بسم اله الرحمن الرحيم

## Palestine Polytechnic University



College of Engineering \&Technology
Mechanical Engineering department
Mechatronics Engineering

Graduation Project

# Rehabilitation and Control of Articulated Hydraulic Manipulator 

## Project Team

Majid Alhroosh
Mujahed Masharqa

Project Supervisor
Dr.Yousef Alswati

Hebron-Palestine

## CERTIFICATION

## Palestine Polytechnic University

## PPU

## Hebron-Palestine

# Rehabilitation and Control of Articulated Hydraulic Manipulator 

## Prepared By

Majid Alhroosh Mujahed Masharqa

In accordance with the recommendations of the project supervisor, and the acceptance of all examining committee members, this project has been submitted to the department of mechanical engineering in the college of engineering and technology in the partial fulfillment of the requirement of department for the degree of Bachelor of Science in engineering.

## Project Supervisor signature

## Committee Member signature

Department Head Signature

## Acknowledgment

We wish to thank our parents for their tremendous contributions and support both morally and financially towards the completion of this project.
We also grateful to our project supervisor Dr. Yousef Alswati who without his help and guidance this project would not have been completed.
I also show my gratitude to my friends and all who contributed in one way or the other in the course of the project.


#### Abstract

The articulated hydraulic manipulator is a manipulator with three revolute joints and attached gripper. Each joint in this robot is driven by a hydraulic actuator. This robot is belongs to the mechatronics laboratory of Palestine Polytechnic University (ppu), and it has a five degree of freedom.

We will use the logic controller (plc) to control the movement of the manipulator end-effectors in the working envelop, so by using the HMI touch screen the coordinate will be entered and the end effecter move to the desired location in the workspace of manipulator, hence we will design a robotic system to make the manipulator pick an object from one location and move it to another.

In order to make this, many challenging problems will be covered in this project; these problems are kinematic, dynamitic, actuation, motion planning, control and programming.


## Table of Content

| Section | Title | Page Number |
| :---: | :---: | :---: |
|  | Cover page |  |
|  | Certification | IV |
|  | Acknowledgment | IV |
|  | Abstract | III |
|  | Table of Content | IV |
|  | List of Tables | IV |
|  | List of Figures | IV |
| Chapter One : | Introduction | 1 |
| 1.1 | Robotics overview | 2 |
| 1.2 | Robotic mechanical structure | 2 |
| 1.3 | Robotic systems | 3 |
| 1.4 | Classification of robots | 4 |
| 1.5 | project overview | 6 |
| 1.6 | Project Schedule | 8 |
| 1.7 | Report Content | 9 |
| Chapter Two : | Forward and Inverse Kinematic | 10 |
| 2.1 | Overview | 11 |
| 2.2 | Position and orientation representation | 11 |
|  | 2.2.1 Position and displacement | 12 |
|  | 2.2.2 Orientation and Rotation matrix | 13 |
|  | 2.2.3 Homogeneous Transformation | 15 |
| 2.3 | Parameterization of rotation: Euler angles | 15 |
| 2.4 | Forward kinematic | 16 |



|  | 5.2.1 Physical System Description | 59 |
| :---: | :---: | :---: |
|  | 5.2.2 Functional description | 60 |
|  | 5.2.3 Hydraulic Description | 60 |
|  | 5.2.4 Electrical Description | 62 |
| 5.3 | Closed Loop Control | 63 |
|  | 5.3.1 Feedback Sensors | 64 |
|  | 5.3.2 Controller and software | 68 |
| Chapter Six | Hardware and Software Description | 72 |
| 6.1 | Introductions | 73 |
| 6.2 | Hardware and Software Components | 73 |
| 6.3 | Sequence function chart -SFC (State Graph) | 74 |
| 6.4 | PID functions on Plc. | 79 |
|  | 6.4.1 Introduction | 79 |
|  | 6.4.2 The PID Controller Model | 80 |
|  | 6.4.3 Operating Principles | 83 |
|  | 6.4.4 Principal of the Regulation Loop | 85 |
|  | 6.4.5 Role and Influence of PID Parameters | 86 |
| 6.5 | Touch Screen and Vijeo Designer | 89 |
| Chapter Seven | Experiments and Results | 92 |
| 7.1 | Introduction | 93 |
| 7.2 | Experimental results | 93 |
| 7.3 | Conclusion | 95 |
| 7.4 | Future Work | 95 |References96

Appendices ..... 97

## List of Tables

| Table | Title | Page Number |
| :---: | :--- | :---: |
| 1.1 | First semester Project Schedule | 8 |
| 1.2 | second semester Project Schedule | 9 |
| 2.1 | DH parameters for the articulator manipulator | 20 |
| 2.2 | The axis rang for each joint | 8 |
| 7.1 | experimental results of the manipulator | 94 |

## List of Figures

| Figure | Title | Page Number |
| :---: | :---: | :---: |
| 1.1 | Robotic system | 3 |
| 1.2 | Articulated manipulator | 6 |
| 1.3 | The articulated manipulator in the mechatronics lab (PPU) | 7 |
| 2.1 | Representing the rigid body in space as position and orientation | 13 |
| 2.2 | The symbolic representation of articulated manipulator | 17 |
| 2.3 | Coordination frames showing DH1 and DH2 | 19 |
| 2.4 | Coordinates frame assigned for the articulated manipulator | 20 |
| 2.5 | Coordinates of $O_{\boldsymbol{c}}$ relative to the base frame | 23 |
| 2.6 | Projection of the wrist center onto $x_{0}-y_{0}$ plane. | 24 |
| 2.7 | Projecting onto the plan formed by links 2 and 3 | 25 |
| 2.8 | Elbow-up position and Elbow-down position | 26 |
| 2.9 | Workspace for the articulated manipulator | 27 |
| 3.1 | The articulated manipulator showing $O_{c}$ | 42 |
| 3.2 | Elbow singularities of the articulated manipulator | 43 |
| 4.1 | Cubic Polynomial Trajectory, Velocity profile and Acceleration profile | 52 |
| 4.2 | linear segments with parabolic Blends | 53 |
| 4.3 | Linear segments with parabolic blends trajectory, velocity and Acceleration. | 56 |
| 5.1 | the robot control system | 58 |


| 5.2 | Hydraulic rotary actuator for the joints | 59 |
| :--- | :--- | :--- |
| 5.3 | Hydraulic circuits for the system | 61 |
| 5.4 | Simplifies electric circuits for 220 volt power | 62 |
| 5.5 | Plc and HMI touch screen | 63 |
| 5.6 | Figure Generic concept of joint space control | 65 |
| 5.7 | The principle of norm controller | 66 |
| 5.8 | State graph for the articulated manipulator | 57 |
| 6.1 | PID controller model | 72 |
| 6.2 | PID Operating Principals on PLC | 80 |
| 6.3 | principal of a regulation loop | 83 |
| 6.4 | Influence of Proportional Action | 85 |
| 6.5 | Influence of Integral Action | 87 |
| 6.6 | Influence of Derivative Action | 88 |
| 6.8 |  |  |

# Chapter One 

## Introduction

1.1 Robotics overview
1.2 Robotic mechanical structure
1.3 Robotic systems
1.4 Classification of robots
1.5 project overview

### 1.6 Project Schedule

1.7 Report Content

### 1.1 Robotics overview

Robotics is concerned with the study of those machines that can replace human being in the execution of a task, as regards both physical activity and decision making.

