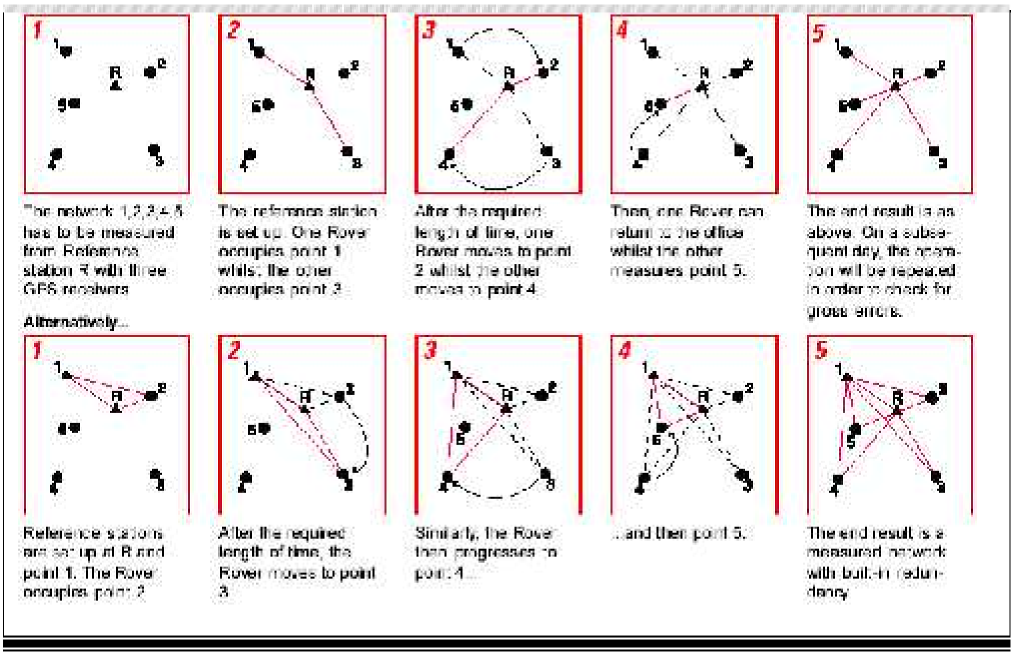


Fig 3-4 : method of fast static surveying[1]

Rapid static procedures may be used on baselines up to 20 kilometers in length, A minimum of three receivers shall be used simultaneously during all rapid static GPS sessions and A minimum of 5 satellites shall be observed simultaneously for a minimum of 5 minutes, plus one minute per kilometer of base line length per session. Typical observation times range from 5-20 minutes , Data sampling shall have an epoch time interval of 5 seconds or less .[1]

3-7-3 Stop-and-Go GPS Surveying (Kinematic GPS Surveying)

The Kinematic technique is typically used for detail surveying, recording trajectories etc., although with the advent of RTK its popularity is diminishing. The technique involves a moving Rover whose position can be calculated relative to the Reference.

Initialization

Firstly, the Rover has to perform what is known as an initialization. This is essentially the same as measuring a Rapid Static point and enables the post processing software to resolve the ambiguity when back in the office. The Reference and Rover are switched on and remain absolutely stationary for 5-20 minutes, collecting data. (The actual time depends on the baseline length from the Reference and the number of satellites observed).[1]

After this period, the Rover may then move freely. The user can record positions at a predefined recording rate, can record distinct positions, or record a combination of the two. This part of the measurement is commonly called the kinematic chain.

A major point to watch during kinematic surveys is to avoid moving too close to objects that could block the satellite signal from the Rover receiver. If at any time, less than four satellites are tracked by the Rover receiver, you must stop, move into a position where 4 or more satellites are tracked and perform an initialization again before continuing.[1]

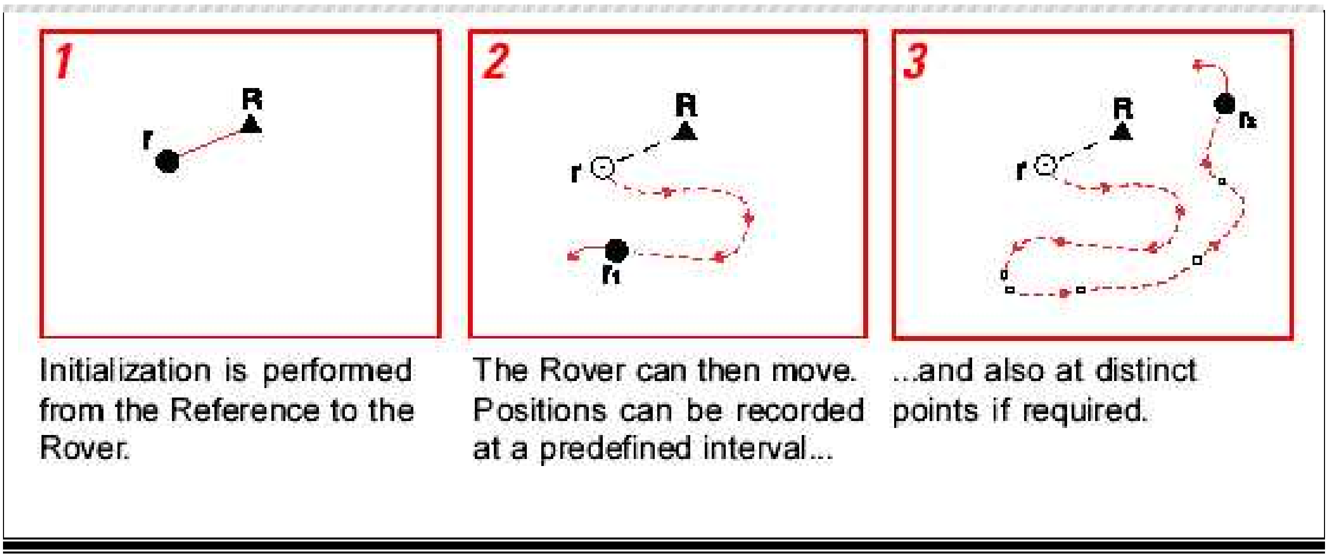


Fig 3-5 : method of kinematic surveying[1]

3-7-4 Kinematic on the Fly

This is a variation of the Kinematic technique and overcomes the requirement of initializing and subsequent reinitialization when the number of observed satellites drops below four.

Kinematic on the Fly is a processing method that is applied to the measurement during post-processing. At the start of measurement, the operator can simply begin walking with the Rover receiver and record data. If they walk under a tree and lose the satellites, upon emerging back into satellite coverage, the system will automatically reinitialize.[1]

A minimum of two receivers shall be used simultaneously during all stop and go GPS sessions. Two receivers shall occupy reference stations and one receiver will be the rover. This procedure shall be limited to baselines of 5 kilometers or less.A minimum of 5 satellites shall be observed simultaneously for a minimum of 5 epochs, Initialization of the roving receiver can be accomplished by occupying a known point for a minimum of 5 epochs or making a rapid static observation of at least 5 minutes on the first point and then moving to other points to be surveyed , and Data sampling shall have an epoch time interval of 5 seconds or less. A minimum of 5 epochs must be recorded for each point located, its Typical achieved accuracy: 1 to 2 cm + 1 ppm.[1]

3-7-5 RTK GPS

RTK stands for Real Time Kinematic. It is a Kinematic on the Fly survey carried out in real time. The Reference Station has a radio link attached and rebroadcasts the data it receives from the satellites. The Rover also has a radio link and receives the signal broadcast from the Reference. The Rover also receives satellite data directly from the satellites via its own GPS Antenna. These two sets of data can be processed together at the Rover to resolve the ambiguity and therefore obtain a very accurate position relative to the Reference receiver.

Once the Reference Receiver has been set up and is broadcasting data through the radio link, the Rover Receiver can be activated. When it is tracking satellites and receiving data from the Reference, it can begin the initialization process. This is similar to the initialization performed in a post-processed kinematic on the fly survey, the main difference being that it is carried out in real-time, Once the initialization is complete, the ambiguities are resolved and the Rover can record point and coordinate data. At this time, baseline accuracies will be in the 1 – 5cm range .

It is important to maintain contact with the Reference Receiver; otherwise the Rover may lose the ambiguity. This results in a far less accurate position being calculated , Additionally, problems may be encountered when surveying close to obstructions such as tall buildings, trees etc. as the satellite signal may be blocked. .[11]

RTK is quickly becoming the most common method of carrying out high precision, high accuracy GPS surveys in small areas and can be used for similar applications as a conventional total station. This includes detail surveying, stakeout, COGO applications etc.

The project area shall contain and be enclosed with RTK control base stations , A minimum of two receivers shall be used simultaneously during all RTK GPS sessions. One base receiver shall occupy a reference point and one or more receivers shall be used as rovers , and the Initialization of the roving receiver(s) shall be made on a known point to validate the initial vector solution. A check shot shall be observed by the rover unit(s) before the base station is taken down.

Each RTK point shall have 2 different independent occupations based on a time offset , The second occupation is recommended to be made from a different base station , To ensure good local accuracies between new RTK points and nearby existing stations, all previously established base stations, control points, and stations pair are to be RTK positioned, when feasible, for consistency, Typical achieved accuracy (averages): 1 to 2 cm + 1 ppm (horizontal) and 1-2.5 times greater (vertical).

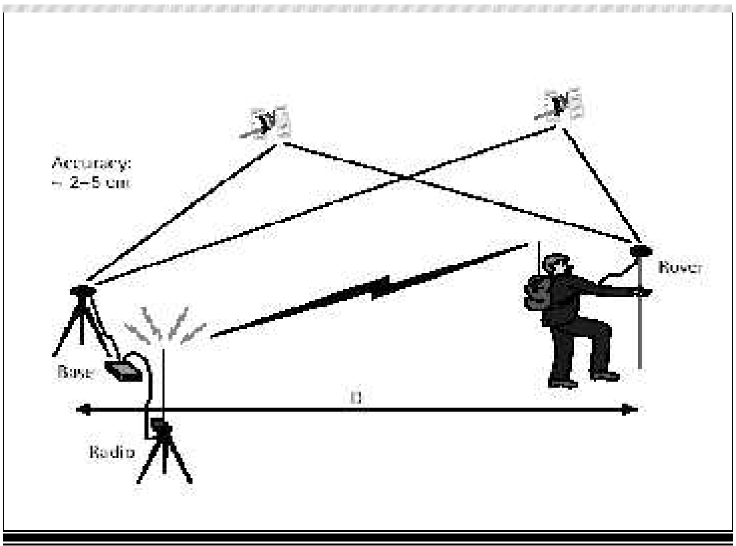


Fig3-6 (RTK GPS surveying)[1]

Table (3.1) : Comparison between GNSS modes

Concept	Requirements	Applications	Accuracy
Static (Post-processing)	• L1 or L1/L2 GPS receiver • Computer for post-processing • 45 min to 1 hr minimum observation time[1]	• Control surveys (that require high accuracy)	• Subcentimeter level
Rapid Static (Post-processing)	• L1/L2 GPS receiver • 5-20 min observation time[1]	• Control surveys (that require medium to high accuracy)	• Subcentimeter level
Stop & Go Kinematic[2] (Post-processing)	• L1 GPS receiver • Computer for post-processing	• Medium accuracy control surveys	• Centimeter level
Real Time Kinematic/OTF Kinematic[3] (Real-time or post-processing)	For post-processing: • L1/L2 GPS receiver • Computer For real-time: • L1/L2 GPS receiver • Internal or external processor (computers) • Radio/modem data link set	• Real-time high accuracy surveys • Location surveys • Medium accuracy control surveys • Photo control • Continuous topo	• Sub decimeter level

CHAPTER FOUR

THE GRAVITY FIELD OF THE EARTH

4-1 Introduction

The significance of the external gravity field of the earth in geodesy may be described comprehensively as follows:

The external gravity field is the reference system for the overwhelming part of the measured quantities in geodesy. This field must be known in order to reduce the quantities into geometrically defined systems . If the distribution of gravity values on the surface of the earth is known, then in combination with other geodetic measurements, the shape of this surface may be determined .

The most important reference surface for height measurements, the geoid ,as an idealized ocean surface is a level surface of the gravity field. The analysis of the external gravity field yields information on the structure and characteristics of the interior of the earth. In making the corresponding gravity field parameter availed [11].

4-2 Gravitational Potential of the earth

According to Newton's law of gravitation (1687), two point masses m_1 and m_2 attract each other with the gravitational force (attractive force)[13]:

$$F = G\frac{m_1 m_2}{l^2} \qquad (4\text{-}1)$$

Here, G is Newton's gravitational constant with the value of (6.67259) × 10^{-11} m3 kg^{-1} s^{-2}. The attraction force F is symmetric. To study how a mass m attracts other masses, the attracted masses assumed to be a unit mass (m=1) .The force attracting the unit mass at point P(X,Y,Z) by the mass m at P0 (X0,Y0,Z0)separated by a distance l is[13]:

$$F = G\frac{m}{l^2} \qquad (4\text{-}2)$$

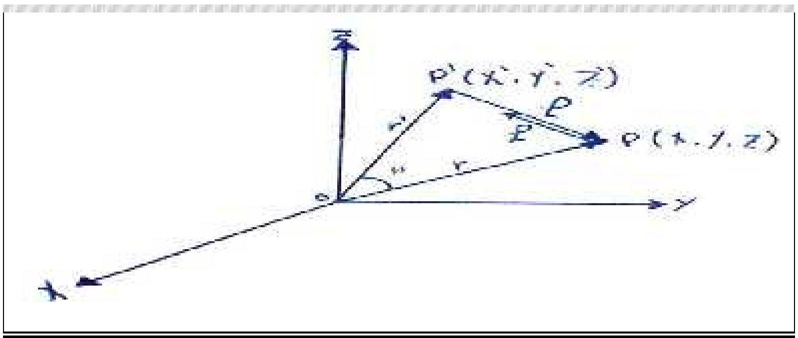


Fig4-1 : Gravitation) [13]

The force F is represented by a vector from P_0 to P. The vector of the gravitational force Fcan be defined by its magnitude F and 3D components of the unit vector is given by :

$$\bar{F} = \begin{matrix} F_X \\ F_Y \\ F_Z \end{matrix} = -F \begin{matrix} (X-X_0)/l \\ (Y-Y_0)/l \\ (Z-Z_0)/l \end{matrix} = -\frac{GM}{l^2} \begin{matrix} (X-X_0)/l \\ (Y-Y_0)/l \\ (Z-Z_0)/l \end{matrix} \qquad (4\text{-}3)$$

The gravitational potential is a conservative, which satisfies the Laplace differential equation outside the Earth .A scalar force generating potential exists. This function is called the gravitational potential V (X, Y, Z) , where V reads:

$$V \; X,Y,Z \; = \frac{\text{GM}}{\text{l}} \qquad (4\text{-}4)$$

The unit mass related force vector F in equation (٤-3) can be rewritten in terms of V as follows:

$$\bar{F} = grad(V) \qquad (4\text{-}5a)$$

$$\bar{F} = \begin{matrix} F_X \\ F_Y \\ F_Z \end{matrix} = \begin{matrix} \frac{\partial V}{\partial X} \\ \frac{\partial V}{\partial Y} \\ \frac{\partial V}{\partial Z} \end{matrix} \qquad (4\text{-}5b)$$

Assuming a system of point masses $m_1, m2 \ldots mn$ are attracting the point P, and separated from the point P by distances l1 , l2 ,…,ln, then the gravitational potential V is the summation of all single potentials. The total gravitational potential is[13]:

$$V\ x, y, z\ = \sum_{i=1}^{n} V_i = \sum_{i=1}^{n} \frac{Gm_i}{l_i} \quad (4\text{-}6)$$

If the point P is influenced by a solid body with a volume v and a density of ρ(X,Y,Z), then the potential V is calculated by a superimposing infinite number of point masses dm. The point mass can be calculated by the volume of point mass dv and the density ρ, reading :

$$dm = \rho dv \quad (4\text{-}7)$$

The total gravitational potential by the solid body is calculated by the integration over the whole volume of the solid body V is given by:

$$V = \quad dv = G \quad \frac{\rho\ X,Y,Z\ dv}{l} \quad (4\text{-}8)$$

4-3 Laplace's equation of Spherical Harmonics

the Laplace equation = 0 in Cartesian and in spherical coordinates. The former solution will be useful for planar, regional applications. The latter solution is a global solution. Both of them will lead to series developments in terms of orthogonal base functions: Fourier1 series and spherical harmonic series respectively[13].

The solution of the Laplace equation is the most important step in solving the boundary value problem. The second step will be to express the boundary function in the same series development and determine the series' coefficients.