At the present time, the industrial robots have a significant impact on the industry, such that the robot can improve the quality of life by freeing workers from dirty, boring, dangerous and heavy labor.

An official definition of such a robot comes from the Robotic Institute of America (RIA): a robot is a re-programmable multi-functional manipulator designed to move materials, parts, tools, or specialized devices through variable programmed motions for the performance of a variety of tasks.

### 1.2 Robotic mechanical structure

Robots are classified as those with fixed base "robot manipulators", and those with mobile base "mobile robots", in our project we have a robot with fixed base.

The mechanical structure of a robot manipulator consist of rigid link connected by a joint to form the kinematic chain, the joint can be revolute (rotary) or linear (prismatic).

In the case of revolute joint, this joint allows relative rotation between two links, these displacements are called joint angles .while prismatic joint allows a linear relative motion between tow links, which called the joint offset.

To construct the manipulator, the first link in a chain is connected to the base and the last link is connected to the end effecter, this end effecter can be anything from a welding device to a mechanical hand used to manipulate the environment. The kinematic chain of manipulator is characterized by number of degree of freedom (DOF).

### 1.3 Robotic systems

The basic component of robotic system is

- Manipulator (robotic arm)
- the end effecter (which is part of the manipulator)
- power supply
- the controller

And this can be viewed in figure (1.1)


The manipulator, which is the robotic arm, consists of segments joined together with axes capable of motion in various directions allowing the robot to perform work.

The end effecter, which is the gripper tool, a special device, or fixture attached to the robotic arm, actually performs the work.

The power supply provides and regulates the energy that is converted to motion by robotic actuator, and it may be either electric m pneumatic, or hydraulic.

The controller initiate, terminates, and coordinates the motion of sequences of a robot. Also it accepts the necessary input to the robot and provides the outputs to interface with the outside world.

### 1.4 Classification of robots

Robotic manipulators can be classified by several categories, such as their power source, geometry, application area, or their method control. Such classification is useful primarily in order to determine Which robot is righty for given task. For example, a hydraulic robot would not be suitable for food handling or clean room application.

Power source: Most robots are eclectically, hydraulically, or pneumatically powered. The advantage of use hydraulic power is that the hydraulic actuators are unrivaled in their speed of response and torque producing capability. These hydraulic robots are used primary for lifting heavy loads

Application area: Robots are often classified by application into assembly and non-assembly robots.

Method of control: Robots are classified by control method into servo (high technology) and non-servo (low technology) robots.

Geometry: Robot manipulators are usually classified cinematically on the basis of the first three joints of the arm. The majority of these manipulators fall into one of the five geometry types: articulated (RRR), spherical (RRP), SCARA (RRP), cylindrical (RPP), or Cartesian (PPP).

The common industrial manipulator is often referred to as a robot arm, with links and joints described in similar terms. Manipulators which emulate the characteristics of a human arm are called articulated arms (articulated manipulator). All their joints are rotary (or revolute).

The motion of articulated robot arms differs from the motion of the human arm. While robot joints have fewer degrees of freedom, they can move through greater angles. For example, the elbow of an articulated robot can bend up or down whereas a person can only bend their elbow in one direction with respect to the straight arm position


Figure 1.2: Articulated manipulator

## 1.5 project overview

Our work is concerned with rehabilitation and control of the articulated hydraulic robot that has five degree of freedom (figure 1.2). Each joint in this robot is driven by a hydraulic actuator. This robot is belongs to the mechatronics laboratory at Palestine Polytechnic University (ppu).

We will use a programmable logic controller (plc), specifically Schneider plc, to control the movement and orientation of the endeffecter in the working envelop, so by using the HMI touch screen the coordinate will be entered and the end effecter move to the desired location in the envelop.

This project also includes analysis, study of the kinematics, dynamics, trajectory planning, actuation and programming of the robot under consideration.


Figure 1.3: the articulated manipulator in the mechatronics lab (PPU)

### 1.6 Project Schedule

- First Semester

| Process | Week |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 |  | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| Selected the project |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Collection of the needed data for the project |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Modeling of Articulated hydraulic robot |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Writing the Documentation |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

Table1.1: first semester Project Schedule

- Second Semester

| Process | Week |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 |  | 5 |  |  |  |  |  | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| Building Manual Control Circuits |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Programming |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Building PLC circuit |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Testing and Writing the Documentation |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

Table1.2: seconded semester Project Schedule

### 1.7 Report Content

Now we provide a brief description of each chapter:

Chapter one: introduce an overview to robotic systems, classification of robots, and the project goal.

Chapter two: present solutions to the forward kinematics problems using Denevative-Haetenberg convention and to the inverse kinematics problem using Geometric Approach.

Chapter Three: introduce forward and inverse velocity kinematics using geometric Jacobean matrix, also this chapter provide a solution to singularities which is configurations that make the manipulator loss one or more degree of freedom.

Chapter Four: is concerned with describing motion of the manipulator in terms of trajectories through space.

Chapter Five: present we study the methods of controlling the manipulator (by digital controller) so that it will tack a desired position trajectory through space.

## Chapter Two

## Forward and Inverse Kinematics

2.1 Overview
2.2 Position and orientation representation

### 2.2.1 Position and displacement

2.2.2 Orientation and Rotation matrix
2.2.3 Homogeneous Transformation
2.3 Forward kinematic
2.4 Parameterization of rotation: Euler angles
2.5 Inverse kinematic
2.6 The Workspace

### 2.1 Overview

Kinematics is the branch of classical mechanics that describes the motion of points or bodies without consideration of the causes of motion. To describe motion, kinematics studies the trajectories of points, lines and other geometric objects and their differential properties such as velocity and acceleration.

For the articulated manipulator we have to consider the forward and inverse kinematic. First consider the forward kinematic problem which is to determine the position and orientation of the end-effectors by given the values of joint variables of the robot. Then we solve the inverse kinematics problem which is to determine the values of the joint variables given the end-effectors position and orientation.

To perform the kinematics analysis, we must establish various coordinate frames to represent the position and orientations of rigid body objects, and with transformations among these coordinate frames.

### 2.2 Position and orientation representation

A rigid body (robot link) is completely described in space by its position and orientation with respect to reference frame. A coordinate reference frame $i$ consist of an origin, denoted $O_{i}$, and a triad of mutually orthogonal bases vectors, denoted $\left(x_{i} y_{i} z_{i}\right)$, that are all fixed within a particular body. The pose of a body will always be expressed relative to some other body, so it can be expressed as the pose of one coordinate frame relative to another. Similarly, rigid-body displacements can be expressed as displacements between two coordinate frames, one of which
may be referred to as moving, while the other may be referred to as fixed. This indicates that the observer is located in a stationary position within the fixed reference frame, not that there exists any absolutely fixed frame.

### 2.2.1 Position and displacement

The position of body in coordinate frame denote by coordinate vector $P$ and if we have two coordinate frames $i$ and $j$, the position of the origin of coordinate frame $i$ relative to coordinate frame $j$ can be denoted by $3 \times 1$ vector

$$
p_{i}^{j}=\begin{align*}
& p_{i}^{i} x  \tag{2.1}\\
& p_{i}^{i} y \\
& p_{i}^{i} z
\end{align*}
$$

The components of this vector are the Cartesian coordinates of $O_{i}$ in the jframe.

A translation is a displacement in which no point in the rigid body remains in its initial position and all straight lines in the rigid body remains parallel to their initial orientations. The translation of a body in space can be represented by the combination of its positions prior to and following the translation. Conversely, the position of a body can be represented as a translation that takes the body from a position in which the coordinate frame fixed to the body coincides with the fixed coordinate frame to the current position in which the two frames are not coincident. Thus, any representation of position can be used to create a representation of displacement, and vice versa.

### 2.2.2 Orientation and Rotation matrix

In order to describe the orientation of body we will attach a coordinate frame to the body and then give a description of this coordinate system relative to the reference frame. In figure 2.1 coordinate frame ( $x_{i} y_{i} z_{i}$ ) haze been attached to the body in a known way. A description of frame $\left(x_{i} y_{i} z_{i}\right)$ relative to frame ( $x_{j} y_{j} z_{j}$ ) now suffices to give the orientation of the body. Thus, position of points is described with vectors and orientations of bodies are described with an attached coordinate frame. One way to describe the body-attached coordinate frame $\left(x_{i} y_{i} z_{i}\right)$, is to write the unit vectors of its three principal axes in terms of the coordinate frame $\left(x_{j} y_{j} z_{j}\right)$.


Figure 2.1: representing the rigid body in space as position and orientation

## Rotation matrix

The orientation of coordinate frame $i$ relative to coordinate frame $\boldsymbol{j}$ can bedenoted by expressing the bases vectors $\left(x_{i} y_{i} z_{i}\right)$ in terms of the bases vectors $\left(x_{j} y_{j} z_{j}\right)$. This yields $\left(x_{i}^{j} y_{i}^{j} z_{i}^{j}\right)$, which can when written together as $3 \times 3$ matrix is known as the rotation matrix. The components of $R_{i}^{j}$ are the dot products of basis vectors of the two coordinate frames.

$$
R_{i}^{j}=\begin{array}{lll}
x_{i} \cdot x_{j} & y_{i} \cdot x_{j} & z_{i} \cdot x_{j}  \tag{2.2}\\
x_{i} \cdot y_{j} & y_{i} \cdot y_{j} & z_{i} \cdot y_{j} \\
x_{i} \cdot z_{j} & y_{i} \cdot z_{j} & z_{i} \cdot z_{j}
\end{array}
$$

Because the basis vectors are unit vectors and the dot product of any two unit vectors is the cosine of the angle between them, the components are commonly referred to as direction cosines. Thus, the columns of $R_{i}^{j}$ specify the direction cosines of the coordinate axis of $\left(x_{i} y_{i} z_{i}\right)$ relative to coordinate axis of $\left(x_{j} y_{j} z_{j}\right)$

The set of $\mathrm{n} \times \mathrm{n}$ rotation matrices is known as the special orthogonal of order $n$, and is denoted by $S O(n)$. for any $R \in S O(3)$ the following properties hold
$>\mathrm{R}^{\mathrm{T}}=\mathrm{R}^{-1}$
$>$ The columns (and therefore the rows) of R are mutually
$>$ Each column ( and therefore each row) of R is a unit vector
$>\operatorname{det} \mathrm{R}=1$

Rotation matrices are combined through simple matrix multiplication such that the orientation of frame k relative to frame j can be expressed as

$$
\begin{equation*}
R_{k}^{j}=R_{i}^{j} R_{k}^{i} \tag{2.3}
\end{equation*}
$$

### 2.2.3 Homogeneous Transformation

Homogeneous transformations combine rotation and translation onto one matrix. A homogeneous transformation has the form of

$$
\mathrm{H}=\begin{array}{ll}
R & P  \tag{2.4}\\
0 & 1
\end{array}, \mathrm{R} \in \mathrm{SO}(3), \mathrm{P} \in R^{3}, \mathrm{H} \in R^{4 x 4}
$$

Where $R$ is the rotation matrix and $P$ is the translational matrix.
Homogeneous transformation matrices can be used to perform coordinate Transformations between frames that differ in orientation and translation.

### 2.3 Parameterization of rotation: Euler Angles

A common method for specifying a rotation matrix in three independent quantities is to use Euler angles. Consider the fixed coordinate frame $o_{0} x_{0} y_{0} z_{0}$ and the rotated frame $o_{1} x_{1} y_{1} z_{1}$ we can specify the orientation of frame $o_{1} x_{1} y_{1} z_{1}$ relative to the frame $o_{0} x_{0} y_{0} z_{0}$ by three angles $(\emptyset, \theta, \psi)$ called Euler angles, and obtained by three successive rotation as follow. First rotate about the $z$-axis by an angle $\emptyset$, next rotate about the current $y$-axis by the angle $\theta$,finally rotate about the current $z$-axis by an angle $\psi$. In terms of the basic rotation matrices the resulting rotational transformation can be generated as the product:
$R_{z y z}=R_{z, \phi} R_{y, \theta} R_{z, \psi}$

$$
=\begin{array}{ccc}
c_{\Phi} c_{\theta} c_{\psi}-s_{\Phi} S_{\psi} & -c_{\Phi} c_{\theta} S_{\psi}-s_{\Phi} c_{\psi} & c_{\Phi} S_{\theta}  \tag{2.5}\\
s_{\Phi} c_{\theta} c_{\psi}+c_{\Phi} S_{\psi} & -S_{\Phi} c_{\theta} S_{\psi}+c_{\Phi} c_{\psi} & S_{\Phi} S_{\theta} \\
-S_{\theta} c_{\psi} & s_{\theta} S_{\psi} & c_{\theta}
\end{array}
$$

For the articulated manipulator the Euler angles are the angles of the wrist rotation so:
$\emptyset=\theta_{5}, \theta=\theta_{4}, \psi=0$
The rotation matrix of frame $o_{5} x_{5} y_{5} z_{5}$ relative to wrist frame $o_{3} x_{3} y_{1} z_{3}$

$$
R_{5}^{3}=\begin{array}{ccc}
c_{\Phi} c_{\theta} & -S_{\Phi} & c_{\Phi} S_{\theta}  \tag{2.6}\\
S_{\Phi} c_{\theta} & c_{\Phi} & S_{\Phi} s_{\theta} \\
-s_{\theta} & 0 & c_{\theta}
\end{array}
$$

### 2.4 Forward kinematic:

A robot manipulator is composed of a set of links connected together by joints, the joints may be simple, such as a revolute joint, or a prismatic joint or they can be more complex such as a ball and socket joint.

A robot manipulator with n joints will have $n+1$ links, since each joint connects two links, we number the joints from 1 to n , and we number the links from 0 to n , starting from the base. by convention joints $i$ connects link $i-1$ to link $i$. We will consider the location of joint $i$ to be fixed with respect to $i-1$. When joint $i$ is actuated ,link $i$ moves. Therefore ,link 0 (the first link )is fixed ,and does not move when the joints are actuated.