For a function V(X, Y, Z), the Laplace equation for this function is the Laplace operator Δ(.)=0:

$$\triangle\ V\ = \frac{d^2V}{dX^2} + \frac{d^2V}{dY^2} + \frac{d^2V}{dZX^2} \quad (4\text{-}9)$$

Using spherical coordinates (r , ϕ , λ) as defined in fig (4-2), Laplace's equation can be transformed to:

$$r^2 \frac{d^2V}{dr^2} + 2r \frac{d^2V}{dr} + \frac{d^2V}{d\phi^2} - \tan\phi \frac{dV}{d\phi} + \frac{d^2V}{\cos^2\phi\, d\lambda^2} = 0 \quad (4\text{-}10)$$

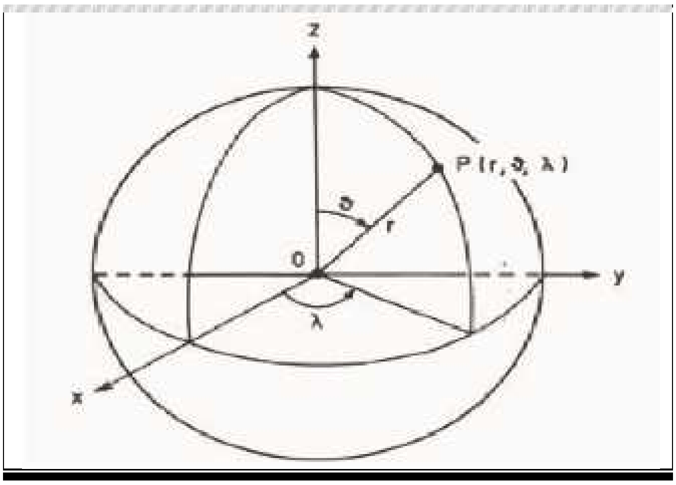


Fig 4-2 : Geographic coordinates (r , ϕ , λ) and the spherical coordinates (r , λ, ϕ)[13]

Assuming that the density ρ is constant (ρ is given the value of the average density of the Earth) and dv is the same for all elements, then only l is changing for each element. The Laplace operator for the gravitational potential in equation (4-8) is given by[13]:

$$\triangle V = \triangle G \int_v \frac{\rho dv}{l} = g \int_v \triangle \left(\frac{1}{l}\right) \rho dv = 0 \qquad (4\text{-}11)$$

As $\Delta\left(\frac{1}{l}\right) = 0$, V is a harmonic function. The solution of Laplace's equation is found by separating the variables r, λ and $\bar{\emptyset}$ using the substitution in equation (4-12) reading:

$$V(r, \bar{\emptyset}, \lambda) = \sum_{n=0}^{\infty} \frac{1}{r^{n+1}} \sum_{m=-n}^{n} \bar{A}_{nm} \bar{Y}_{nm}(\bar{\emptyset}, \lambda) \qquad (4\text{-}12a)$$

$$f_1 = \frac{1}{r^{n+1}} \qquad n = 0,1,2,\ldots \qquad (4\text{-}12b)$$

$$f_2(\bar{\emptyset}) = P_{nm}(\sin\bar{\emptyset}) \qquad n = 0,1,2,\ldots n-1, n \qquad (4\text{-}12c)$$

$$f_3(\lambda) = \cos m\lambda \quad or \quad \sin m\lambda \qquad m = 0,1,2,\ldots n-1, n \qquad (4\text{-}12d)$$

In equation (4-15), $P_{nm}(\sin\bar{\phi})$ are the Legendre functions of degree n and order m. Assuming $(\sin\bar{\phi}) = \tau$, the Legendre function is generally defined by the differential formula in equation (4-16) :

$$P_{nm}(t) = \frac{1}{2^n n!}(1-t)^{m/2} \frac{d^m p_{nm}(t)}{dt^m}(t^2-1)^n \qquad (4\text{-}13)$$

As the differential equation (4-10) is linear, for each integer n there is a solution. The summation of all solutions is also a solution for Laplace's equation. The potential V can be written in terms of surface Spherical Harmonics (SH) in equation (4-15).

$$V(r,\bar{\phi},\lambda) = \sum_{n=0}^{\infty}\frac{1}{r^{n+1}}\sum_{m=-n}^{n} A_{nm}Y_{nm}(\phi,\lambda) \quad (4\text{-}14)$$

$$Y_{nm}(\phi,\lambda) = \begin{cases}\cos m\lambda P_{n|m|}(\sin\bar{\phi}) & ,m\le 0\\ \sin m\lambda P_{nm}(\cos\bar{\phi}) & ,m>0\end{cases} \quad (4\text{-}15a)$$

$$A_{nm} = \begin{cases}a_{nm} & ,m\le 0\\ b_{nm} & ,m>0\end{cases} \quad (4\text{-}15b)$$

Equation (4-14) can be reformulated as double summation. In this case V reads:

$$V(r,\bar{\phi},\lambda) = \sum_{n=0}^{\infty}\frac{1}{r^{n+1}}\sum_{m=0}^{n}(a_{nm}\cos m\lambda + b_{nm}\sin m\lambda)P_{nm}(\sin\bar{\phi}) \quad (4\text{-}16)$$

4-3-1 The normalized SH

As shown above, the gravitational potential V satisfies the Laplace equation. In equation (4-14),V was modeled to solve the Laplace equation in terms of SH. When higher degrees and orders Legendre functions $P_{nm}(t)$ are calculated, instability problems appear in the calculations. To avoid these issues, a normalized form of equation (4-14) is introduced in equation (4-17) using the normalized Legendre functions $\bar{P}_{nm}(t)$ [13].

$$V(r,\bar{\phi},\lambda) = \sum_{n=0}^{\infty}\frac{1}{r^{n+1}}\sum_{m=-n}^{n} A_{nm}Y_{nm}(\phi,\lambda) \quad (4\text{-}17a)$$

$$\bar{Y}_{nm}(\bar{\phi},\lambda) = f_{nm}Y_{nm}(\bar{\phi},\lambda) \quad (4\text{-}17b)$$

$$\bar{P}_{nm}(t) = f_{nm}P_{nm}(t) \quad (4\text{-}17c)$$

$$\bar{A}_{nm}(t) = \frac{A_{nm}}{f_{nm}} \quad (4\text{-}17d)$$

Finally, the potential V reads:

$$V(r,\bar{\phi},\lambda) = \sum_{n=0}^{\infty}\frac{1}{r^{n+1}}\sum_{m=0}^{n}(\bar{a}_{nm}\cos m\lambda + \bar{b}_{nm}\sin m\lambda)\bar{P}_{nm}(\sin\bar{\phi}) \quad (4\text{-}18)$$

The normalizing function f_{nm}in equation (4-17) reads :

$$f_{nm} = \begin{cases} \sqrt{2n+1} & ,m=0 \\ \sqrt{2(2n+1)\frac{n-m\,!}{n+m\,!}} & ,m\neq 0 \end{cases} \qquad (4\text{-}19)$$

The coefficients $\bar{a}_{nm}$ and $\bar{b}_{nm}$ are constants, which have to be determined .They are generally called the spherical harmonic coefficients.

4-3-2 The normalized Legendre functions

Substituting the normalizing function in equation (4-19) in the recursive formula of Legendre function P_{nm} in equation (4-13), the fully normalized Legendre function in equation(4-20) is realized. $P_{nm}(\sin\bar{\phi})$is the fully normalized associated Legendre function.$P_{nm}(\sin\bar{\phi})$ can be calculated by the recursive formulas (4-20), with the abbreviations t = sin ϕ and υ = cos ϕ as follows[13]:

$$\bar{P}_{n,m} = a_{nm} t \bar{P}_{n-1,m} - b_{nm}\bar{P}_{n-2,m} \qquad (4\text{-}20a)$$

$$a_{nm} = \sqrt{\frac{2n-1\ (2n+1)}{n-m\ (n+m)}} \qquad (4\text{-}20b)$$

$$b_{nm} = \sqrt{\frac{2n+1\ \ n+m-1\ (n-m-1)}{n-m\ \ n+m\ (2n-3)}} \qquad (4\text{-}20c)$$

$$\bar{P}_{0,0} = 1 \quad , \quad \bar{P}_{1,0} = \sqrt{3}t \quad , \quad \bar{P}_{1,1} = \sqrt{3}u \qquad (4\text{-}20d)$$

If n=m, then $\bar{P}_{n,m}$ reads:

$$\bar{P}_{m,m} = u\sqrt{\frac{2m+1}{2m}}\bar{P}_{m-1,m-1} \qquad (4\text{-}20e)$$

In general from for the earth in normalized from :

$$V\ r,\bar{\phi},\lambda\ = \frac{GM}{r} + \frac{GM}{a}\sum_{n=2}^{\infty}(\frac{a}{r})^{n+1}\sum_{m=0}^{n}(\bar{C}_{nm}\cos m\lambda + \bar{S}_{nm\,\sin m\lambda})\,P_{nm}(\sin\bar{\phi}) \qquad (4\text{-}21a)$$

$$V\ r,\bar{\phi},\lambda\ = \frac{GM}{r} + \frac{GM}{r}\sum_{n=2}^{\infty}(\frac{a}{r})^{n+1}\sum_{m=0}^{n}(\bar{C}_{nm}\cos m\lambda + \bar{S}_{nm\,\sin m\lambda})\,P_{nm}(\sin\bar{\phi}) \qquad (4\text{-}21b)$$

4-3-3 Normal gravity of the earth

The normal gravity field of the earth is generated by an ellipsoid of revolution with semi-major axis (a) and semi-minor axis (b), such that[11]:

- the total mass of the reference ellipsoid is equal to that of the Earth;
- the reference ellipsoid is rotating around its minor axis at the same angular velocity as the earth rotation;
- the surface of the reference ellipsoid is an equipotential surface. The normal potential UO on the reference ellipsoid is equal to the geopotential WO on the geoid.

Many ellipsoids were defined by physical definition depending on the principle of normal gravity. Examples of physically defined ellipsoids are GRS67, GRS80, and WGS84. The defining parameters for GRS80 ellipsoid are shown in table 4.1:

Table (4.1): The defining parameters of the GRS80 ellipsoid

Notation	Constant	Unit	Numerical value
A	Semi-major axis	M	6378137.000
GM	Product of G and total mass M	m^3s^{-2}	$0.3986005.10^{15}$
J2	Dynamic from factor $\frac{C-A}{Ma^2}$		0.00108263
ω	Angular velocity	s^{-1}	$0.72921151.\ 10^{-4}$
B	Semi-minor axis	Meter	6356752.3141
f	Geometrical flattening		0.003352810681 1/298.257222101
e^2	First eccentricity squared		0.006694380023
e'^2	second eccentricity squared		0.006739496775
U_0	Normal potential on the ellipsoid	$m.sec^2$	62636860.850
γ_P	Normal gravity on the pole	Gal	983.21863685
γ_e	Normal gravity on the equator	Gal	978.03267715
$f*$	Gravity flattening		0.005302440112 1/188.592417552
K	$(b\gamma_P - a\gamma_e)/a\gamma_e$		0.001931851353
M	$\omega^2a^2b/(GM)$		0.003449786003 1/289.873052743
γ_{45}	Normal gravity at latitude 45^0	Gal	980.6199203
$\bar{\gamma}$		Gal	979.7644656

As the ellipsoid is defined as an equipotential surface, it has a potential U, that is a result of the ellipsoidal gravitational potential V', and the centrifugal potential Ω[11].

$$U = V' + \Omega \qquad (4\text{-}22)$$

Where:

V' : Gravitational potential

$$V' \; r, \bar{\emptyset}, \lambda \; = \frac{GM}{r} \;\; 1 + \sum_{n=1}^{\infty} \;\; \frac{a}{r} \;^{2n} \; \bar{J}_{2n} \bar{P}_{2n}(sin\bar{\emptyset}) \qquad (4\text{-}23)$$

$$J_{2n} = (-1)^n \frac{3.e^{2n}\sqrt{4n+1}}{2n+1 \;\, (2n+3)} \;\; 1 - n + \frac{5n}{e^2} . J_2 \qquad (4\text{-}24)$$

$$\Omega = \frac{1}{2}\omega^2(x^2 + y^2) \qquad (4\text{-}25)$$

Approximate Calculation for U:

$$U \;\; x, y, z \;\; = U \;\; u, \beta \;\; = \frac{GM}{E} \tan^{-1}\frac{E}{u} + \frac{1}{2}\omega^2 a^2 \frac{q}{q_0} \;\; \sin^2\beta - \frac{1}{3} \;\; + \frac{1}{2}\omega^2(u^2 + E^2)\cos^2\beta \qquad (4\text{-}26)$$

$$E^2 = a^2 - b^2 \qquad (4\text{-}27)$$

$$q = \frac{1}{2} \;\; \frac{1}{2} \;\; 1 + 3\frac{u^2}{E^2} \;\; \tan^{-1}\frac{E}{u} - \frac{1}{3}\frac{u}{E} \qquad (4\text{-}28)$$

$$q_0 = \frac{1}{2} \;\; \frac{1}{2} 1 + 3\frac{b^2}{E^2} \;\; \tan^{-1}\frac{E}{b} - \frac{1}{3}\frac{b}{E} \qquad (4\text{-}29)$$

The Reference gravity potential U_0 , can be calculated by assuming u=b:

$$U_0 = U \;\; b, \beta \;\; = \frac{GM}{E}\tan^{-1}\frac{E}{b} - \frac{1}{3}\omega^2 a^2 \qquad (4\text{-}30)$$

The normal gravity at the surface of the ellipsoid can be calculated as follows:

$$\gamma = \gamma_e \frac{1 + K\sin^2\phi}{\sqrt{1 - e^2\sin^2\phi}} \qquad (4\text{-}31)$$

$$k = \frac{b\gamma_p}{a\gamma_e} - 1 \qquad (4\text{-}32)$$

The gravity at a given height h above the ellipsoid:

$$\gamma_h = \gamma \;\; 1 - \frac{2}{a} \;\; 1 + f + m - 2f\sin^2\phi \;\; h + \frac{3}{a^2}h^2 \qquad (4\text{-}33)$$

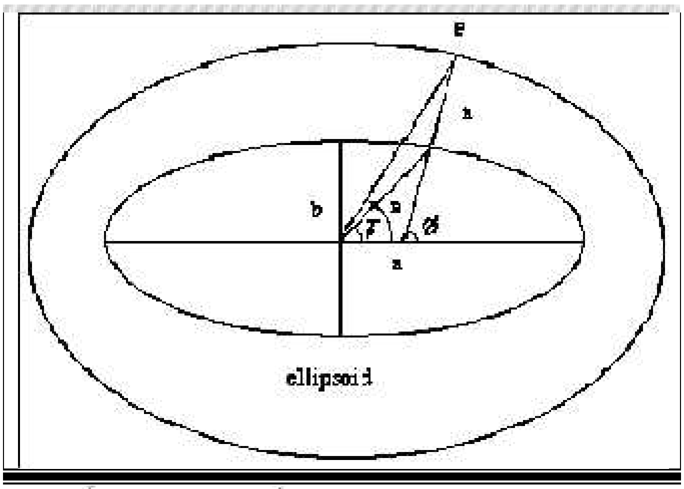


Fig 4-3: ϕ the reduced latitude of ϕ[11]

$$\tan\bar{\phi}\,(1-e^2)\tan\phi \qquad (4\text{-}34)$$

$$tan\beta = \frac{b}{a} tan\phi \qquad (4\text{-}35)$$

4-3-4 Derivatives of the potential of the Earth

A point P on the Earth's surface is subjected to two types of acceleration. The first type is the gravitational acceleration part g_1 due to the Earth's mass M. The second type z is the centrifugal acceleration due to the Earth's rotation. The total acceleration g is the vector summation of both gravitational and centrifugal accelerations , which represent the actual gravity vector[13]:

$$\bar{g} = \bar{g} + \bar{z} \qquad (4\text{-}36)$$

The relationship between the accelerations in equation (4-36) and their related potential is given in equation (4-37). The total gravity potential W, created by the total acceleration g , is the summation of the gravitational potential V and the centrifugal potential Ω . This total gravity

$$W = V + \Omega \qquad (4\text{-}37)$$

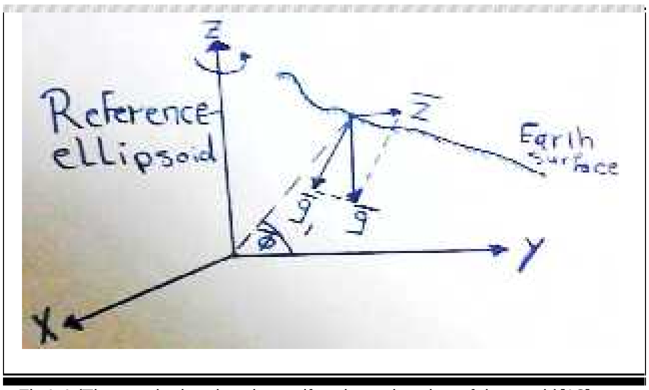


Fig4-4 (The gravitational and centrifugal acceleration of the earth)[13]

The centrifugal potential is caused by rotation of the Earth around its minor axis. The centrifugal acceleration vector will therefore have only two components in the X and Y directions. As the angular velocity of the Earth around its minor axis is 0.7292115 × 10–4 s –1 as defined by the GRS80 , the centrifugal potential reads[13]:

$$\Omega = 0.5\omega^2 r^2 \cos\bar{\phi} = \frac{1}{2}\omega^2(X^2 + Y^2) \quad (4\text{-}38)$$

Its related centrifugal acceleration vector and magnitude are:

$$\bar{z} = grad\ \Omega = \begin{matrix} \omega^2 X \\ \omega^2 Y \\ 0 \end{matrix} + \begin{matrix} \omega^2 r \cos\bar{\phi}\cos\lambda \\ \omega^2 r \cos\bar{\phi}\sin\lambda \\ 0 \end{matrix} \quad (4\text{-}39a)$$

$$z = |\bar{z}| = \sqrt{\frac{\partial\Omega}{\partial X}^2 \frac{\partial\Omega}{\partial Y}^2 \frac{\partial\Omega}{\partial Z}^2} = \omega^2\sqrt{X^2 Y^2} = \omega^2 r\cos\bar{\phi} \quad (4\text{-}39b)$$

4-3-5 Gravity field anomalies

To fined the difference between the two equipotential surfaces; the Geoid and the ellipsoid, that is called the Geoid undulation we define the following[11]:

1. Disturbing potential TP :

$$T_p = W_p - U_p \quad (4\text{-}40a)$$

$$T_p = V_p + \Omega_p - V'_p + \Omega_p = V_p - V'_p \quad (4\text{-}40b)$$

2. Gravity disturbance:

$$g_p = g_p - \gamma_q \quad (4\text{-}41)$$

4. Gravity anomaly:

$$\Delta g_p = g_p - \gamma_q \quad (4\text{-}42)$$

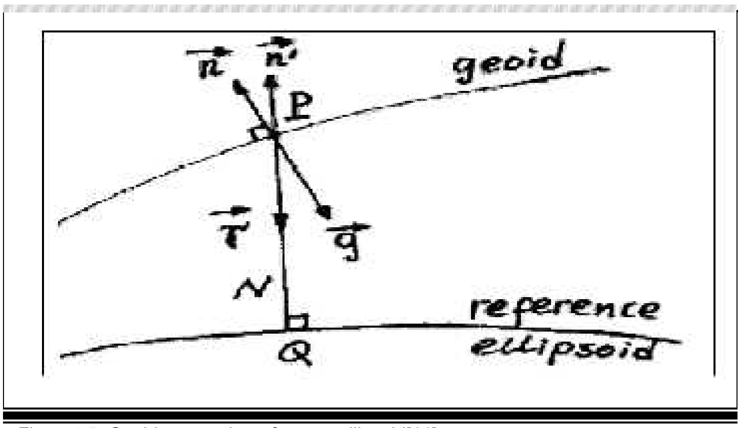


Figure4-5: Geoid versus the reference ellipsoid[11]

If P is the point in the Geoid and Q is the same point in the ellipsoid in the ellipsoid, where WP = Uq , then we can fined the normal potential of point P using the normal potential of point Q[11]:

$$N = \frac{T_P}{\gamma_q} \tag{4-43}$$

Uq Is equal to U_0 because Q is on the ellipsoid surface (h=0), this can be done by the use of Tylor series:

Gravity disturbance:

$$\delta g_p = -\frac{\partial T_P}{\partial r} \tag{4-44}$$

Gravity anomaly:

$$\Delta g_p = -\frac{\partial T_p}{\partial r} - \frac{2}{r} T_p \tag{4-45}$$

The deflection of vertical:

$$\begin{pmatrix} \eta \\ \xi \end{pmatrix} = \begin{pmatrix} -\frac{dN}{dx} \\ -\frac{dN}{dy} \end{pmatrix} = -\begin{pmatrix} \frac{dN}{R\cos\phi d\lambda} \\ \frac{dN}{Rd\phi} \end{pmatrix} \tag{4-46}$$

$$\begin{pmatrix} \eta \\ \xi \end{pmatrix} = -\frac{1}{\gamma_q} \begin{pmatrix} \frac{dT}{R\cos\phi d\lambda} \\ \frac{dT}{Rd\phi} \end{pmatrix} \tag{4-47}$$

4-3-6 Global geoid / gravity models

The common way for representing the gravitational potential V in a global model is to use the SH Presently, there are many global gravity potentialfield models available from various sources and with different spatial resolutions. The International Center for Global Gravity Models (ICGEM) provides access to the various satellite only or combined models on behalf of the International Association of Geodesy (http://icgem.gfz-potsdam.de/ICGEM/ICGEM.html) .Examples of these models are shown in table (4.2)[13].