Joint 1 is called base, joint 2 is called shoulder, joint 3 is called elbow as shown in figure 2.2


Figure 2.2: the symbolic representation of articulated manipulator

To perform the kinematic analysis, we attach the coordinate frame rigidly to each link .In particular, we attach $o_{i} x_{i} y_{i} z_{i}$ to link $i$. This means that, whatever motion the robot executes, the coordinates of each point on link $i$ are constant when expressed in the $i^{t h}$ coordinate frame. Furthermore, when joint $i$ is actuated, link $i$ and its attached frame $o_{i} x_{i} y_{i} z_{i}$, experience a resulting motion. The frame $o_{0} x_{0} y_{0} z_{0}$, which attached to the robot base, is referred to as the inertial frame. We will assign the coordinates frames that satisfy the Denative_Hartenberg convention.

## The Denative_Hartenberg convention:

This convention is concerned about assigning a coordinate frame to each link in the manipulator .In this convention, each homogenous transformation $A_{i}$ is representing as a product of four basic transformations.
$A_{i}=$ Rot $_{z, \theta_{l}} \operatorname{Trans}_{z, d_{l}} \operatorname{Trans}_{x, \alpha_{l}}$ Rot $_{x, \alpha_{l}}$

$$
=\begin{array}{cccc}
C_{\theta_{i}} & -S_{\theta_{i}} C_{\alpha_{i}} & S_{\theta_{i}} S_{\alpha_{i}} a_{i} C_{\theta_{i}}  \tag{2.7}\\
S_{\theta_{i}} & C_{\theta_{i}} C_{\alpha_{i}} & -C_{\theta_{i}} S_{\alpha_{i}} a_{i} S_{\theta_{i}} \\
0 & S_{\alpha_{i}} & C_{\alpha_{i}} & d_{i} \\
0 & 0 & 0 & 1
\end{array}
$$

Where the four quantities $\theta_{i}, a_{i}, d_{i}$ and $\alpha_{i}$ are parameters associated with link $i$ and joint $i$.So if the DH convention is satisfied then the transformation matrix can be written as the above form.

The attached frame must have the following features according to the DH convention:
(DH1) the axis $x_{i}$ is perpendicular to the axis $z_{i-1}$.
(DH2) the axis $x_{i}$ intersects the axis $z_{i-1}$.

The tow features are shown in figure 2.3.


Figure 2.3: coordination frames showing DH1 and DH2.

Below we will describe the DH parameters that used in the transformation:
$a_{i}:$ is the distance between the axis $z_{i}$ and $z_{i-1}$, and its measured along the axis $x_{i+1}$
$\alpha_{i}:$ is the angle between the axis $z_{i}$ and $z_{i-1}$, measued in a plan normal to $x_{i+1}$
$\theta_{i}$ : is the angle from the axis $x_{i}$ and $x_{i-1}$, measured in a plan normal to $z_{i}$.
$d_{i}$ : The distance from the origin $o_{i-1}$ to the interconnection between $x_{i}$ axis and $z_{i-1}$ measured along the $z_{i}$ axis.

For the articulated manipulator we assign the coordinate frames that satisfy the DH convention and this is seen in figure 2.3.


Figure 2.3: coordinates frame assigned for the articulated manipulator The DH parameters for the articulated manipulator are shown in table 2.1:

| Link | $a_{i}$ | $\alpha_{i}$ | $d_{i}$ | $\theta_{i}$ |
| :--- | :---: | :---: | :---: | :---: |
| 1 | 0 | $\pi^{\prime} 2$ | $a_{1}$ | $\theta_{1}$ |
| 2 | $a_{2}$ | 0 | 0 | $\theta_{2}$ |
| 3 | $a_{3}$ | 0 | 0 | $\theta_{3}$ |

Table 2.1: DH parameters for the articulator manipulator.

The A-matrices are obtained using equation 2.5:
$A_{1}=\begin{array}{cccc}C_{1} & 0 & S_{1} & 0 \\ S_{1} & 0 & -C_{1} & 0 \\ 0 & 1 & 0 & d_{1} \\ 0 & 0 & 0 & 1\end{array}$

$$
A_{2}=\begin{array}{ccc}
C_{2} & -S_{2} & 0 a_{2} C_{2} \\
S_{2} & C_{2} & 0 a_{2} S_{2} \\
0 & 0 & 1 \\
0 \\
0 & 0 & 0
\end{array}
$$

$$
A_{3}==\begin{array}{cccc}
C_{3} & -S_{3} & 0 a_{3} C_{3}  \tag{2.8}\\
S_{3} & C_{3} & 0 a_{3} S_{3} \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{array}
$$

The homogenous transformation "T-matrices" are thus given by:

$$
\begin{gather*}
T_{1}^{0}=A_{1} \\
T_{2}^{0}=A_{1} A_{2}=\begin{array}{cccc}
C_{1} C_{2} & -S_{2} C_{1} & S_{1} & a_{2} C_{1} C_{2} \\
S_{1} C_{2} & S_{1} S_{2} & -C_{1} & a_{2} S_{1} C_{2} \\
S_{2} & C_{1} & 0 & a_{2} S_{2}+d_{1} \\
0 & 0 & 0 & 1 \\
\\
T_{3}^{0}=A_{1} A_{2} A_{3}= & \\
& C_{1} C_{23} & -S_{23} C_{1} & S_{1} a_{3} C_{1} C_{23}+a_{2} C_{1} C_{2} \\
S_{1} C_{23} & S_{1} S_{23} & -C_{1} a_{3} S_{1} C_{23}+a_{2} S_{1} C_{2} \\
S_{12} & C_{23} & 0 & a_{2} S_{2}+d_{1}+a_{3} S_{23} \\
0 & 0 & 0 & 1
\end{array} \tag{2.9}
\end{gather*}
$$

Notice that the first three entries of the last column of $T_{3}^{0}$ are the $x, y$ and $z$ component of the origin $O_{3}$ with respect to the base frame; that is,

$$
\begin{align*}
& x=a_{3} C_{1} C_{23}+a_{2} C_{1} C_{2} \\
& y=a_{3} S_{1} C_{23}+a_{2} S_{1} C_{2} \tag{2.10}
\end{align*}
$$

$$
z=a_{2} S_{2}+d_{1}+a_{3} S_{23}
$$

are the coordinate of the end effecter with respect to the base frame. The rotational part of $T_{3}^{0}$ gives the orientation of the frame $O_{3}$ relative to the base frame.

### 2.5 Inverse kinematic:

The inverse kinematic problem is to find the joint variables in terms of the end effecter's position and orientation, the joint variable is the angle $\theta_{i}$ in the case of revolute joint, or $d_{i}$ in the case of prismatic joint. Her we need a geometric approach to find $\theta_{1}, \theta_{2}, \theta_{3}$ that corresponds to a given position of the end effecter, called wrist center, and is represented by the point $O_{c}$.