Table(4.2):some of the common global gravity models with their data sources .

Model	Year Degree		Data
EIGEN06c	2011	1420	S(GOCE, GRACE, LAGEOS), G, A
EIGEN051c	2010	359	S(GRACE, CHAMP), G, A
EIGEN05c	2008	360	S(GRACE, LAGEOS), G, A
EGM2008	2008	2190	S(GRACE), G, A
EIGEN-GL04c	2006	360	S(GRACE, LAGEOS), G, A
GGM02c	2004	200	S(GRACE), G, A
EIGEN-CG01c	2004	360	S(CHAMP, GRACE), G, A
PGM2000A	2000	360	S, G, A
EGM96	1996	360	S, G, A

Data: S=Satellite gravity data, G = Gravity data, A = Altimetry data

The calculation of the SH coefficients can only be solved by means of global data coverage .This could only be achieved after the first geodetic satellite missions (like the LAGEOS,GRACE, GOCE and CHAMP missions).. The combination of satellite observations with terrestrial measurements led. The SH can be calculated by two methods: the first is the integration method that keeps the orthogonality conditions of the SH, and second is the least squares estimation .

The integration methods have several problems. One is that the data have to be downward continued to the zero level (geoid) resulting in the so-called surface SH; the other is that weighting of observations of different sources is not possible. The integration formulas to calculate the spherical harmonic coefficients using the gravity anomalies Δg and the geoid heights N are given :

$$\begin{matrix}\bar{C}_{nm}\\ \bar{S}_{nm}\end{matrix} = \frac{1}{4\pi GM} \; {}_{\sigma} \; r\gamma \; \frac{r}{a} \; {}^{n} N\bar{P}_{nm} \begin{matrix}\cos m\lambda\\ \sin m\lambda\end{matrix} \; d\sigma \qquad (4\text{-}48a)$$

$$\begin{matrix}\bar{C}_{nm}\\ \bar{S}_{nm}\end{matrix} = \frac{1}{4\pi GM} \; {}_{\sigma} \; \frac{r^2}{n-1} \; \frac{r}{a} \; {}^{n} \Delta g\bar{P}_{nm} \begin{matrix}\cos m\lambda\\ \sin m\lambda\end{matrix} \; d\sigma \qquad (4\text{-}48b)$$

In the least squares solution, the introduction of the variance and covariance matrices is possible for each group of data or for any single observation .

4-4 Gravimetric Geoid (stocks formula)

Stocks formula is one of the most fundamental formulas in physical geodesy. It gives us the possibility to determine the Geoid height N from terrestrial gravity measurements[11].

Stocks assumptions:

1. No mass outside the geoid.
2. Gravity measurements all over the world.

Where , $S \; r, \psi$ is called the extended Stokes function.

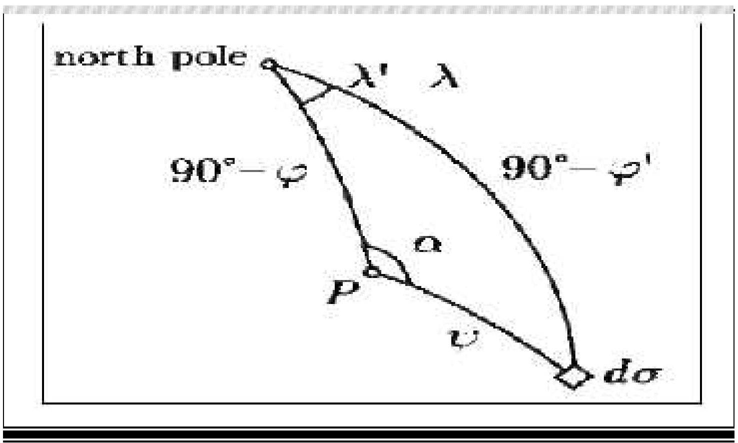


Figure4-6: spherical distance ψ[16]

The stock's function is defined as a weighting function, and it depends on the spherical distance ψ between the point P and th.

When r → R , we get the disturbing potential on the geoid:

$$T\ R,\emptyset,\lambda\ = \frac{R}{4\pi}\ _{\sigma}\ S\ \psi\ .\Delta g \qquad (4\text{-}49)$$

The geoid height N is then obtained by the famous stocks formula:

$$N = \frac{T(R,\emptyset,\lambda)}{\gamma} = \frac{R}{4\pi\gamma}\ _{\sigma}\ S\ \psi\ .\Delta g \qquad (4\text{-}50)$$

Where γ denotes the normal gravity and Sψ is called stokes function:

$$S\psi = S(r,\psi)\ _{r\rightarrow R} = \sum_{n=2}^{\infty}\frac{2n+1}{n-1}P_n(\cos\psi) \qquad (4\text{-}51)$$

Also:

$$S\ \psi\ = \frac{1}{sin\frac{\psi}{2}} - 6sin\frac{\psi}{2} + 1 - 5\cos\psi - 3\cos\psi \ln\left(\sin\frac{\psi}{2} + \sin^2\frac{\psi}{2}\right) \qquad (4\text{-}52)$$

If we introduce a new variable t = cosψ , stokes function can also be written as:

$$s\ \psi\ = \ \sqrt{\frac{2}{1-t}} - 6\ \sqrt{\frac{1-t}{2}} + 1 - 5t - 3t\ln\ \ \sqrt{\frac{1-t}{2}} + \frac{1-t}{2} \qquad (4\text{-}53)$$

Stokes' formula may be numerically evaluated by the grid method in the following way, see figure 6.5[11]:

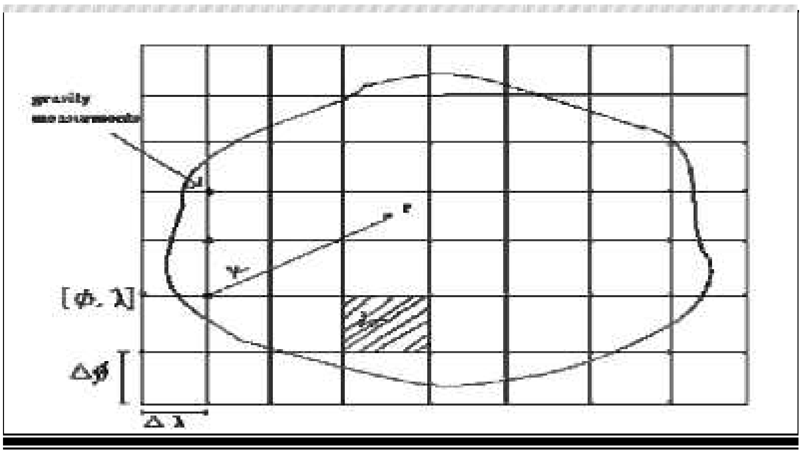


Figure4-7: stocks formula (grid method)[13]

$$N = \frac{R}{4\pi\gamma}\ _{\sigma}\Delta\ gS\ \psi\ d\sigma = \frac{R}{4\pi\gamma}\sum_I\ \sum_J\ \ _{\sigma IJ}\Delta gS\ \psi\ .d\sigma_{ij} \qquad (4\text{-}54)$$

$$\approx \frac{R}{4\pi\gamma}\sum_I\ \sum_I\ \ _{\sigma IJ}\Delta\bar{g}_{ij}S\ \psi_{IJ}\ .d\sigma_{ij} = \frac{R}{4\pi\gamma}\sum_I\ \sum_I\ \Delta\bar{g}_{ij}S\ \psi_{IJ}\ \ _{\sigma IJ}d\sigma \qquad (4\text{-}55)$$

$$N = \frac{R}{4\pi\gamma} \sum_i \sum_i (\Delta\bar{g}_{ij} S(\psi_{ij}) A_{ij}) \quad (4\text{-}56)$$

Where:

R = mean radius of the earth.

Γ = normal gravity of the reference ellipsoid.

$\Delta\bar{g}_{ij}$= mean gravity anomaly for block σ ij.

Ψ ij = spherical distance from the computation point (φ , λ) to the block center of σ ij.

Φ min , λ min = the minimum latitude and minimum longitude of the integration area.

Δφ , Δλ = block sizes (latitude/longitude difference of a block).

A ij = area of block σ ij.

Some of the above quantities can be computed as follows:

$$\cos\psi_{ij} = \sin\phi \sin\phi_i + \cos\phi \cos\phi_i \cos(\lambda - \lambda_{j)} \quad (4\text{-}57)$$

$$\phi_i = \phi_{min} + (i - \tfrac{1}{2})\Delta\phi \quad (4\text{-}58)$$

$$\lambda_i = \lambda_{min} + (j - \tfrac{1}{2})\Delta\lambda \quad (4\text{-}58)$$

$$A_{ij} = \quad_{\sigma ij} d\sigma = 2.\Delta\lambda.\sin\frac{\Delta\phi}{2}\cos\theta_i \quad (4\text{-}59)$$

4-5 GNSS/Leveling

The GNSS/GPS leveling can be directly used in the defining the eight reference surface (HRS) by measuring the ellipsoidal heights (h) of points with known orthometric height (H) or normal height (H*). The ellipsoidal heights are measured directly by means of GPS/GNSS. The height anomaly (ζ=h-H*) or the geoid height (N=h-H) at a given is directly determined[13]

CHAPTER FIVE

GEOID CALCULATION

5-1 Introduction

This chapter discusses measurement and calculation to achieve this project the GNSS observations introduce the height above the WGS84 ellipsoid to get the orthometric height , the geoid undulation N is required . the geoid height can be obtained from local regional or global geoid / gravity models where the general formula for the gravitational potential reads . [19]

$$V\ r,\bar{\phi},\lambda\ = \frac{GM}{r} + \frac{GM}{a}\sum_{n=2}^{\infty}\left(\frac{a}{r}\right)^{n+1}\sum_{m=0}^{n}\left(\bar{C}_{nm}\cos m\lambda + \bar{S}_{nm\sin m\lambda}\right)P_{nm}(\sin\bar{\phi}) \qquad \textbf{(5-1)}$$

And the geoid heights read

$$N = \frac{V - \acute{V}}{\gamma} \qquad \textbf{(5-2)}$$

V' : the ellipsoidal gravitational potential

V : is the gravitational potential

γ : the normal gravity for ellipsoid

5-2 Reference points

Triangulation point sites exploned and visited, Triangulation points with knowing the coordinates were reached using the guidance of a hand held GNSS (garmin) receiver. After reaching the sites of each triangulation point , the latitudes , longitudes and heights above ellipsoid were measured . the visited sites were triangulation points in Yatta ,Bani Naim , Nuba , Sourif , Tarqoumia , Halhul , Beit Oula, Hebron, Dura, Aldahriyeh, Alsamoua', ALshyoukh, and Ethna , as shown in figure (1-5) .[19]

fig 5-1: Triangulation points

the data are installed from data collector . these data were installed in different coordinates system . the first coordinates system is the WGS84 (latitude , longitude and ellipsoidal heights) .the second system was the Palestinian coordinates (Palestine 1923 Grid) with easting , northing , and orthometric height , the other system is Israel grid coordinates .[19]

5-3 Gravity / Geoid Model

Different Global Geoid gravity models are available online for free . these models are created by different institutions different sources of data are used for each models , the data can be satellite gravity height , terrestrial gravity data , and height fitting points the models are with different degrees and orders , table (4.2) shows the geoid models used in this project and the degree and the order . [19]

Table(4.2):some of the common global gravity models with their data sources .

Model	Year	Degree	Data
EIGEN06c	2011	1420	S(GOCE, GRACE, LAGEOS), G, A
EIGEN051c	2010	359	S(GRACE, CHAMP), G, A
EIGEN05c	2008	360	S(GRACE, LAGEOS), G, A
EGM2008	2008	2190	S(GRACE), G, A
EIGEN-GL04c	2006	360	S(GRACE, LAGEOS), G, A
GGM02c	2004	200	S(GRACE), G, A
EIGEN-CG01c	2004	360	S(CHAMP, GRACE), G, A
PGM2000A	2000	360	S, G, A
EGM96	1996	360	S, G, A

Data: S=Satellite gravity data, G = Gravity data, A = Altimetry data

5-3-1 EIGEN-5C

EIGEN is European Improved Gravity model of the Earth by New techniques,The EIGEN-5C is a new global combined high-resolution GRACE-based gravity field model of the GFZ-GRGS cooperation.

A new combined gravity field model EIGEN-5C has been obtained from the combination of GRACE & LAGEOS satellite data and surface data .The new EIGEN-5Cmodel shows the following improvements compared to previously released models better orbit fits for GRACE and SLR satellites, smoother spectral behavior, better reduction of meridional stripes, and better fit in GPS/Leveling comparisons the calculated geoid height for the west bank using EIGEN-5c are shown in figure (5- 2). [16].

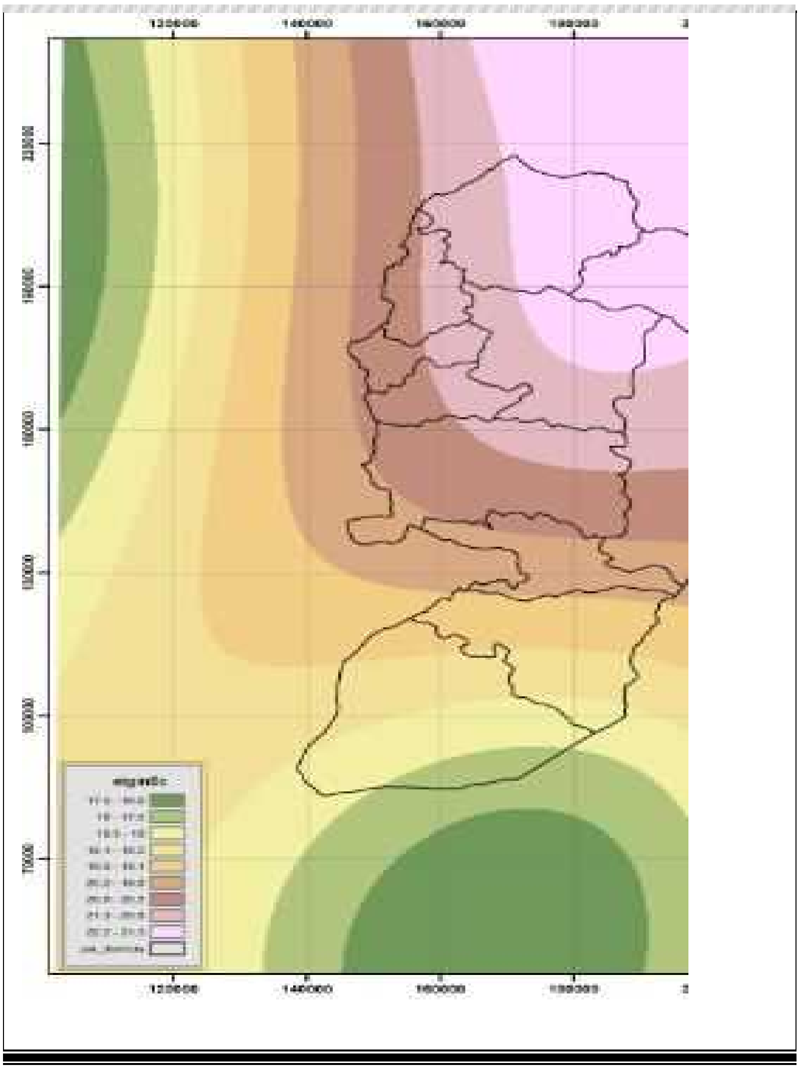


fig 5-2: EIGEN-5C raster

5-3-2 EIGEN-5S

EIGEN is European Improved Gravity model of the Earth by New techniques, EIGEN-5S a special band-limited normal equation combination method has been applied in order to preserve the high accuracy from the satellite data in the lower frequency band of the geopotential and to form a smooth transition to the high frequency information coming from the surface data. The satellite only model EIGEN-5S has been selected as standard for European Space Agency ESA's official data processing of the upcoming gradiometer satellite mission GOCE**,** the calculated geoid height for the west bank using EIGEN-5S are shown in figure (5- 3).[16]

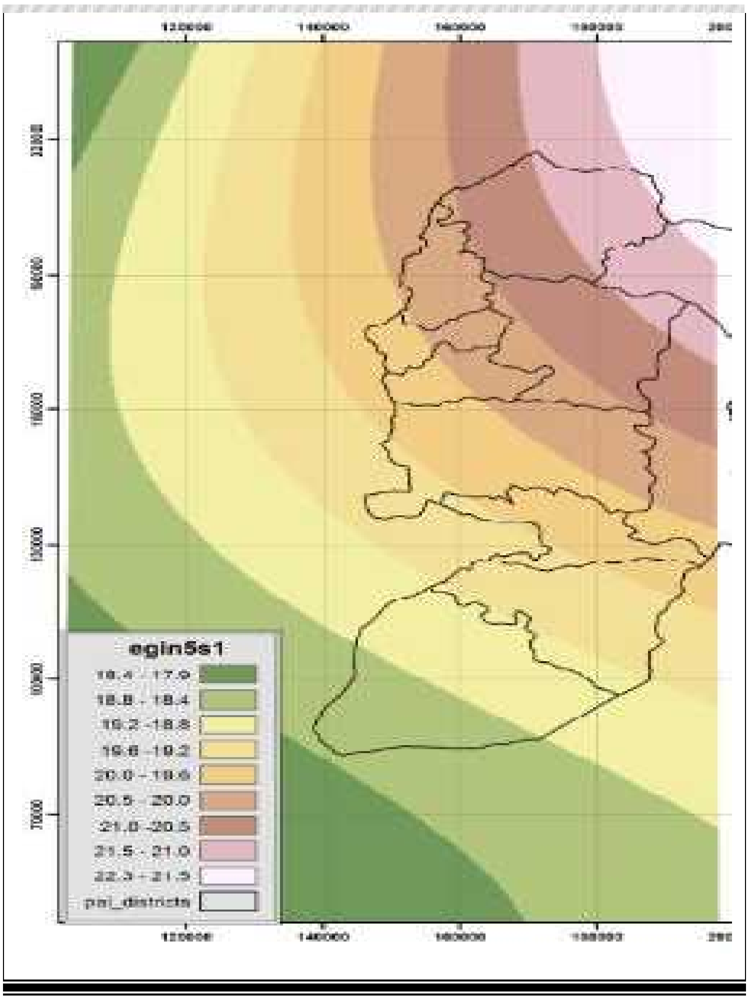


Fig 5-3: EIGEN-5S raster

5-3-3 EIGEN-6C3stat

High Resolution Global Combined Gravity Field Model, based on the 4th Release of the GOCE Direct Approach, is a static pre-version of the new Global Combined Gravity Field model to degree/order 1949, This model has been inferred from the combination of LAGEOS,GRACE, GOCE and ground data on the continents for wavelengths beyond sphere. harm. Degree 235, EIGEN-6C3 will contain time variable parameters for all spherical Harmoin coeffients up to degree 50 (drift parameters, annual and semi-annual terms) [17]. the calculated geoid height for the west bank using EIGEN-6c3 are shown in figure (5-4).[17]

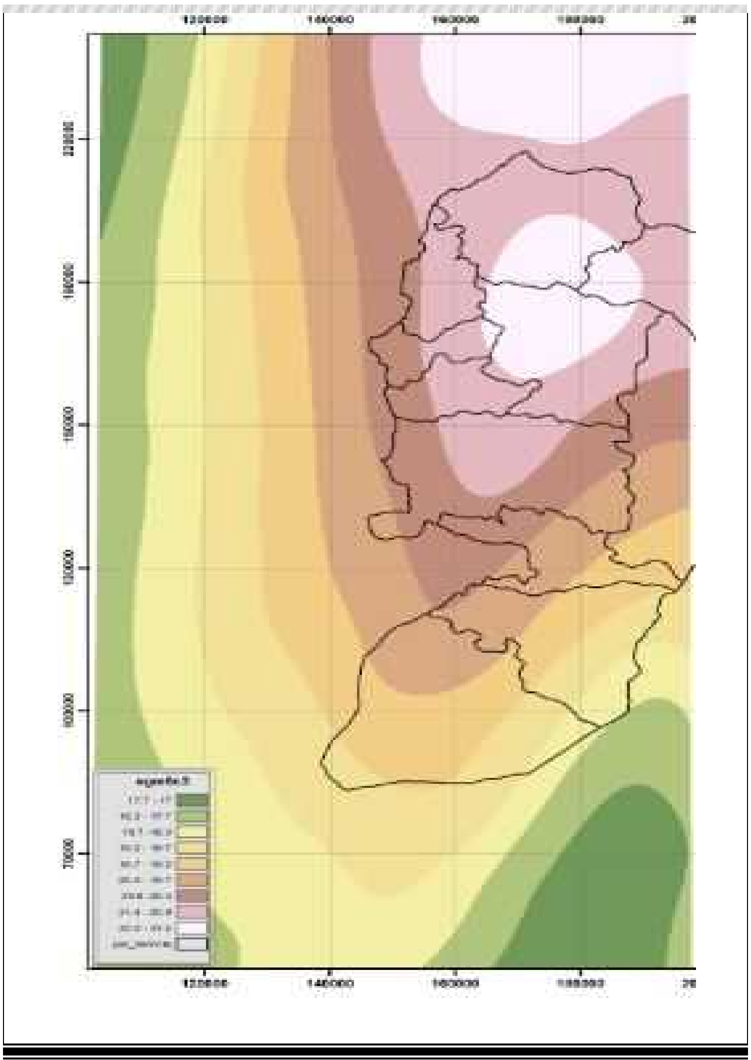


Fig 5-4: EIGEN-6C3stat raster

5-3-4 EIGEN-6S

is a new combined gravity field model from the EIGEN-6S satellite data and the DTU10 global gravity anomaly grid of a maximum degree 1420.Over land and beyond degree 240, EIGEN-6S is in principle a reconstruction of EGM2008 .

EIGEN-6S contain time variable parameters for all sphere. harm. Coffient up to degree 50 (drift, annual and semiannual terms) ,GOCE-only models are not as good as GRACE models for GOCE orbit computation .The best GOCE orbit fit results are obtained with combined GRACE+GOCE models. the calculated geoid height for the west bank using EIGEN-6S are shown in figure (5- 5).[17]

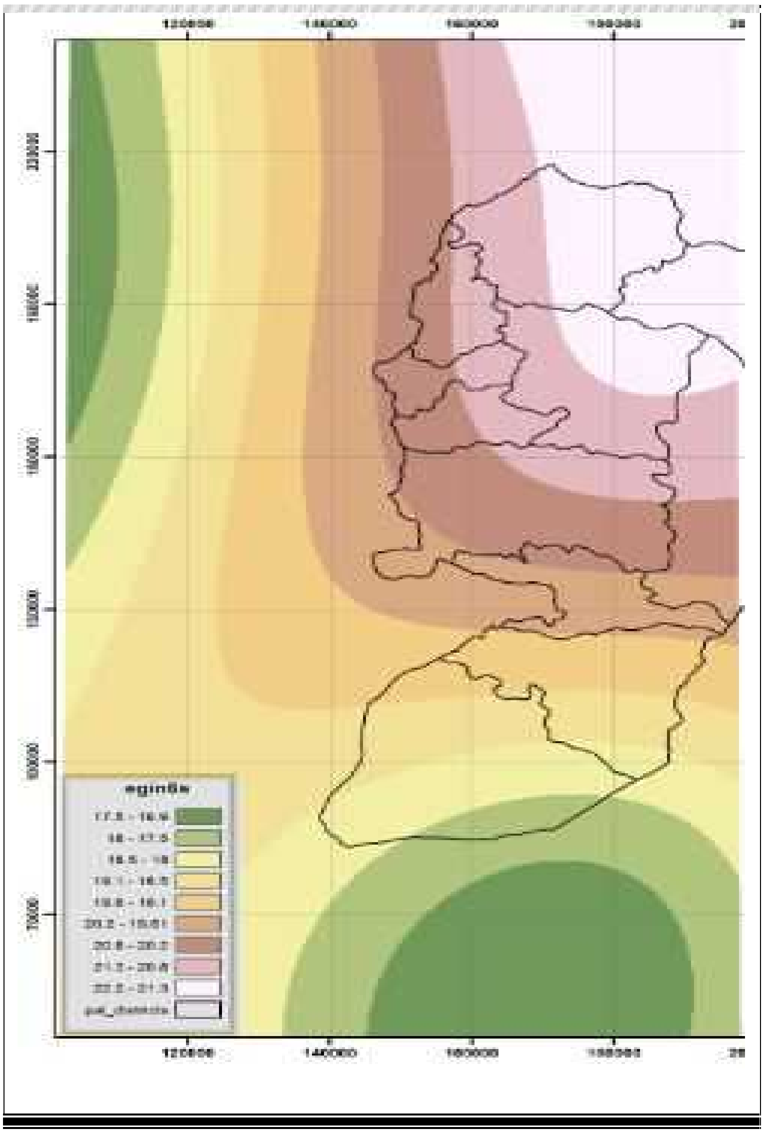


Fig 5-5: EIGEN-6C raster

5-3-5 EGM2008

The official Earth Gravitational Model EGM2008 has been publicly released by the National Geospatial-Intelligence Agency (NGA) EGM Development Team. This gravitational model is complete to spherical harmonic degree and order 2159, and contains additional coefficients extending to degree 2190 and order 2159. Full access to the model's coefficients and other descriptive files with additional details about EGM2008 are provided within these web pages. the calculated geoid height for the west bank using EGM2008 are shown in figure (5-6).[17]

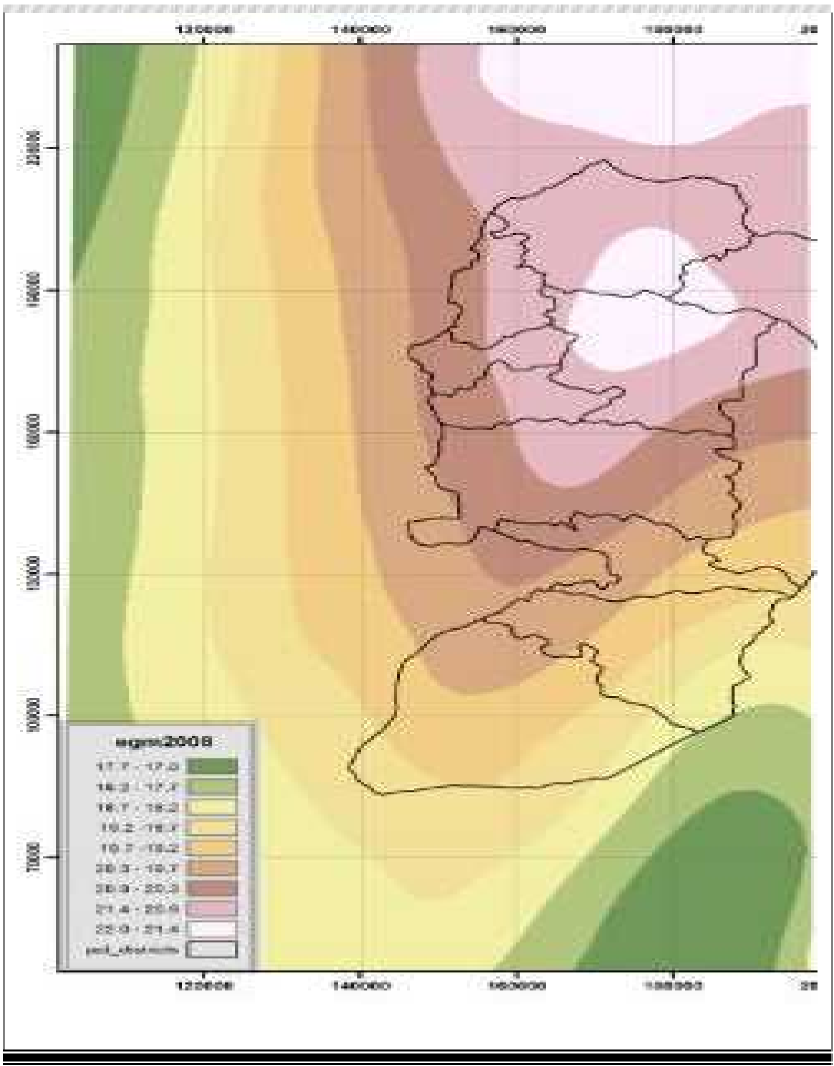


Fig 5-6: EGM2008 raster

5-3-6 EGM96

EGM96 (Earth Gravitational Model 1996) is a geopotential model of the Earth consisting of spherical harmonic coefficients complete to degree and order 360 , the calculated geoid height for the west bank using EGM96 are shown in figure (5- 7). [16]

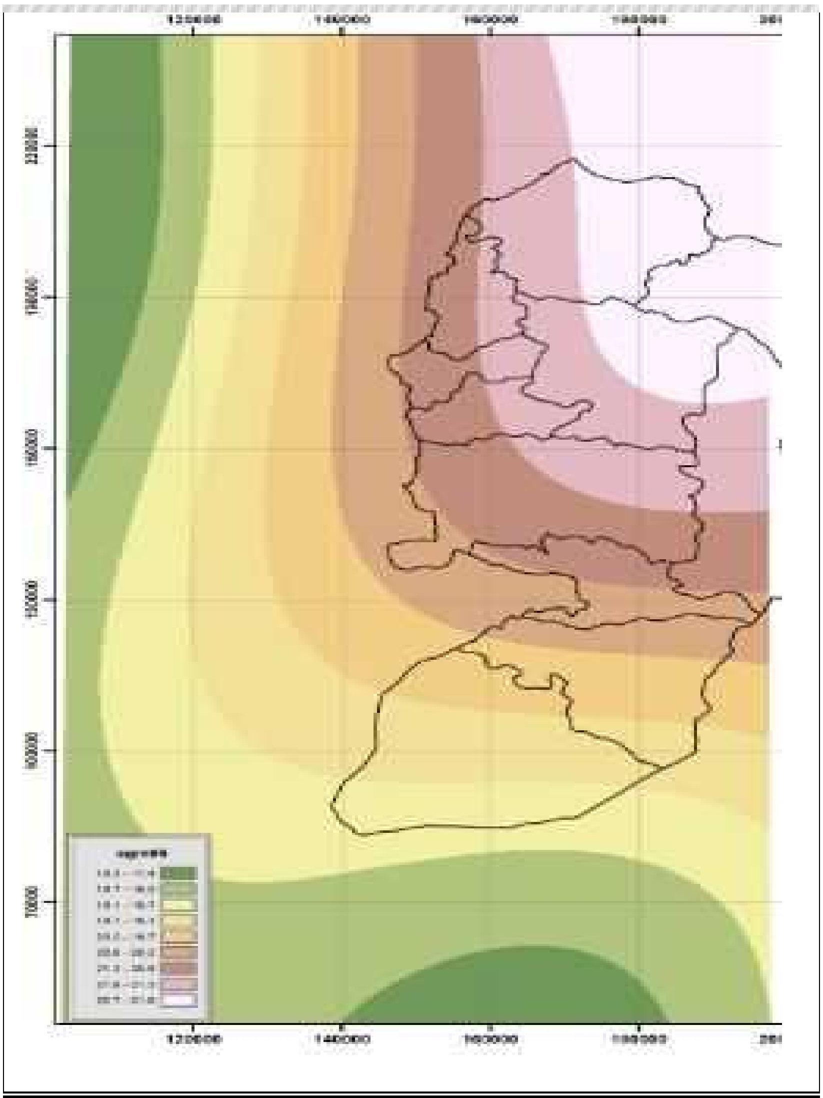


Fig 5-7: Egm96 raster

5-4 Calculation of Geoid height

The calculation of the geoid heights using different model was applied using the calculation service by the GFZ-Potsdam's. This is an interactive Java Applet to calculate a selected gravity field functional for a set of gridded points on a reference ellipsoid. one can select one of the model files offered by this service.

The computed grid file is usually available after few seconds or few minutes depending on the functional the maximum degree of the model and the number of grid points. Calculating the functional gravity disturbance, gravity anomaly, gravity anomaly and gravity needs more time than the calculations of the geoid height because they use DEMS for the calculations . A plot of the calculated grid can be also provided as postscript [14].

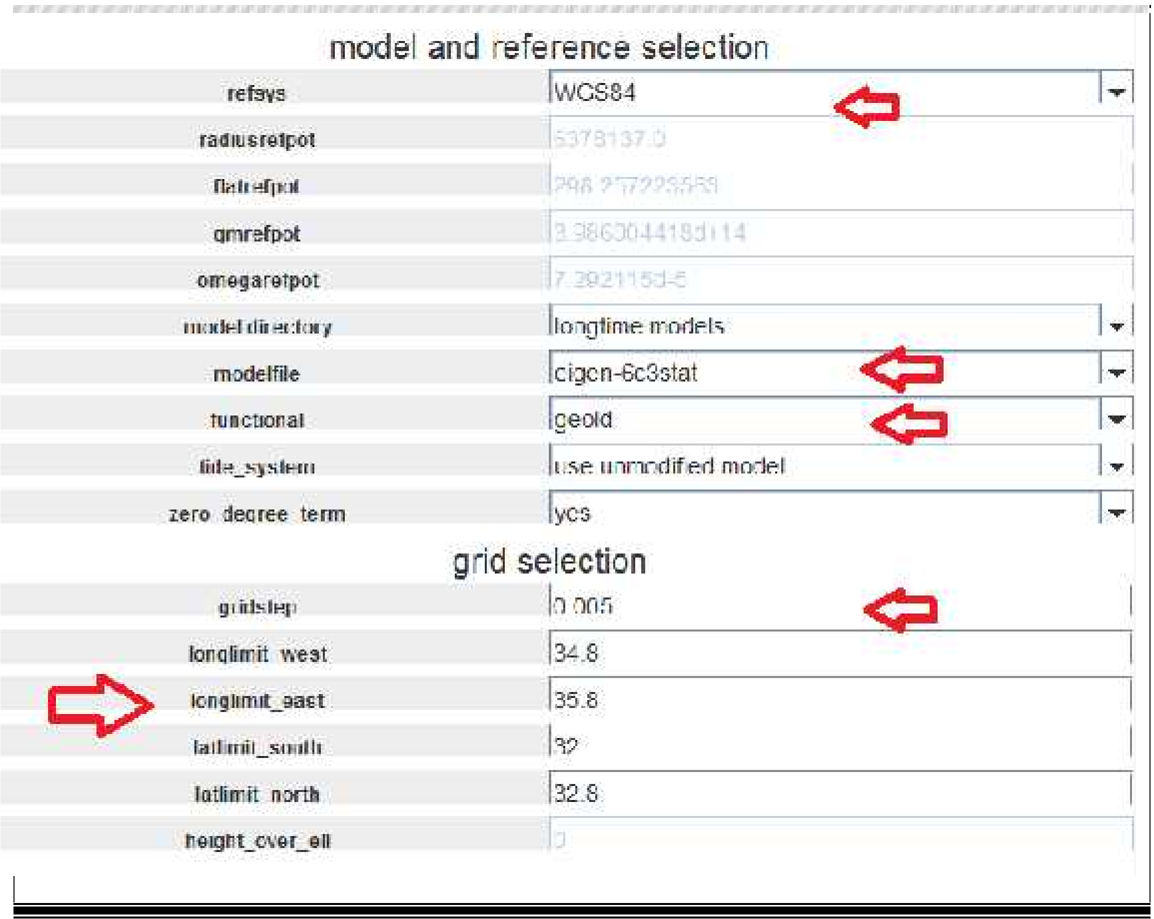


Fig 5-8: Gravity Field Functionals on Ellipsoidal Grids

The result of using different model in Palestine are discussed in the following section.