The wrist center is the point between the arm and the end effecter and it has the coordinates of $x_{c}, y_{c}, z_{c}$, as shown in figure 2.4


Figure 2.4: coordinates of $O_{c}$ relative to the base frame

The general idea of the geometric approach is to solve for joint variable $\theta_{i}$ for a revolute joint by projecting the manipulator onto the $x_{i-1}-y_{i-1}$ plan and solving a simple trigonometry problem. For example, to solve for $\theta_{1}$ we project the arm onto the $x_{0}-y_{0}$ plane and use trigonometry to find $\theta_{1}$ as shown in figure 2.5.


Figure 2.5: projection of the wrist center onto $x_{0}-y_{0}$ plane.

We see from this projection that:

$$
\begin{equation*}
\theta_{1}=\operatorname{Atan} 2\left(x_{c}, y_{c}\right) \tag{2.11}
\end{equation*}
$$

In which Atan2 $(x, y)$ denotes the two argument arctangent function.

To find the angles $\theta_{2}, \theta_{3}$ for the articulated manipulator given $\theta_{1}$, we consider the plane formed by the second and the third links as shown in figure 2.6. Since the motion of the second and third link is planar:


Figure 2.6: projecting onto the plan formed by links 2 and 3.
$\theta_{2}$ Is given by:

$$
\begin{align*}
\theta_{2} & =\operatorname{Atan} 2 \gamma, s-\operatorname{Atan} 2 a_{2}+a_{3} c_{3}, a_{3} s_{3}  \tag{2.12}\\
& =\operatorname{Atan} 2\left(\bar{x}_{c}^{2}+y_{c}^{2}-d^{2}, z_{c}-d_{1}-\operatorname{Atan} 2\left(a_{2}+a_{3} c_{3}, a_{3} s_{3}\right)\right)
\end{align*}
$$

Using the law of cosines we see that the angle $\theta_{3}$ is given by:

$$
\begin{align*}
\cos \theta_{3} & =\frac{\gamma^{2}+S^{2}-a_{2}^{2}-a_{3}^{2}}{2 a_{2} a_{3}} \\
& =\frac{x_{c}^{2}+y_{c}^{2}-d^{2}+z_{c}-d_{1}-a_{2}^{2}-a_{3}^{2}}{2 a_{2} a_{3}}=D \tag{2.13}
\end{align*}
$$

Since $\gamma^{2}=x_{c}^{2}+y_{c}^{2}-d^{2}$ and $s=\left(z_{c}-d_{1}\right)$. Hence, $\theta_{3}$ is given by

$$
\begin{equation*}
\theta_{3}=\operatorname{Atan} 2\left(D, \pm \sqrt{1-D^{2}}\right) \tag{2.14}
\end{equation*}
$$

The tow solutions for $\theta_{3}$ is corresponds to the elbow-down position and elbow-up position, respectively, as shown in figure 2.7.


Figure 2.7: Elbow-up position and Elbow-down position

For the articulated manipulator we can use the inverse orientation techniques to find the final two joint variables using the rotation matrix described in equation 2.6:
$\theta_{4}=\operatorname{Atan} 2\left(c_{1} c_{23} r_{13}+s_{1} c_{23} r_{23}+s_{23} r_{33}\right.$,

$$
\left.-c_{1} s_{23} r_{13}-s_{1} s_{23} r_{23}+c_{23} r_{33}\right)
$$

$\theta_{5}=\operatorname{Atan} 2 \quad s_{1} r_{13}-c_{1} r_{23}, \pm \overline{1-s_{1} r_{13}-c_{1} r_{23}{ }^{2}}$

Where $r_{i j}$ are the elements of Euler rotation matrix $R_{5}^{3}$.

### 2.6 The Workspace:

The workspace of a manipulator is the total volume swept out by the end effecter as the manipulator executes all possible motions. The workspace is constrained by the geometry of the manipulator as well as mechanical constraints of the joints. For example, a revolute joint may be limited to less than a full $360^{\circ}$ of motion. The mechanical limits in the articulated manipulator limit the motion of a revolute joint to the values that appears in table 2.2. These values are measured experimentally.

| Axis Movement | Axis Rang |
| :--- | :---: |
| Axis 1: base rotation | $-67<\bar{\theta}_{1}>113$ |
| Axis 2 : Shoulder rotation | $-45<\bar{\theta}_{2}>45$ |
| Axis 3 : Elbow rotation | $-135<\bar{\theta}_{3}>-45$ |

Table 2.2: the axis rang for each joint

The workspace for the articulated manipulator I shown in figure 2.8:


Figure 2.8: Workspace for the articulated manipulator

## Chapter Three

Velocity Kinematic - The Jacobian

3.1 Overview
3.2 Skew symmetry matrices
3.3 The derivative of a Rotation Matrix
3.4 Angular Velocity : The Fixed Axis Case
3.5 Angular Velocity : the General Case
3.6 Linear velocity Of A Point Attached To Moving Frame
3.7 Derivation Of Jacobian
3.8 The Analytical Jacobian
3.9 Singularities
3.10 Inverse Velocity and Acceleration

### 3.1 Overview

In this chapter we derive the velocity relations, relating the linear and angular velocities of the end effecter to the joint velocities. First we consider the forward kinematics of velocity which is to determine the linear and angular velocities of the end effectors by giving the joint velocities, and then we solve the inverse kinematic of velocity which is to determine the joint velocities that produce the desired end effectors velocities.

To determine the velocities relationships we need to attach coordinate frame rigidly to each link and find the forward kinematic equations that define a function between the space of Cartesian positions and orientations and the space of joint positions as we done in the previous chapter. Then the velocities relationships are determined by the Jacobian of this function.

The Jacobian is a matrix that generalizes the notion of the ordinary derivative of a scalar function. The derivative of kinematic equations is done with aid of skew symmetric matrices.

### 3.2 SKEW SYMMETRY MATRICES

This section derives the properties of rotation matrices that can be used to compute relative velocity transformations between coordinate frames.

An $n \times n$ matrix $S$ is said to be skew symmetric if and only if

$$
S^{T}+S=0
$$

We denote the set of all $3 \times 3$ skew symmetric matrices by so 3 .
The skew symmetric matrix contains only three independent entries and every $3 \times 3$ skew symmetric matrix has the form

$$
S=\begin{array}{ccc}
0 & -s_{3} & s_{2}  \tag{3.1}\\
s_{3} & 0 & -s_{1} \\
-s_{2} & s_{1} & 0
\end{array}
$$

So if $a=a_{x}, a_{y}, a_{z}{ }^{T}$ is a 3-vector then we define the skew symmetric matrix as:

$$
\text { S } a=\begin{array}{ccc}
0 & -a_{z} & a_{y}  \tag{3.2}\\
a_{z} & 0 & -a_{x} \\
-a_{y} & a_{x} & 0
\end{array}
$$

## Properties of skew symmetric matrices

1. The operator $S$ in linear, that is

$$
S \alpha a+\beta b=\alpha S a+\beta S b
$$

For any vectors $a$ and $b$ belonging to $R^{3}$ and scalar $\alpha$ and $\beta$
2. For any vector $a$ and $p$ belonging to $R^{3}$

$$
\text { S a } p=a \times p
$$

3. For Reso 3 and $a \in R^{3}$

$$
R S a R^{T}=S R a
$$

4. For an $n \times n$ skew symmetric matrix $S$ and any vector $X \in R^{n}$

$$
X^{T} S X=0
$$

### 3.3 The derivative of a Rotation Matrix

If a rotation matrix $R$ is a function of single variable $\theta$. Hence, $R=R \theta$ Eso 3 For every $\theta$, the derivative of $R$ is:

$$
\begin{equation*}
\frac{d}{d \theta} R=S R \theta \tag{3.3}
\end{equation*}
$$