5-4-1 EIGEN-5C

using EIGEN-5C geoid model, grid of points in Hebron district was get, the calculated grids were in ASCII format. These grids converted to raster format in ARCGIS as shown in figure (5-9) .[19]

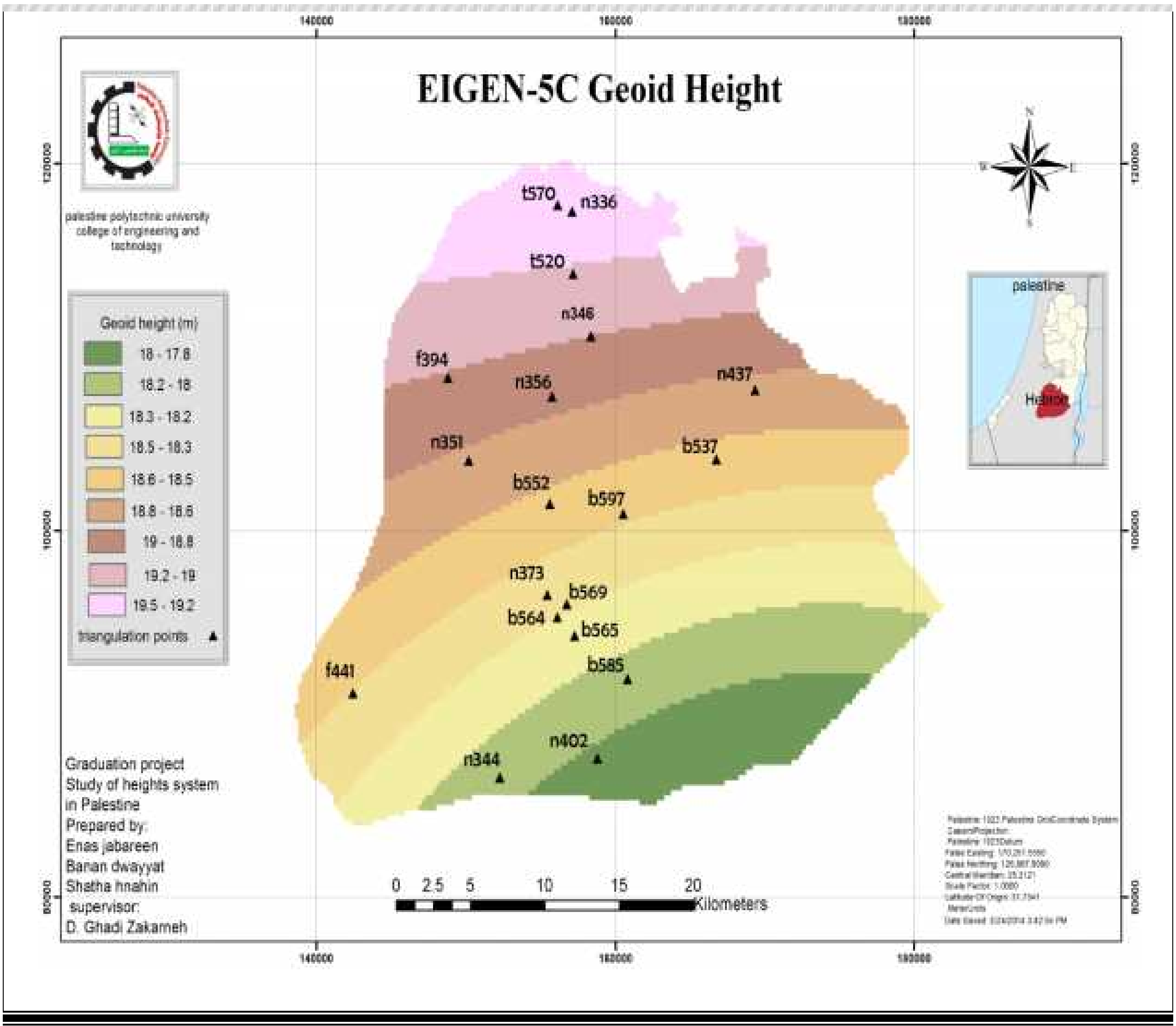


Fig 5-9: geoid height (m) in hebron district using EIGEN-5C

5-4-2 EIGEN-5S

Using EIGEN-5S geoid model, grid of points in Hebron district was get, the calculated grids were in ASCII format. These grids converted to raster format in ARCGIS as shown in figure (5-10) .[19]

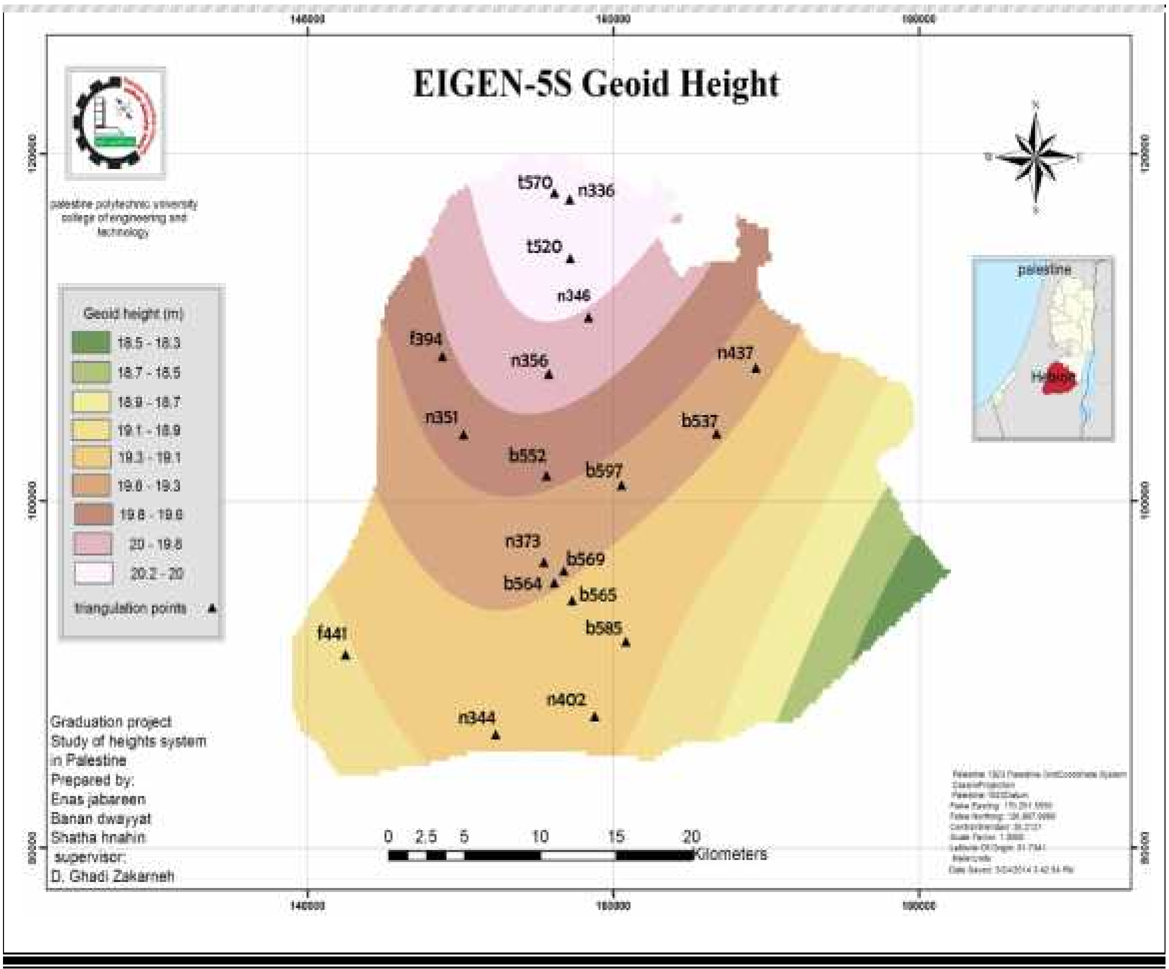


Fig 5-10: geoid height (m) in hebron district using EIGEN-5S

5-4-3 EIGEN-6C3stat

Using EIGEN-6C3stat geoid model, grid of points in Hebron district was get, the calculated grids were in ASCII format. These grids converted to raster format in ARCGIS as shown in figure (5-11) .[19]

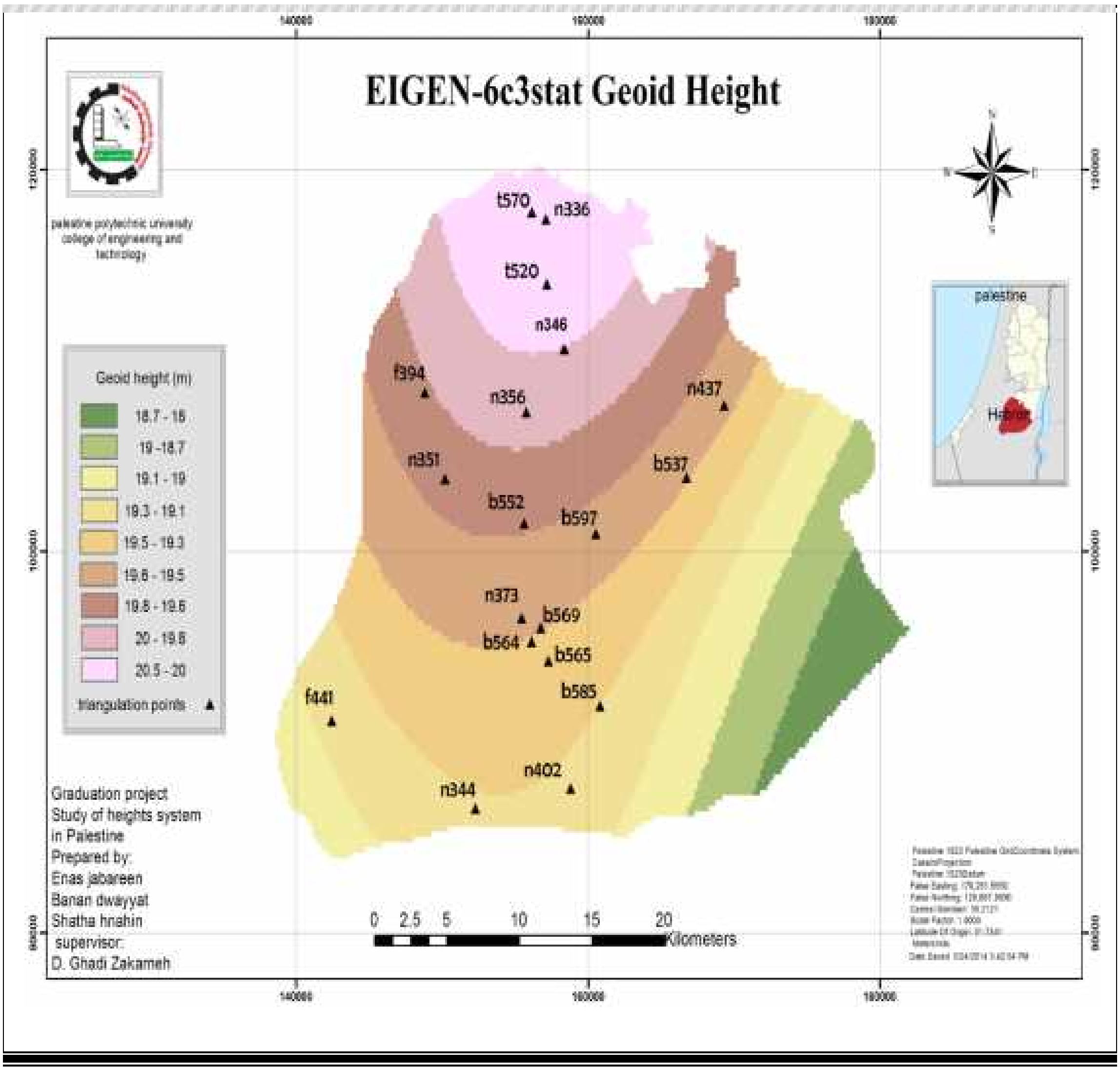


Fig 5-11: geoid height (m) in hebron district using EIGEN-6C3Stat

5-4-4 EIGEN-6S

Using EIGEN-6S geoid model, grid of points in Hebron district was get, the calculated grids were in ASCII format. These grids converted to raster format in ARCGIS as shown in figure (5-12) .[19]

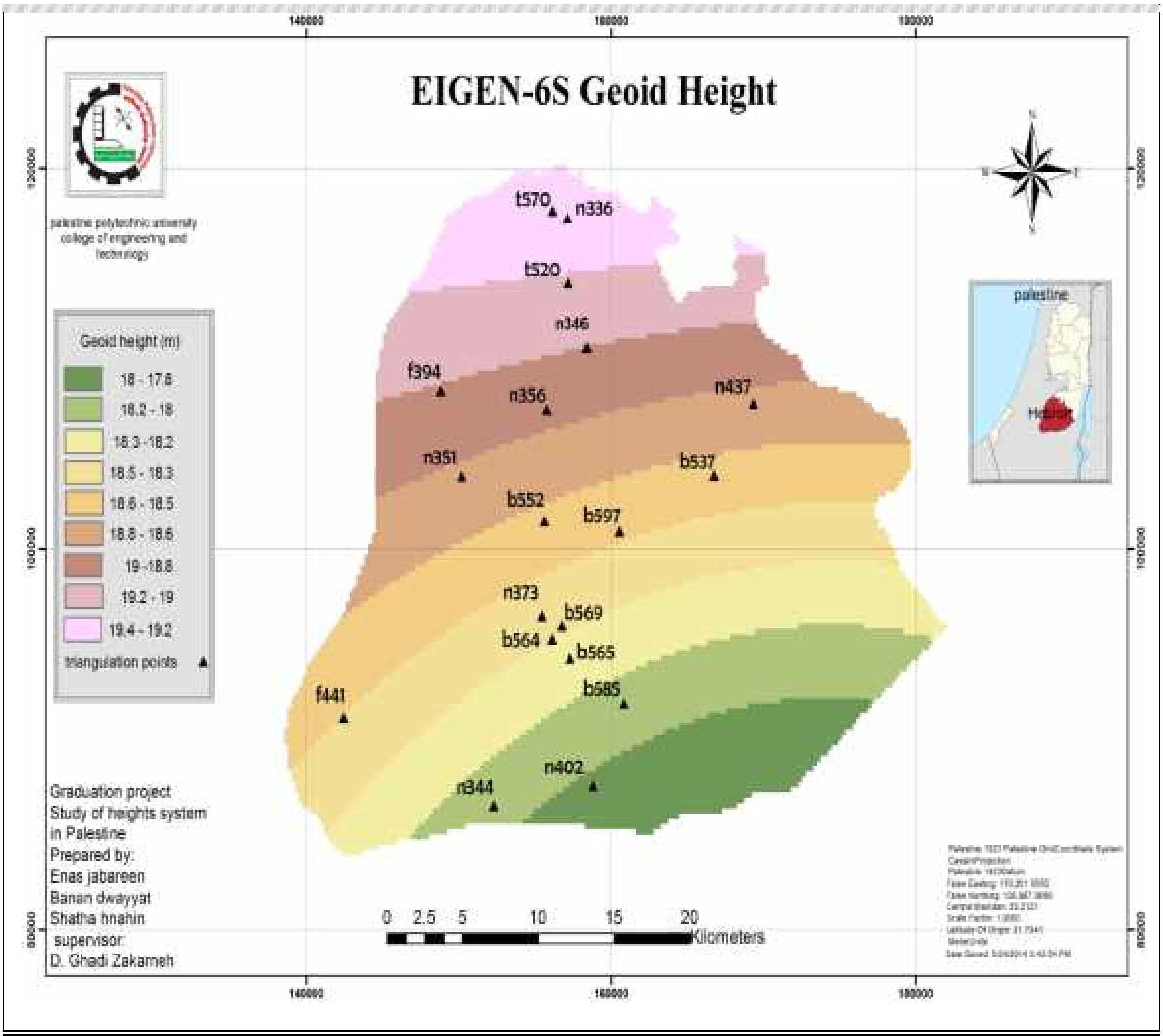


Fig 5-12: geoid height (m) in hebron district using EIGEN-6C

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5-4-5 EGM2008

Using EGM2008 geoid model, grid of points in Hebron district was get, the calculated grids were in ASCII format. These grids converted to raster format in ARCGIS as shown in figure (5-13) .[19]

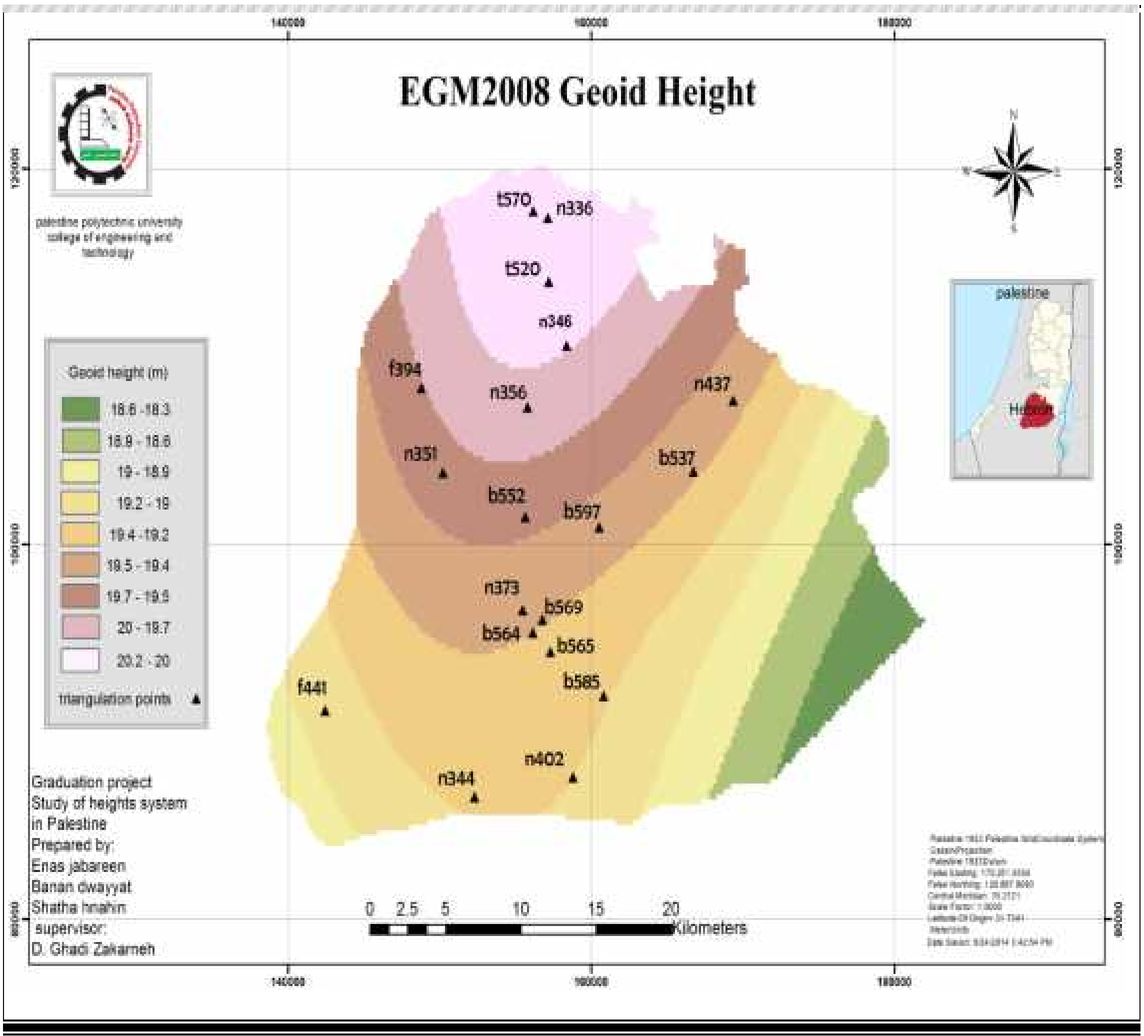


Fig 5-13: geoid height (m) in hebron district using EGM2008

5-4-6 EGM96

Using EGM96 geoid model, grid of points in Hebron district was get, the calculated grids were in ASCII format. These grids converted to raster format in ARCGIS as shown in figure (5-14) .[19]

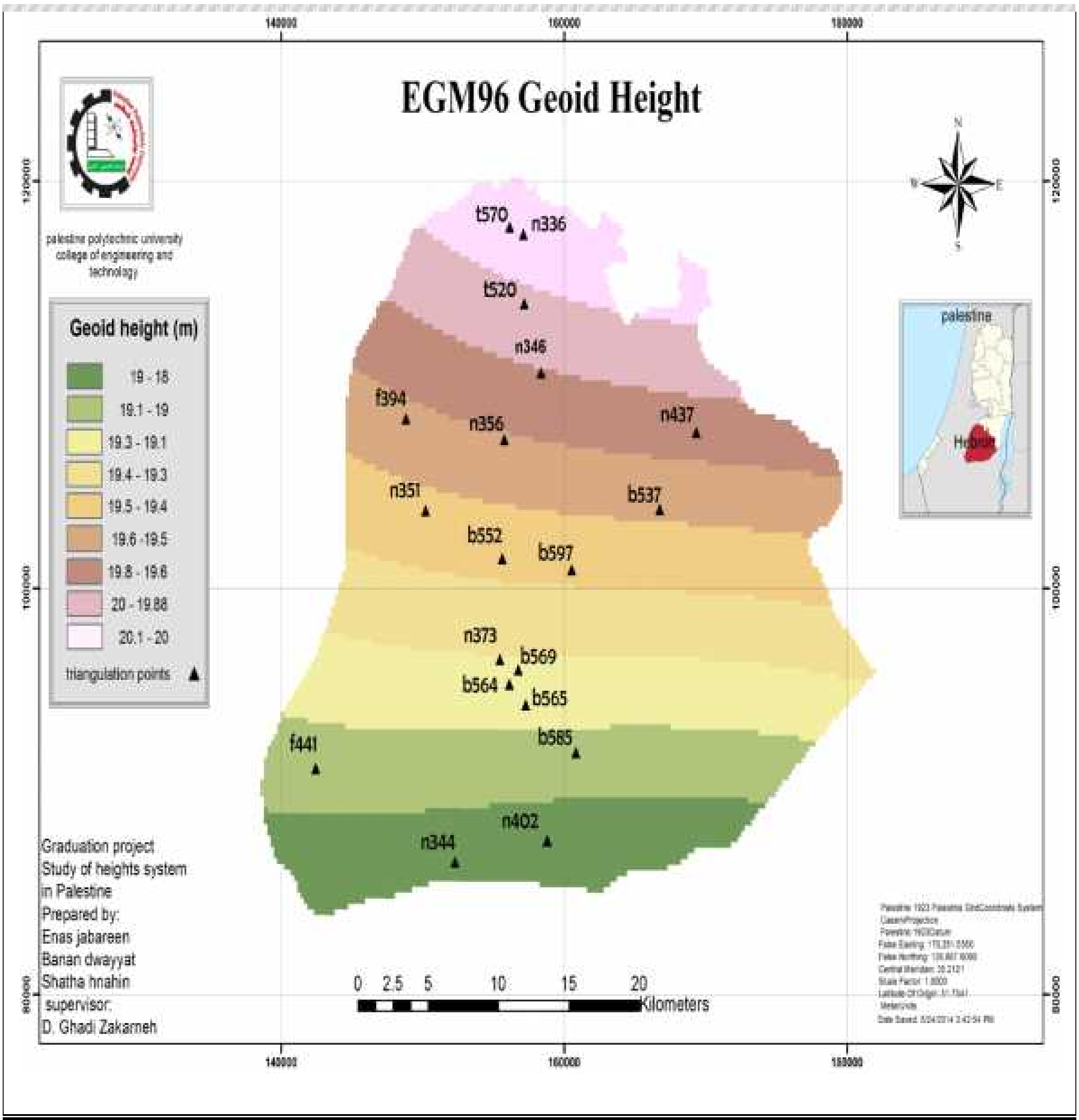


Fig 5-14: geoid height (m) in hebron district using EGM٩٦

CHAPTER SIX

GEIOD MODEL EVALUATION

6-1 Introduction

In this chapter, comparison between different Geoid Models are done. Here , compare each model by using the Triangulation point with known ellipsoidal and orthometric height ,The difference between both height is the Geoid undulation , see figure (6-1)

$$N = h - H_{Orthometric} \quad (6\text{-}1)$$

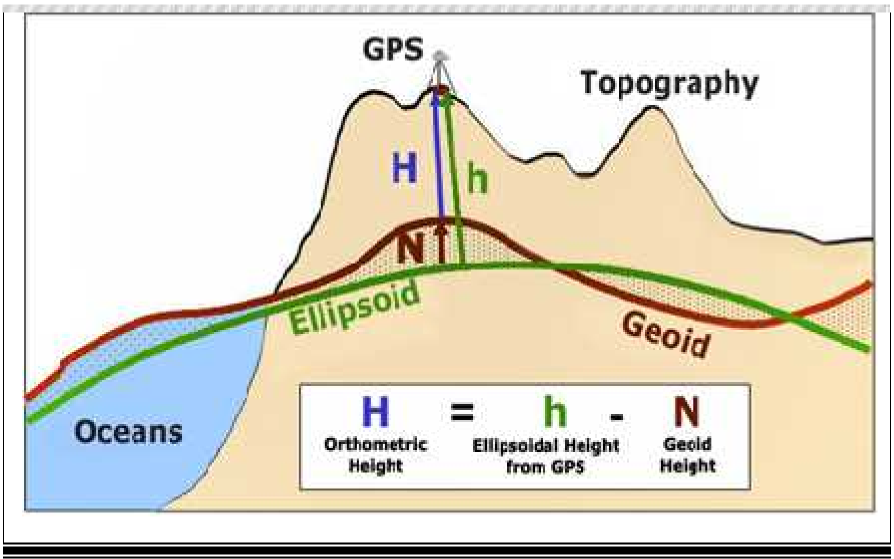


Figure (6-1) difference height between ellipsoid geoid [13]

These Geoid undulations are compare to the Geoid undulation from different Geoid models. The test and result are discussed in the following sections[19] .

6-2 EIGEN-5C

The EIGEN-5C Geoid height were compared to the geoid height of the control point (triangulation point) .The orthometric heights of the control point were calculated again using EIGEN-5C Geoid model as shown in table (6-1) and equation (6- 2) .

$$H_{Orthometric} = h_{WGS} - N_{EIGEN\text{-}5C} \tag{6-2}$$

Table(6.1):The Orthometric Height of EIGEN-5C

Trigs_name	hight	N_Geoid	H=h-N
b585	813.313	18.132515	795.180485
b569	829.664	18.293547	811.400453
b565	793.07	18.356758	774.713242
b564	793.202	18.374723	774.827277
f394	525.871	18.986366	506.884634
n344	634.002	18.103819	615.898181
n346	1001.763	19.000393	982.762706
n336	658.207	19.310026	638.896974
t570	609.623	19.327652	590.295348
b597	921.66	18.533855	903.126145
n373	758.47	18.419035	740.050965
f441	661.38	18.501923	642.878077
t520	868.75	19.1604	849.5896
n356	895.15	18.877272	876.272728
b552	933.5	18.626276	914.873724
n351	748.9	18.798239	730.101761
n402	814.76	17.994505	796.765495
n437	843.09	18.783781	824.306219

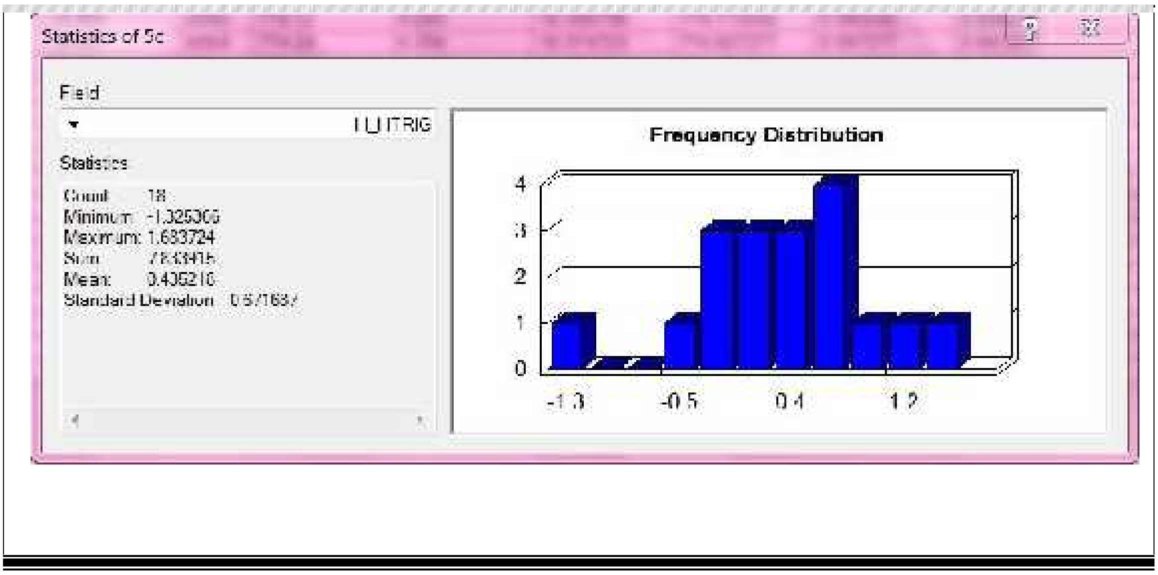
The difference between the official othometric heights and the orthometric heights calculate from EIGEN-5C were used to evaluation this model . the deifference are calculated give :

$$H_{\text{EIGEN-5C}} - H_{Trig} = N - N_{\text{EIGEN-5C}} = \Delta N \quad (6\text{-}3)$$

Table(6.2): Orthomatric height a given Form the official height

name	H	hight_trig	H_HTRIGS
b585	795.180485	794.32	0.860485
b569	811.400453	810.69	0.710453
b565	774.713242	774.12	0.593242
b564	774.827277	774.24	0.587277
f394	506.884634	508.21	1.325366-
n344	615.898181	614.98	0.918181
n346	982.762706	982.75	0.012607
n336	638.896974	638.89	0.006974
t570	590.295348	588.94	1.355348
b597	903.126145	902.79	0.336145
n373	740.050965	739.5	0.550965
f441	642.878077	643.29	0.411923-
t520	849.5896	849.42	0.1696
n356	876.272728	875.27	1.002728
b552	914.873724	913.19	1.683724
n351	730.101761	730.17	0.068239-
n402	796.765495	796.08	0.685495
n437	824.306219	824.2	0.106219

The statistics Orthomatric height a given Form the official height of EIGEN 5C show in Figure (6-2).



Fgure (6-2) ststistics Orthomatric height agiven Form the official height of EIGEN 5C

Table(6.3): Orthomatric height For GNSS measured orthometric height

name	Hi	N_Geoid	Hi_N
b585	18.944	18.132515	0.811485
b569	19.14	18.293547	0.846453
b565	19.193	18.356758	0.836242
b564	19.216	18.374723	0.841277
f394	19.267	18.986366	0.280634
n344	19.441	18.103819	1.337181
n346	19.714	19.000393	0.713607
n336	19.756	19.310026	0.445974
t570	19.722	19.327652	0.394348
b597	20.29	18.533855	1.756145
n373	20.29	18.419035	1.870965
f441	20.34	18.501923	1.838077
t520	20.34	19.1604	1.1796
n356	20.32	18.877272	1.442728
b552	20.31	18.626276	1.683724
n351	20.34	18.798239	1.541761
n402	20.26	17.994505	2.265495
n437	20.27	18.783781	1.486217

The statistics Orthomatric height For GNSS measured orthometric height of EIGEN 5C show in Figure (6-3).

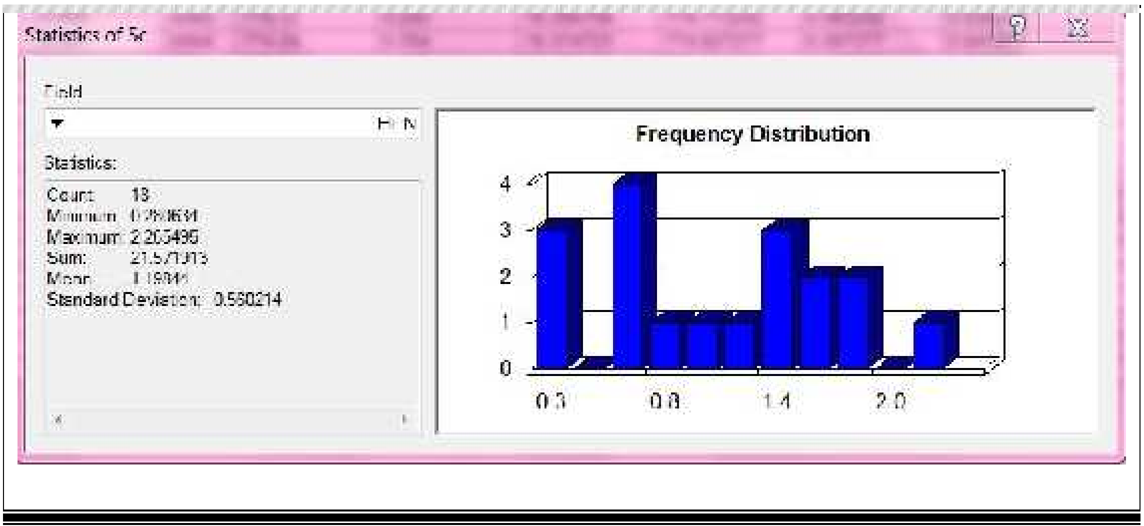


Figure (6-3) statistics Orthomatric height For GNSS measured orthometric height of EIGEN 5C

The calculated differences of the EIGEN-5C Geoid were test against blunders at 95% probability as given in equation (6-4) .

$$\bar{X} + 1.96 * \delta < \Delta N < \bar{X} + 1.96 * \delta \qquad (6\text{-}4)$$

For the official height

$-0.8812 < \Delta N < 1.751$

For GNSS measured orthometric height

$-0.1004 < \Delta N <$

These result shown that no point were considered as blunders , and the final accuracy of the model can be calculated using RMSE without using the blunders as shown in equation (6-5).

$$RMSE = \sqrt[2]{\frac{\Sigma\Delta N^2}{n}} \qquad (6\text{-}5)$$

For the official height

$$RMSE = .758241$$

These result shown that two point were considered as blunders , and the final accuracy of the model can be calculated using RMSE without using the blunders as shown in equation (6-5).

For GNSS measured orthometric height

$$RMSE = 1.32291$$

6-3 EIGEN-5S

The EIGEN-5S Geoid height were compared to the geoid height of the control point (triangulation point) .

The orthometric heights of the control point were calculated again using EIGEN-5S Geoid model as shown in table (6.4) and equation (6- 6)

$$H_{\text{Orthometric}} = h_{\text{WGS}} - N_{\text{EIGEN-5S}} \qquad (6\text{-}6)$$

Table(6.4):The Orthometric Height of EIGEN-5S

Trigs_name	hight	N_Geoid	H=h-N
b585	813.313	18.775126	794.537874
b569	829.664	18.770655	810.893345
b565	793.07	18.772383	774.297617
b564	793.202	18.789663	774.412337
f394	525.871	18.882765	506.988235
n344	634.002	18.614182	615.387819
n346	1001.763	19.043659	982.719341
n336	658.207	19.146143	639.060857
t570	609.623	19.138693	590.484307
b597	921.66	18.908918	902.751082
n373	758.47	18.782393	739.687607
f441	661.38	18.570412	642.809588
t520	868.75	19.086803	849.663197
n356	895.15	18.954815	876.195185
b552	933.5	18.859543	914.640457
n351	748.9	18.830055	730.069945
n402	814.76	18.69404	796.06596
n437	843.09	19.13353	823.95647

The difference between the official othometric heights and the orthometric heights calculate from EIGEN-5S were used to evaluation this model . the deifference are calculated give :

$$H_{\text{EIGEN-5S}} - H_{Trig} = N - N_{\text{EIGEN-5S}} = \Delta N \qquad (6\text{-}7)$$

Table(6.5): Orthomatric height a given Form the official height

name	H	hight_trig	H_Htrig
b585	794.537874	794.32	0.217874
b569	810.893345	810.69	0.203345
b565	774.297617	774.12	0.177617
b564	774.412337	774.24	0.172337
f394	506.988235	508.21	1.221765-
n344	615.387819	614.98	0.407818
n346	982.719341	982.75	0.030659-
n336	639.060857	638.89	0.170857
t570	590.484307	588.94	1.544307
b597	902.751082	902.79	0.038918-
n373	739.687607	739.5	0.187607
f441	642.809588	643.29	0.480412-
t520	849.663197	849.42	0.243197
n356	876.195185	875.27	0.925185
b552	914.640457	913.19	1.450457
n351	730.069945	730.17	0.100055-
n402	796.06596	796.08	0.01404-
n437	823.95647	824.2	0.24353

The statistics Orthomatric height a given Form the official height of EIGEN 5S show in Figure (6-4) .

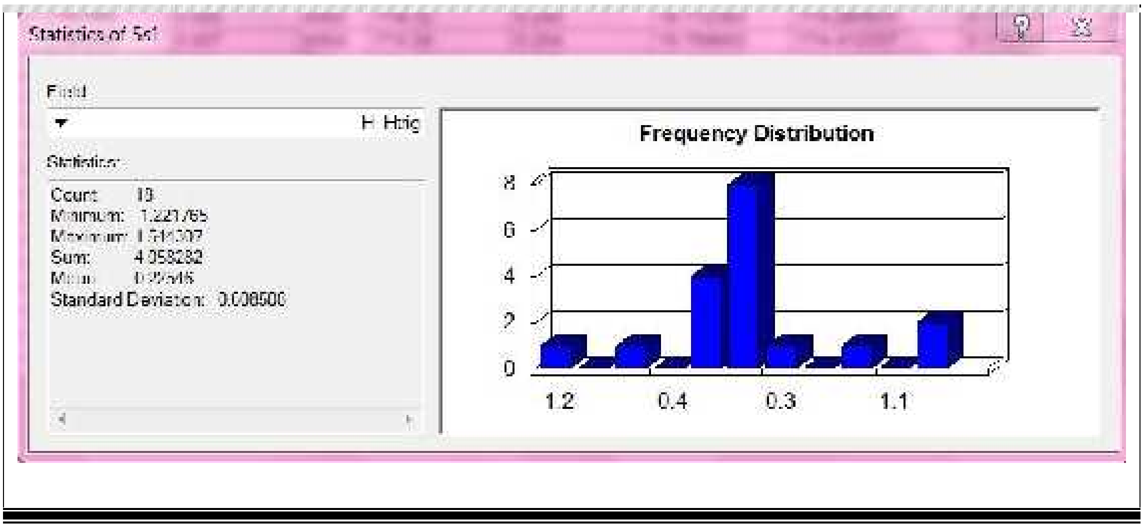


Figure (6-4) statistics Orthomatric height a given Form the official height of EIGEN 5S

Table(6.6): Orthomatric height For GNSS measured orthometric height

name	Hi	N_Geoid	Hi_N
b585	18.944	18.775126	0.168874-
b569	19.14	18.770655	0.369345-
b565	19.193	18.772383	0.420617-
b564	19.216	18.789663	0.426337-
f394	19.267	18.882765	0.384235-
n344	19.441	18.614182	0.826818-
n346	19.714	19.043659	0.670341-
n336	19.756	19.146143	0.609857-
t570	19.722	19.138693	0.583307-
b597	20.29	18.908918	1.381082-
n373	20.29	18.782393	1.507607-
f441	20.34	18.570412	1.769588-
t520	20.34	19.086803	1.253197-
n356	20.32	18.954815	1.365185-
b552	20.31	18.859543	1.450457-
n351	20.34	18.830055	1.509945-
n402	20.26	18.69404	1.56596-
n437	20.27	19.13353	1.13647-

The statistics Orthomatric height For GNSS measured orthometric height of EIGEN 5S show in Figure (6-5).