So for the basic rotation matrices:
If $R=R_{x, \theta}$, then $\frac{d}{d \theta} R_{x, \theta}=S i R_{x, \theta}, S i=\begin{array}{ccc}0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0\end{array}$
If $R=R_{y, \theta}$, then $\frac{d}{d \theta} R_{y, \theta}=S j R_{y, \theta}, S j=\begin{array}{ccc}0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0\end{array}$
If $R=R_{z, \theta}$, then $\frac{d}{d \theta} R_{z, \theta}=S j R_{z, \theta}, S k=\begin{array}{ccc}0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0\end{array}$

### 3.4 Angular Velocity : The Fixed Axis Case

When a body moves in pure rotation about fixed axis, and if $k$ is a vector in the direction of the axis of rotation, then the angular velocity is given by

$$
\begin{equation*}
\omega=\dot{\theta} k \tag{3.4}
\end{equation*}
$$

In which $\dot{\theta}$ is the time derivative of $\theta$. And the linear velocity of any point on the body is given by

$$
\begin{equation*}
v=\omega \times r \tag{3.5}
\end{equation*}
$$

In which $r$ is a vector from the origin to the point.

### 3.5 Angular Velocity : the General Case

Suppose that a Rotational matrix $R$ is time varying, so that $R=R t \in s o 3$ For every $t$, the derivative of $R$ is

$$
\begin{equation*}
\dot{R} t=S \omega t \quad R(t) \tag{3.6}
\end{equation*}
$$

Where the matrix $S \omega t$ is a skew symmetric. The vector $\omega t$ is the angular velocity of the rotating frame with respect to fixed frame at time $t$. The previous equation shows the relationship between angular velocity and the derivative of rotation matrix.

We are often interested in finding the resultant angular velocity due to the relative rotation of several coordinate frames. The angular velocities can be added once they expressed relative to the same coordinate frame, suppose that we are given

$$
R_{n}^{o}=R_{1}^{o} R_{2}^{1} \ldots R_{n}^{n-1}
$$

Extending the above reasoning we obtain

$$
\begin{equation*}
\dot{R}_{n}^{o}=S \omega_{0, n}^{0} R_{n}^{0} \tag{3.7}
\end{equation*}
$$

In which

$$
\begin{align*}
\omega_{0, n}^{0} & =\omega_{0,1}^{0}+R_{1}^{0} \omega_{1,2}^{1}+R_{2}^{0} \omega_{2,3}^{2}+\cdots+R_{n-1}^{0} \omega_{n-1, n}^{n-1} \\
& =\omega_{0,1}^{0}+\omega_{1,2}^{0}+\omega_{2,3}^{0}+\cdots+\omega_{n-1, n}^{0} \tag{3.8}
\end{align*}
$$

The symbol $\omega_{i, j}^{k}$ denoted the angular velocityvector corresponding to the derivative of $R_{j}^{i}$, expressed relative to frame $k$.

### 3.6 Linear velocity of a Point attached to moving Frame

We now consider the linear velocity of a point that is rigidly attached to a moving frame. Suppose we have two coordinate frames $o_{0} x_{0} y_{0} z_{0}$ and $o_{1} x_{1} y_{1} z_{1}$, and that the homogenous transformation relating the two frames is time dependent, so that

$$
H_{1}^{0}(t)=\begin{array}{cc}
R_{1}^{0}(t) & o_{1}^{0}(t) \\
0 & 1
\end{array}
$$

If a point $p$ is rigidly attached to frame $o_{1} x_{1} y_{1} z_{1}$, and $o_{1} x_{1} y_{1} z_{1}$ is rotating relative to the frame $o_{0} x_{0} y_{0} z_{0}$ then

$$
p^{0}=R p^{1}+o
$$

Differentiating the above expression gives

$$
\begin{align*}
\dot{p}^{0} & =\dot{R} P^{1}+\dot{o} \\
& =s w R P^{1}+o \\
& =\omega \times r+v \tag{3.9}
\end{align*}
$$

Where $r=R p^{1}$ is the vector from $\boldsymbol{o}_{1}$ to $p$ expressed in the orientation of the frame $o_{0} x_{0} y_{0} z_{0}$, and $\boldsymbol{v}$ is the rate at which origin $\boldsymbol{o}_{\boldsymbol{1}}$ is moving.

### 3.7 Derivation Of Jacobian

Consider an $n$ - link manipulator with the joint variables $q_{1}, \ldots, q_{n}$. Let

$$
T_{n}^{0}=\begin{array}{cc}
R_{n}^{0}(q) & o_{n}^{0}(q) \\
0 & 1
\end{array}
$$

denote the transformation from the end effectors frame to the base frame, where $\mathrm{q}=\mathrm{q}_{1}, \ldots, \mathrm{q}_{\mathrm{n}}{ }^{\mathrm{T}}$ is the vector of joint variables. As the robot moves about, both the joint variables $q_{i}$ and the end effectors position $\mathrm{o}_{\mathrm{n}}^{0}$ and the orientation $\mathrm{R}_{\mathrm{n}}^{0}$ will be function of time. The object of this section is to relate the linear and angular velocity of the end effectors to the vector of joint velocities $\dot{q}(\mathrm{t})$. Let

$$
\begin{equation*}
S \omega_{n}^{0}=\dot{T}_{n}^{0}\left(R_{n}^{0}\right)^{T} \tag{3.10}
\end{equation*}
$$

define the angular velocity vector $\omega_{\mathrm{n}}^{0}$ of the end effectors, and let

$$
v_{n}^{0}=\dot{o}_{n}^{0}
$$

denote the linear velocity of the end effectors. We seek expression of the form

$$
\begin{aligned}
v_{n}^{0} & =J_{v} \dot{q} \\
\omega_{n}^{0} & =J_{\omega} \dot{q}
\end{aligned}
$$

Where $_{v}$ and $J_{\omega}$ are $3 \times n$ matrices, we can writing the previous equation together as

$$
\begin{equation*}
\xi=I \dot{q} \tag{3.11}
\end{equation*}
$$

in which $\zeta$ and $J$ are given by

$$
\xi=\begin{aligned}
& v_{n}^{0} \\
& \omega_{n}^{0}
\end{aligned} \quad \text { and } J=\begin{aligned}
& I_{v} \\
& I_{\omega}
\end{aligned}
$$

The vector $\zeta$ is called a body velocity, and the matrix J is called Manipulator Jacobian or Jacobian for short and sometimes it's called the Geometric Jacobian. Note that $I$ is a $6 \times n$ matrix where $n$ is the number of manipulator links.

To derive the Jacobian for any manipulator, we find the terms $J_{\omega}, J_{v}$ for each link and then we combine the $6 \times n$ matrix. If the joint $i$ is revolute, then the joint variable $q_{i}$ equal to $\theta_{i}$, and the axis of rotation is $z_{i-1}$ and If the joint is prismatic, then the joint variable $q_{i}$ equal to $d_{i}$.