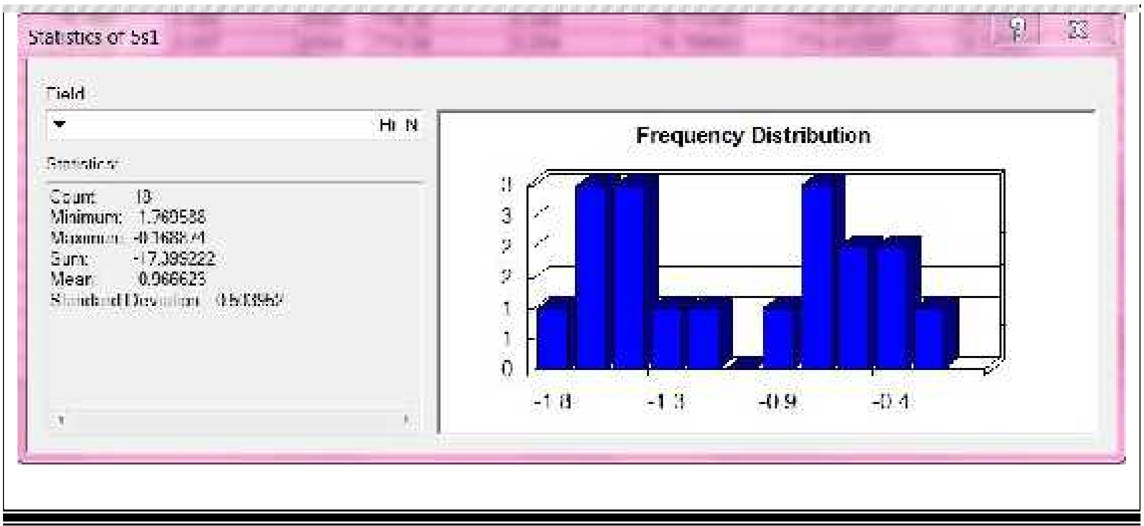


Figure (6-5) statistics Orthomatric height For GNSS measured orthometric height of EIGEN 5S

The calculated differences of the EIGEN-5S Geoid were test against blunders at 95% probability as given in equation (6-8) .

$$\bar{X} + 1.96 * \delta < \Delta N < \bar{X} + 1.96 * \delta \qquad (6\text{-}8)$$

For the official height

$$-0.9672 < \Delta N < 1.418$$

For GNSS measured orthometric height

$$-1.7132 < \Delta N <$$

$$RMSE = \sqrt[2]{\frac{\Sigma \Delta N^2}{n}} \qquad (6\text{-}9)$$

These result shown that two point were considered as blunders , and the final accuracy of the model can be calculated using RMSE without using the blunders as shown in equation (6-9).

For the official height

$$RMSE = .3261279$$

These result shown that two point were considered as blunders , and the final accuracy of the model can be calculated using RMSE without using the blunders as shown in equation (6-9).

For GNSS measured orthometric height

$$RMSE = 1.035897$$

6-4 EIGEN-6C3STAT

The EIGEN-6C3STAT Geoid height were compared to the geoid height of the control point (triangulation point) .

The orthometric heights of the control point were calculated again using EIGEN-6C3STAT Geoid model as shown in table (6.7) and equation (6- 10)

$$H_{\text{Orthometric}} = h_{\text{WGS}} - \text{N}_{\text{EIGEN-6C3STAT}} \qquad (6\text{- }10)$$

Table(6.7):The Orthometric Height of EIGEN-6C3STAT

Trigs_name	hight	N_Geoid	H=h-N
b585	813.313	19.318407	793.994593
b569	829.664	19.43137	810.23263
b565	793.07	19.470251	773.599749
b564	793.202	19.482353	773.719647
f394	525.871	19.836933	506.034067
n344	634.002	19.300497	614.701503
n346	1001.763	20.068209	981.694791
n336	658.207	20.279427	637.927573
t570	609.623	20.27964	589.34336
b597	921.66	19.584522	902.075478
n373	758.47	19.510424	738.959576
f441	661.38	19.211729	642.168271
t520	868.75	20.197002	848.552998
n356	895.15	19.948936	875.201064
b552	933.5	19.690117	913.809883
n351	748.9	19.741003	729.158997
n402	814.76	19.284721	795.475279
n437	843.09	19.519802	823.570198

The difference between the official othometric heights and the orthometric heights calculate from EIGEN-6C3STAT were used to evaluation this model . the difference are calculated give :

$$H_{\text{EIGEN-6C3STAT}} - H_{Trig} = N - N_{\text{EIGEN-6C3STAT}} = \Delta N \qquad (6\text{-}11)$$

Table(6.8): Orthomatric height a given Form the official height

name	H	hight_trig	H_htrigs
b585	793.994593	794.32	0.325407-
b569	810.23263	810.69	0.45737-
b565	773.599749	774.12	0.520251-
b564	773.719647	774.24	0.520353-
f394	506.034067	508.21	2.175933-
n344	614.701503	614.98	0.278497-
n346	981.694791	982.75	1.055209-
n336	637.927573	638.89	0.962427-
t570	589.34336	588.94	0.40336
b597	902.075478	902.79	0.714522-
n373	738.959576	739.5	0.540424-
f441	642.168271	643.29	1.121729-
t520	848.552998	849.42	0.867002-
n356	875.201064	875.27	0.068936-
b552	913.809883	913.19	0.619883
n351	729.158997	730.17	1.011003-
n402	795.475279	796.08	0.604721-
n437	823.570198	824.2	0.629802-

The ststistics Orthomatric height agiven Form the official height of EIGEN-6C3 STAT show in figure (6-6) .

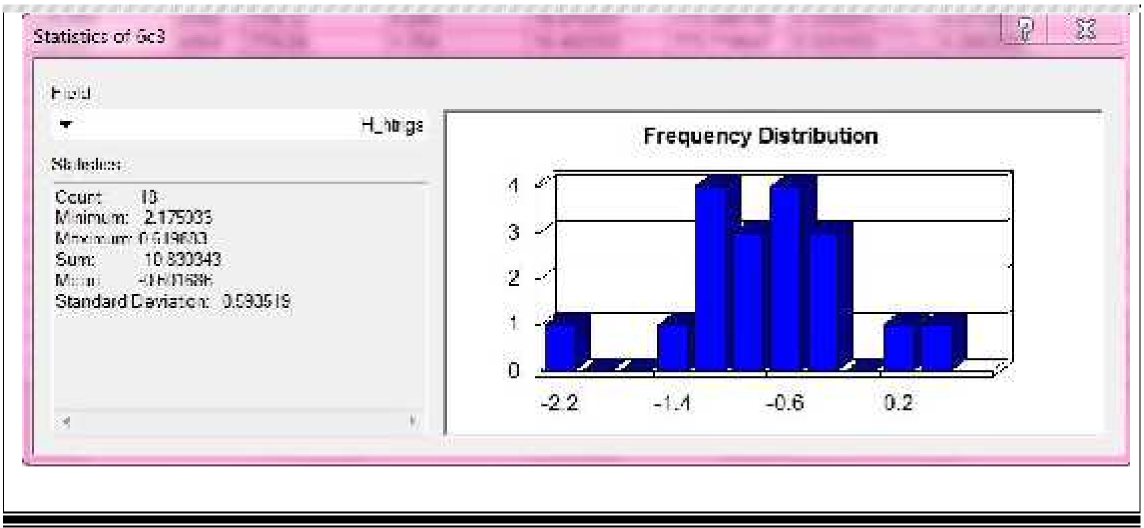


Figure (6-6) statistics Orthomatric height a given Form the official height of EIGEN-6C3STAT

Table(6.9): Orthomatric height For GNSS measured orthometric height

name	Hi	N_Geoid	Hi_N
b585	18.944	19.318407	0.374407-
b569	19.14	19.43137	0.29137-
b565	19.193	19.470251	0.277251-
b564	19.216	19.482353	0.266353-
f394	19.267	19.836933	0.569933-
n344	19.441	19.300497	0.569933-
n346	19.714	20.068209	0.140503
n336	19.756	20.279427	0.354209-
t570	19.722	20.27964	0.523427-
b597	20.29	19.584522	0.55764-
n373	20.29	19.510424	0.705478
f441	20.34	19.211729	0.779576
t520	20.34	20.197002	1.128271
n356	20.32	19.948936	0.142998
b552	20.31	19.690117	0.371064
n351	20.34	19.741003	0.619883
n402	20.26	19.284721	0.598997
n437	20.27	19.519802	0.975279

The statistics Orthomatric height For GNSS measured orthometric height of EIGEN-6C3STAT show in Figure (6-7).

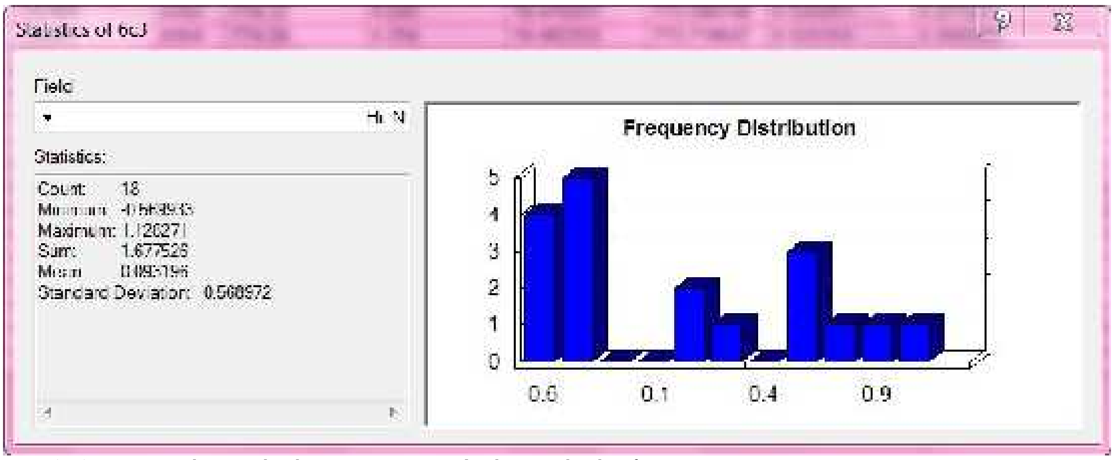


Figure (6-7) statistics Orthomatric height For GNSS measured orthometric height of EIGEN-6C3STAT

The calculated differences of the EIGEN-6C3STAT Geoid were test against blunders at 95% probability as given in elation (6-12) .

$$\bar{X} + 1.96 * \delta < \Delta N < \bar{X} + 1.96 * \delta \qquad (6\text{-} 12)$$

For the official height

$-1.761 < \Delta N < 0.5616$

For GNSS measured orthometric height

$-1.021 < \Delta N <$

$$RMSE = \sqrt[2]{\frac{\Sigma \Delta N^2}{n}} \qquad (6\text{-}13)$$

These result shown that two point were considered as blunders , and the final accuracy of the model can be calculated using RMSE without using the blunders as shown in equation (6-13).

For the official height

$RMSE = 0.69116$

These result shown that two point were considered as blunders , and the final accuracy of the model can be calculated using RMSE without using the blunders as shown in equation (6-13).

For GNSS measured orthometric height

$RMSE = 0.526322315$

6-5 EIGEN-6S

The EIGEN-6S Geoid height were compared to the geoid height of the control point (triangulation point) .The orthometric heights of the control point were calculated again using EIGEN-6S Geoid model as shown in table (6.10) and equation (6- 14).

$$H_{Orthometric} = h_{WGS} - N_{EIGEN-6S} \qquad (6\text{- }14)$$

Table(6.10):The Orthometric Height of EIGEN-6C3STAT

Trigs_name	hight	N_Geoid	H=h-N
b585	813.313	18.132515	795.180485
b569	829.664	18.293547	811.370453
b565	793.07	18.356758	774.713242
b564	793.202	18.374723	774.827277
f394	525.871	18.986366	506.884634
n344	634.002	18.103819	615.898181
n346	1001.763	19.000393	982.762607
n336	658.207	19.310026	638.896974
t570	609.623	19.327652	590.385348
b597	921.66	18.533855	903.126145
n373	758.47	18.419035	740.050965
f441	661.38	18.501923	642.878077
t520	868.75	19.1604	849.5896
n356	895.15	18.877272	876.272728
b552	933.5	18.626276	914.873724
n351	748.9	18.798239	730.101761
n402	814.76	17.994505	796.765495
n437	843.09	18.783781	824.306219

The difference between the official othometric heights and the orthometric heights calculate from EIGEN-6S were used to evaluation this model . the difference are calculated give :

$$H_{EIGEN-6S} - H_{Trig} = N - N_{EIGEN-6S} = \Delta N \qquad (6\text{-}15)$$

Table(6.11): Orthomatric height a given Form the official height

name	H	hight_trig	H_HTRIGS
b585	795.180485	794.32	0.860485
b569	811.400453	810.69	0.710453
b565	774.713242	774.12	0.593242
b564	774.827277	774.24	0.587277
f394	506.884634	508.21	1.325366-
n344	615.898181	614.98	0.918181
n346	982.762706	982.75	0.012607
n336	638.896974	638.89	0.006974
t570	590.295348	588.94	1.355348
b597	903.126145	902.79	0.336145
n373	740.050965	739.5	0.550965
f441	642.878077	643.29	0.411923-
t520	849.5896	849.42	0.1696
n356	876.272728	875.27	1.002728
b552	914.873724	913.19	1.683724
n351	730.101761	730.17	0.068239-
n402	796.765495	796.08	0.685495
n437	824.306219	824.2	0.106219

The ststistics Orthomatric height agiven Form the official height of EIGEN-6S show in figure (6-8) .

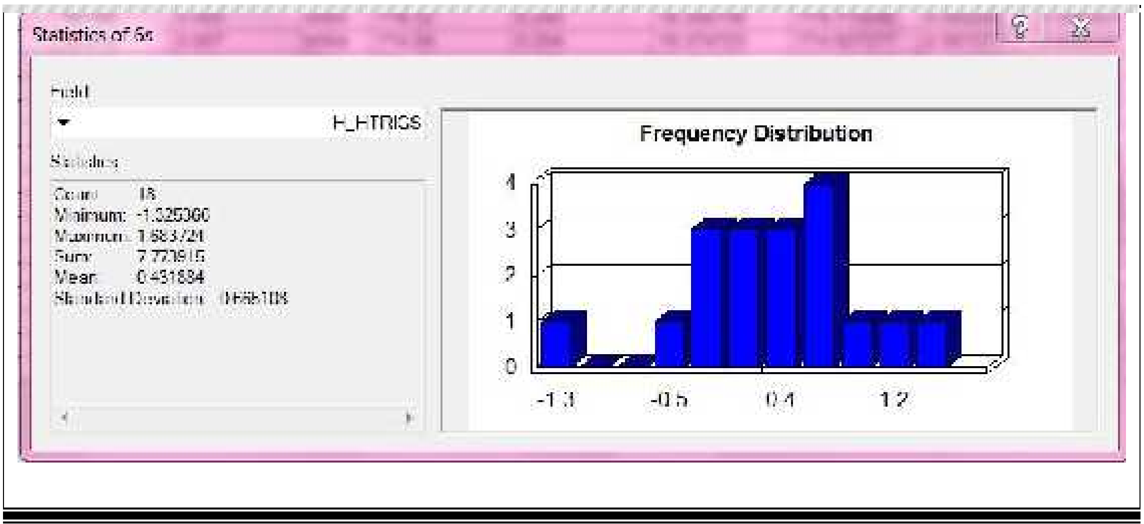


Figure (6-8) statistics Orthomatric height given Form the official height of EIGEN-6S

Table(6.12): Orthomatric height For GNSS measured orthometric height

name	Hi	N_Geoid	Hi_N
b585	18.944	18.132515	1.486219
b569	19.14	18.293547	0.811485
b565	19.193	18.356758	0.846453
b564	19.216	18.374723	0.836242
f394	19.267	18.986366	0.841277
n344	19.441	18.103819	0.280634
n346	19.714	19.000393	1.337181
n336	19.756	19.310026	0.713607
t570	19.722	19.327652	0.445974
b597	20.29	18.533855	0.394348
n373	20.29	18.419035	1.756145
f441	20.34	18.501923	1.870965
t520	20.34	19.1604	1.838077
n356	20.32	18.877272	1.1796
b552	20.31	18.626276	1.442728
n351	20.34	18.798239	1.683724
n402	20.26	17.994505	1.541761
n437	20.27	18.783781	2.265495

The statistics Orthomatric height For GNSS measured orthometric height of EIGEN-6S show in Figure (6-9).

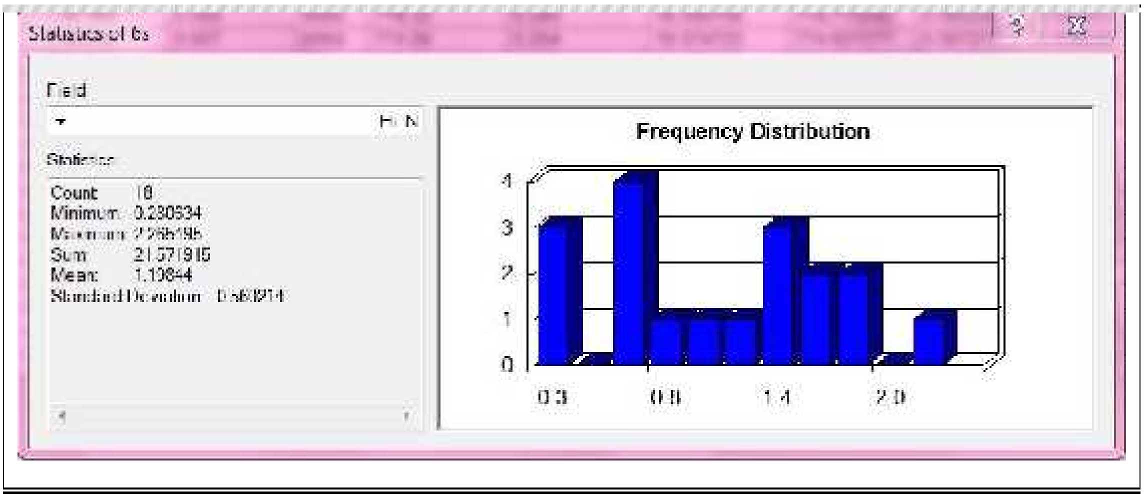


Figure (6-9) statistics Orthomatric height For GNSS measured orthometric height of EIGEN-6S

The calculated differences of the EIGEN-6C3STAT Geoid were test against blunders at 95% probability as given in equation (6-17)

$$\bar{X} + 1.96 * \delta < \Delta N < \bar{X} + 1.96 * \delta \quad (6\text{-}17)$$

For the official height

$$-0.8717 < \Delta N <$$

For GNSS measured orthometric height

$$-0.8412 < \Delta N <$$

$$RMSE = \sqrt[2]{\frac{\Sigma \Delta N^2}{n}} \quad (6\text{-}18)$$

These result shown that two point were considered as blunders , and the final accuracy of the model can be calculated using RMSE without using the blunders as shown in equation (6-18).

For the official height

$$RMSE = 0.50585$$

These result shown that no point were considered as blunders , and the final accuracy of the model can be calculated using RMSE without using the blunders as shown in equation (6-18).

For GNSS measured orthometric height

$$RMSE = 0.841277$$

6-6 EGM2008

The EGM2008 Geoid height were compared to the geoid height of the control point (triangulation point) .

The orthometric heights of the control point were calculated again using EGM2008 Geoid model as shown in table (6.13) and equation (6- 19)

$$H_{\text{Orthometric}} = h_{\text{WGS}} - \text{N}_{\text{EGM2008}} \quad (6\text{- }19)$$

Table(6.13):The Orthometric Height of EGM2008

Trigs_name	hight	N_Geoid	H=h-N
b585	813.313	19.254026	794.058974
b569	829.664	19.359028	810.304972
b565	793.07	19.40085	773.66915
b564	793.202	19.410337	773.791663
f394	525.871	19.775673	506.095327
n344	634.002	19.264757	614.737243
n346	1001.763	20.009888	981.753112
n336	658.207	20.206238	638.000762
t570	609.623	20.205948	589.417052
b597	921.66	19.506178	902.105382
n373	758.47	19.444736	739.025264
f441	661.38	19.153067	642.226933
t520	868.75	20.133989	848.616011
n356	895.15	19.915606	875.234394
b552	933.5	19.642941	913.878571
n351	748.9	19.700756	729.199244
n402	814.76	19.24255	795.51745
n437	843.09	19.456825	823.633175

The difference between the official othometric heights and the orthometric heights calculate from EGM2008 were used to evaluation this model . the difference are calculated give :

$$H_{\text{EGM2008}} - H_{Trig} = N - N_{\text{EGM2008}} = \Delta N \quad (6\text{-}20)$$

Table(6.14): Orthomatric height a given Form the official height

name	H	hight_trig	H_Htrig
b585	794.058974	794.32	0.261026-
b569	810.304972	810.69	0.385028-
b565	773.66915	774.12	0.45085-
b564	773.791663	774.24	0.448337-
f394	506.095327	508.21	2.114673-
n344	614.737243	614.98	0.242757-
n346	981.753112	982.75	0.996888-
n336	638.000762	638.89	0.889238-
t570	589.417052	588.94	0.477052
b597	902.105382	902.79	0.636178-
n373	739.025264	739.5	0.474736-
f441	642.226933	643.29	1.063067-
t520	848.616011	849.42	0.803989-
n356	875.234394	875.27	0.035606-
b552	913.878571	913.19	0.667059-
n351	729.199244	730.17	0.970756-
n402	795.51745	796.08	0.56255-
n437	823.633175	824.2	0.566825-

The statistics Orthomatric height agiven Form the official height of EGM2008 show in Figure (6-10).

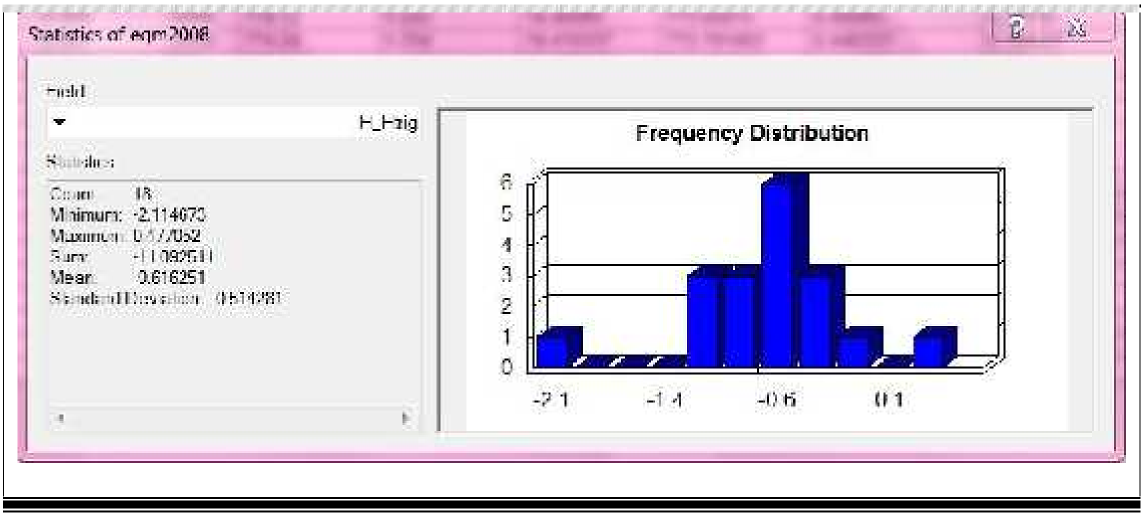


Figure (6-10) ststistics Orthomatric height agiven Form the official height of EGM2008

Table(6.10): Orthomatric height For GNSS measured orthometric height

name	Hi	N_Geoid	Hi_N
b585	18.944	19.254026	0.310026
b569	19.14	19.359028	0.219028
b565	19.193	19.40085	0.20785
b564	19.216	19.410337	0.194337
f394	19.267	19.775673	0.508673
n344	19.441	19.264757	0.176243-
n346	19.714	20.009888	0.295888
n336	19.756	20.206238	0.450238
t570	19.722	20.205948	0.483948
b597	20.29	19.506178	0.78382-
n373	20.29	19.444736	0.845264-
f441	20.34	19.153067	1.186933-
t520	20.34	20.133989	0.206011-
n356	20.32	19.915606	0.404394-
b552	20.31	19.642941	0.667059-
n351	20.34	19.700756	0.639244-
n402	20.26	19.24255	1.01745-
n437	20.27	19.456825	0.813175-

The statistics Orthomatric height For GNSS measured orthometric height of EGM2008 show in Figure (6-11).

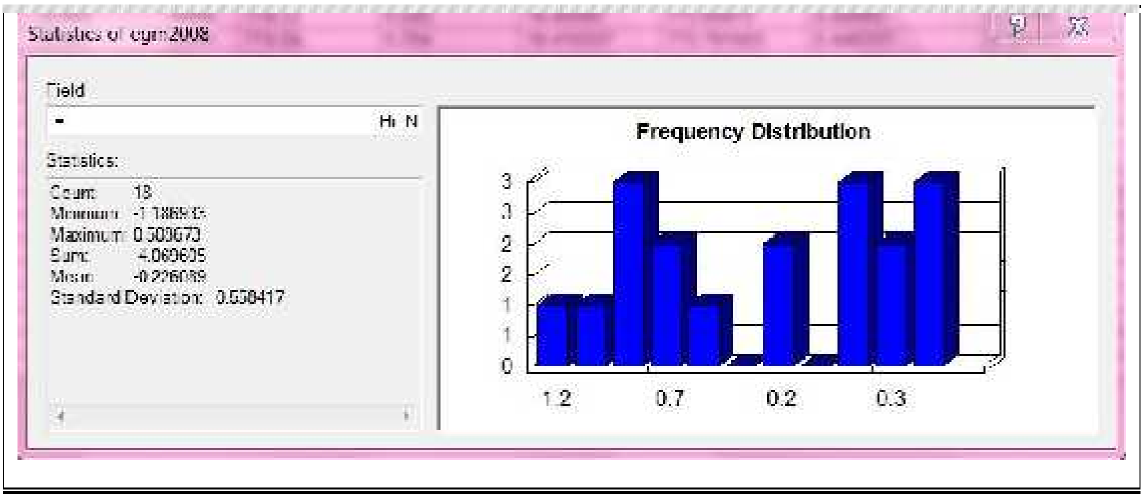


Figure (6-11) **statistics Orthomatric height For GNSS measured orthometric** height of EGM2008

The calculated differences of the EGM2008 Geoid were test against blunders at 95% probability as given in equation (6-21).

$$\bar{X} + 1.96 * \delta < \Delta N < \bar{X} + 1.96 * \delta \qquad (6\text{- }21)$$

For the official height

$$-1.624 < \Delta N <$$

For GNSS measured orthometric height

$$-1.3205 < \Delta N <$$

These result shown that two point were considered as blunders , and the final accuracy of the model can be calculated using RMSE without using the blunders as shown in equation (6-22)

$$RMSE = \sqrt[2]{\frac{\Sigma \Delta N^2}{n}} \qquad (6\text{-}22)$$

For the official height

$$RMSE = 0.65610$$

These result shown that no point were considered as blunders , and the final accuracy of the model can be calculated using RMSE without using the blunders as shown in equation (6-22).

For GNSS measured orthometric height

$$RMSE = 0.59314$$

6-7 EGM96

The EGM96 Geoid height were compared to the geoid height of the control point (triangulation point) .

The orthometric heights of the control point were calculated again using EGM2008 Geoid model as shown in table (6.16) and equation (6- 23)

$$H_{\text{Orthometric}} = h_{\text{WGS}} - \text{N}_{\text{EGM96}} \qquad (6\text{- }23)$$

Table(6.16):The Orthometric Height of EGM96

Trigs_name	hight	N_Geoid	H=h-N
b585	813.313	19.124537	794.188463
b569	829.664	19.210016	810.453984
b565	793.07	19.245031	773.824969
b564	793.202	19.269478	773.932522
f394	525.871	19.647541	506.223459
n344	634.002	18.964725	615.037275
n346	1001.763	19.802525	981.960475
n336	658.207	20.038343	638.168657
t570	609.623	20.040045	589.582955
b597	921.66	19.451006	902.208994
n373	758.47	19.286259	739.183741
f441	661.38	19.10671	642.27329
t520	868.75	19.91721	848.83279
n356	895.15	19.667326	875.482674
b552	933.5	19.459812	914.040188
n351	748.9	19.514032	729.385968
n402	814.76	18.980248	795.779752
n437	843.09	19.736126	823.353874

The difference between the official othometric heights and the orthometric heights calculate from EGM96 were used to evaluation this model . the difference are calculated give :

$$H_{\text{EGM96}} - H_{Trig} = N - N_{\text{EGM96}} = \Delta N \qquad (6\text{-}24)$$

Table(6.17): Orthomatric height a given Form the official height

name	H	hight_trig	H_Htrig
b585	794.188463	794.32	0.131573-
b569	810.453984	810.69	0.236016-
b565	773.824969	774.12	0.295031-
b564	773.932522	774.24	0.307478-
f394	506.223459	508.21	1.986541-
n344	615.037275	614.98	0.057275
n346	981.960475	982.75	0.789525-
n336	638.168657	638.89	0.721343-
t570	589.582955	588.94	0.642955
b597	902.208994	902.79	0.581066-
n373	739.183741	739.5	0.316259-
f441	642.27329	643.29	1.01671-
t520	848.83279	849.42	0.58721-
n356	875.482674	875.27	0.212674
b552	914.040188	913.19	0.850188
n351	729.385968	730.17	0.784032-
n402	795.779752	796.08	0.300248-
n437	823.353874	824.2	0.846126-

The ststistics Orthomatric height agiven Form the official height of EGM96 show in Figure (6-12).

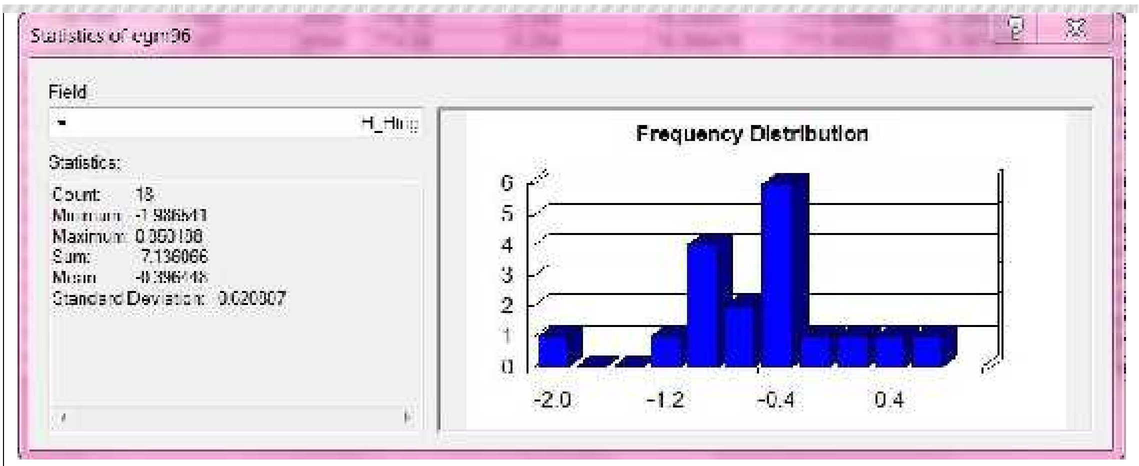


Figure (6-12) ststistics Orthomatric height agiven Form the official height of EGM96

Table(6.18): Orthomatric height For GNSS measured orthometric height

name	HI	N_Geoid	Hi_N
b585	18.944	19.124537	0.180537
b569	19.14	19.210016	0.070016
b565	19.193	19.245031	0.052031
b564	19.216	19.269478	0.053478
f394	19.267	19.647541	0.380541
n344	19.441	18.964725	0.476275-
n346	19.714	19.802525	0.088525
n336	19.756	20.038343	0.282343
t570	19.722	20.040045	0.318045
b597	20.29	19.451006	0.838994-
n373	20.29	19.286259	1.003741-
f441	20.34	19.10671	1.23329-
t520	20.34	19.91721	0.42279-
n356	20.32	19.667326	0.652674-
b552	20.31	19.459812	0.850188-
n351	20.34	19.514032	0.825958-
n402	20.26	18.980248	1.279752-
n437	20.27	19.736126	0.533874-

The statistics Orthomatric height For GNSS measured orthometric height of EGM96 show in Figure (6-1٣).

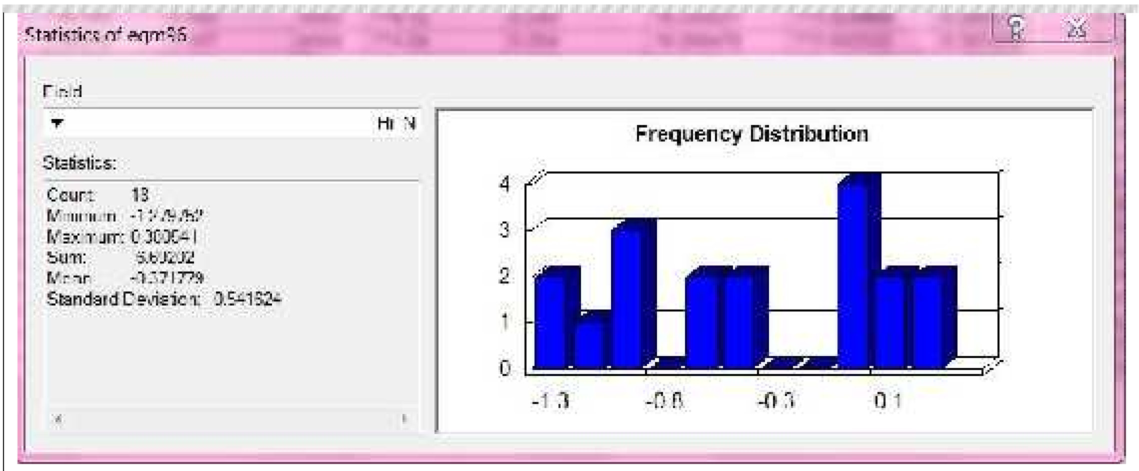


Figure (6-1٣) **statistics Orthomatric height For GNSS measured orthometric** height of EGM96

The calculated differences of the EGM2008 Geoid were test against blunders at 95% probability as given in equation (6-25) .

$$\bar{X} + 1.96 * \delta < \Delta N < \bar{X} + 1.96 * \delta \quad (6\text{- }25)$$

For the official height

$$-1.6132 < \Delta N <$$

For GNSS measured orthometric height

$$-1.440 < \Delta N <$$

$$RMSE = \sqrt[2]{\frac{\Sigma \Delta N^2}{n}} \quad (6\text{-}26)$$

These result shown that two point were considered as blunders , and the final accuracy of the model can be calculated using RMSE without using the blunders as shown in equation (6-26).

For the official height

$$RMSE = 0.71435$$

These result shown that no point were considered as blunders , and the final accuracy of the model can be calculated using RMSE without using the blunders as shown in equation (6-26).

For GNSS measured orthometric height

$$RMSE = 0.65646$$

CHAPTER SEVEN
CONCLUSION AND RESULTS

7-1 Conclusion

The accuracy of several geoid models were examined based on heights that were given by Palestine trigs according to the department of survey with their measured ellipsoidal height, these points were used to test the original orthometric heights and GNSS measured orthomatric height .

The final accuracy of the models is represented by the RMSE in table (7.1) far the official height evaluation and in table (7.2) for GNSS measured orthometric heights.

`

Table(7.1): accuracy of the model far the official height evaluation

Geoid Model	RMSE
EIGEN 6c3	
EIGEN 5c	
EIGEN 6s	
EIGEN 5s	
EGM96	
EGM2008	

Table(7.2): accuracy of the model far GNSS measured orthometric height

Geoid Model	RMSE
EIGEN 6c3	.52632315
EIGEN 5c	1.322912391
EIGEN 6s	.841277
EIGEN 5s	1.035897139
EGM96	.65646215
EGM2008	.593148087

It is clear that the best results came from (EIGIN 5S) model in came of official heights of the triangulation points, In the case of GNSS measured orthometric height the best results come from (EIGIN6C3STAT) model.

7-2 Recommendatin

Using our results, introduce the following recommendation:

The triangulation points should be studied according to their height accuracy and measurement methods (barometric leveling , triangulatur leveling or precise levelling), it recommended to find the location of the Benchmarks in Palestine if not available , new Benchmarks should be located .

One Geoid Model should be selected as an reference surface to be used directly or after some modification to fit the local Benchmarks.

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