For an $n$-link manipulator, the upper half of the jacobian $J_{v}$ is given as

$$
J_{v}=\left[\begin{array}{lll}
I_{v} & \ldots & I_{v_{n}}
\end{array}\right]
$$

In which the $i^{t h}$ column of $V_{v_{i}}$ is

$$
I_{v_{i}}=\begin{array}{lr}
z_{i-1} \times o_{n}-o_{i-1} & \text { for revolute joint } i  \tag{3.12}\\
z_{i-1} & \text { for presmatic joint } i
\end{array}
$$

The lower half f the Jacobian is given as

$$
I_{w}=\left[J_{w_{1}} \ldots I_{w_{n}}\right]
$$

In which the $i^{\text {th }}$ column of is $J_{w_{i}}$

$$
I_{w_{i}}=\begin{array}{lc}
z_{i-1} & \text { for revolute joint } i  \tag{3.13}\\
0 & \text { for presmatic joint } i
\end{array}
$$

Where $\mathrm{z}_{\mathrm{i}-1}=\mathrm{R}_{\mathrm{i}-1}^{0} k$, and $k=[0,0,1]^{T}$

The above formulas make the determination of the Jacobian of any manipulator simple since all the quantities needed are available once the forward kinematic worked out. The coordinate for $z_{i}$ with respect to the base frame are given by the first three elements in the third column of $T_{i}^{0}$. While $o_{\boldsymbol{i}}$ is given by the first three elements in the fourth column of $T_{\boldsymbol{i}}^{0}$.

Thus only third and fourth columns of $T$ matrices are needed in order to evaluate the Jacobian of the manipulator.

For the articulated manipulator: we can express the coordinates for each origin of coordinate frame by the vectors

$$
\begin{array}{cc}
0 \\
O_{0}=\begin{array}{c}
0 \\
0
\end{array} & , \quad O_{1}=\begin{array}{c}
0 \\
d_{1}
\end{array} \\
& \\
O_{2}= & \\
a_{2} C_{1} C_{2} \\
a_{2} S_{1} C_{2} \\
a_{2} S_{2}+d_{1} & \\
a_{3} C_{1} C_{23}+a_{2} C_{1} C_{2} \\
O_{3} & a_{3} S_{1} C_{23}+a_{2} S_{1} C_{2} \\
a_{2} S_{2}+d_{1}+a_{3} S_{23}
\end{array}
$$

Where $O_{i}^{j}$ is a vector that represents the coordinates of the origin of coordinate frame $i$ relative to the base frame.

Also we find

$$
z_{0}=\begin{aligned}
& 0 \\
& 0 \\
& 1
\end{aligned} \quad, \quad z_{1}=\begin{aligned}
& 0 \\
& 0 \\
& 1
\end{aligned} \quad, \quad z_{2}=\begin{gathered}
S_{1} \\
-C_{1} \\
1
\end{gathered}
$$

Now we give the Jacobian components $\left(J_{\omega}, /_{v}\right)$ for each link

For link 1

$$
I_{v 1}=z_{0} \times o_{3}-o_{0}=\begin{gathered}
-a_{3} S_{1} C_{23}-a_{2} S_{1} C_{2} \\
a_{3} C_{1} C_{23}+a_{2} C_{1} C_{2} \\
0
\end{gathered}
$$

$$
I_{\omega 1}=z_{0}=\begin{aligned}
& 0 \\
& 0 \\
& 1
\end{aligned}
$$

For link 2

$$
\begin{gathered}
I_{v 2}=z_{1} \times o_{3}-o_{1}=\begin{array}{c}
-a_{3} S_{1} C_{23}-a_{2} S_{1} C_{2} \\
a_{3} C_{1} C_{23}+a_{2} C_{1} C_{2} \\
0
\end{array} \\
0 \\
I_{\omega 2}=z_{1}=0 \\
1
\end{gathered}
$$

For link 3

$$
\begin{gathered}
I_{v 3}=z_{2} \times o_{3}-o_{2}=\begin{array}{c}
-C_{1} a_{3} S_{23}-a_{3} S_{1} C_{23} \\
a_{3} C_{1} C_{23}-S_{1} a_{3} S_{23} \\
S_{1} a_{3} S_{1} C_{23}+C_{1} a_{3} C_{1} C_{23}
\end{array} \\
I_{\omega 3}=z_{2}=\begin{array}{c}
S_{1} \\
-C_{1} \\
1
\end{array}
\end{gathered}
$$

The Jacobian for the first three links of the articulated manipulator is:

$$
\boldsymbol{J}=\begin{array}{lll}
\boldsymbol{J}_{v 1} & \boldsymbol{J}_{v 2} & \boldsymbol{J}_{v 3} \\
\boldsymbol{I}_{\omega 1} & \boldsymbol{I}_{\omega 2} & \boldsymbol{J}_{\omega 3}
\end{array}
$$


(3.15)

### 3.8 The Analytical Jacobian

The Analytical Jacobian denoted $J_{a} q$ is based on minimal representation for the orientation of the end effectors frame and it different from the Jacobian that we derived previously (its called Geometric Jacobian).

$$
\begin{array}{lc}
\text { Geometric Jacobian } & \zeta=\underset{\omega(q)}{v(q)}=\dot{d}(q) \\
\omega(q) \tag{3.16}
\end{array}=I q \dot{q},
$$

By using Euler's angels for the parameterization of orientation, the analytical Jacobian relates the joint velocities to the time derivative of the pose parameter

$$
\begin{equation*}
X=\frac{d(q)}{\alpha(q)} \quad, \quad \dot{X}=\frac{\dot{d}}{\dot{\alpha}}=I_{a} q \dot{q} \tag{3.17}
\end{equation*}
$$

in which $d q$ is the usual vector from the origin of the base frame to the origin of the end-effecter frame and $\alpha$ denotes the parameterization of the rotation matrix that specifies the orientation of the end-effecter frame relative to the base frame. For Euler angles parameterization, the analytical Jacobian is given by

$$
I_{a} q=\begin{array}{cc}
I & 0  \tag{3.18}\\
0 & B^{-1} \alpha
\end{array} \quad I(q)
$$

In which

$$
B \alpha=\begin{array}{ccc}
c_{\psi} s_{\theta} & -s_{\psi} & 0 \\
s_{\psi} s_{\theta} & c_{\psi} & 0 \\
c_{\theta} & 0 & 1
\end{array}
$$

### 3.9 Singularities

Singularities are configurations in which the manipulator loses one or more degree of freedom of motion. This idea can be made precise in terms of the rank of a Jacobian matrix relating the rates of change of input (joint position) and output (end-effectors position) variables.

The rank of a matrix is not necessary constant. Indeed, the rank of the manipulator Jacobian matrix will depend on the configurationq. Configurations for which rank $j(q)$ is less than its maximum value are called singularities or singular configuration.

In general: A configuration q is singular I and only if

$$
\operatorname{det} I \quad q=0
$$

It's difficult to solve this nonlinear equation, there for we use the method of decupling singularities, which is applicable whenever, for example, the manipulator is equipped with spherical manipulator, so we decouple the determination of singular configuration into two simpler problems. The first is to determine the singularities results from the motion of the arm (arm singularities) while the second is to determine the wrist singularities resulting from the motion of the wrist.

For spherical wrist manipulators, the Jacobian matrix has the block triangle form

$$
j=\begin{array}{cc}
j_{11} & 0  \tag{3.19}\\
j_{21} & j_{22}
\end{array}
$$

With determinates

$$
\operatorname{det} j=\operatorname{det} j_{11} \operatorname{det} j_{22}
$$

The set of singular configurations of the manipulator is the union of the set of arm configurations satisfying $\operatorname{det} j_{11}=0$ (arm configuration) and the set of wrist configurations satisfying det $_{22}=0$ wrist configuration.

For the articulated manipulator with coordinate frames attached as shown in figure 3.1


Figure 3.1: The articulated manipulator showing $O_{C}$

For this manipulator we see that

$$
\begin{array}{ccc}
-a_{2} s_{1} c_{2}-a_{3} s_{1} c_{23} & -a_{2} s_{2} c_{1}-a_{3} s_{23} c_{1} & -a_{3} c_{1} s_{23} \\
j_{11}=a_{2} c_{1} c_{2}+a_{3} c_{1} c_{23} & -a_{2} s_{1} s_{2}-a_{3} s_{1} s_{23} & -a_{3} s_{1} s_{23}  \tag{3.20}\\
0 & a_{2} c_{2}+a_{3} a_{3} c_{23} & a_{3} c_{23}
\end{array}
$$

And the determinate of $j_{11}$ is

$$
\begin{equation*}
\operatorname{det} j_{11}=a_{2} a_{3} s_{3}\left(a_{2} c_{2}+a_{3} c_{23}\right) \tag{3.21}
\end{equation*}
$$

The articulated manipulator is in singular configuration when

$$
s_{3}=0 \quad \text { that is } \theta_{3}=0 \text { or } \pi
$$

This situation is shown in figure 3.2 and it arises when the elbow is fully extended or fully retracted.

And whenever

$$
a_{2} c_{2}+a_{3} c_{23}=0
$$

This configuration occurs when the wrist center intersects the axis of base rotation $z_{0}$ and in this case there is infinity many singular configurations and infinity many solutions to the inverse position kinematics when the wrist center is along this axis. This configuration can be avoided by an offset in the elbow or shoulder.


Figure 3.2: Elbow singularities of the articulated manipulator

### 3.10 Inverse Velocity And Acceleration

The inverse velocity problem is the problem of finding the joint velocities $\dot{q}$ that produce the desired end effector velocity $\dot{X}$ or acceleration $\ddot{X}$.

For manipulators that have six joints and the Jacobian matrix is square and nonsingular ( $\operatorname{det} J_{a} q \neq 0$ ), this problem can be solved by simply inverting the Jacobian matrix to give

$$
\begin{equation*}
\dot{q}=J^{-1} \zeta \tag{3.22}
\end{equation*}
$$

For manipulators with $n>6$ we can solve for $\dot{q}$ (joint velocities) using the right pseudoinverse of $j$. To construct this pseudoinverse, we use the fact that when $j \in R^{m \times n}$, if $\mathrm{m}<\mathrm{n}$ and $\operatorname{rank} j=m$, then $\left(j j^{+}\right)^{-1}$ exist. in this case $\left(j j^{T}\right) \in R^{m \times n}$, and has a rank $m$. using this we can find that:

$$
\begin{equation*}
j^{+}=J^{T}\left(J J^{T}\right)^{-1} \tag{3.23}
\end{equation*}
$$

which is the right Pseudoinverse of the j , and if we multiply it by j this will give the identity matrix $I$, that to say $I J^{+}=I$ but $I^{+} I \neq I$ since the matrix multiplication is not cumulative.

By using the right pseudoinverse we can find a solution for $\dot{q}$ (joint velocity) for nonsingular configurations as

$$
\begin{equation*}
\dot{q}=j^{+} \xi \tag{3.24}
\end{equation*}
$$

We can apply a similar approach when the analytical Jacobian is used in place of manipulator Jacobian. The joint velocities and the end-effectors velocities are related by the analytical Jacobian as

$$
\begin{equation*}
\dot{X}=J_{a}(q) \dot{q} \tag{3.25}
\end{equation*}
$$

Thus, the inverse velocity problem becomes one of solving the linear system given by the above equation.

Differentiating equation (3.23) yield an expression for the acceleration:

$$
\begin{equation*}
\ddot{X}=J_{a} q \ddot{q}+\left(\frac{d}{d t} J_{a} q\right) \dot{q} \tag{3.26}
\end{equation*}
$$

Thus, given vector $\ddot{X}$ of end-effecter, the instantaneous joint acceleration vector is given as a solution of

$$
\begin{equation*}
I_{a} q q=\ddot{X}-\left(\frac{d}{d t} I_{a} q\right) \dot{q} \tag{3.27}
\end{equation*}
$$

# Chapter Four 

## Trajectory Planning

### 4.1 Introduction

### 4.2 Joint space Trajectory

### 4.3 Point to point motion

### 4.3.1 Cubic Polynomial

4.3.2 Linear segment with parabolic blends (LSPB

### 4.1 Introduction

Trajectory planning relates to the way a robot is moved from one location to another in a controlled manner. So that in this chapter we will plan a trajectory; a trajectory refers to the time history of position, velocity and acceleration for each joint in the manipulator. Trajectory planning requires the use of kinematic and dynamic equation of the manipulator.

When we dealing with trajectory there are many constants we expect to see in solving this problem, these constrain could be:

1. Spatial constrain, if we have on obstacle in the environment that we don't want to collide with. We will neglect this constrain in our project and assume that there is no obstacle in the workplace of the robot arm.
2. Time constrain, if the motion ha to be done in particular time.
3. Smoothness, we want the manipulator to have a smooth motion because that uses less energy and easy to control.

The trajectory planning can be done in two main spaces, joint space and Cartesian space .In joint pace it easy to go through point, there is no problems with singularities and it requires less calculation. In the other hand, the actual end-effectors path of this approach can't be predicted and can't follow straight lines.

The trajectory planning in Cartesian space may involve problems difficult to solve. However, Cartesian space is more computationally expensive to execute since at run time, inverse kinematics must be solved at the path update rate. Other major problems that we may
face in Cartesian space is singularity ;if there are some points on the path that the manipulator should follow are in singular configuration, but in this space we can specify the shape of the path between path points.

In Our project we will use the joint space trajectory planning for the articulated manipulator, because we want to move the end-effectors from initial position to final position regardless of the path to follow.

### 4.2 Joint space Trajectory

The joint space is a method of path generation in which the path shapes (in space and in time) are described in terms of function of joint angles.

Each path joint is usually specified in terms of a desired position and orientation of the tool frame, relative to the base frame, each of these points in converted into a set of desired joint angels by application of inverse kinematics.Then a smooth function is found for each of the n joints to describe the motion between the initial and final joint.

Through the remaining of this chapter we interest in establishing formulas for the angels of each DOF as a function of time in the case of the initial and final points on the path and traveling time are specified (point-to-point)

### 4.3 Point to point motion

In the trajectory planning, a complete description of all location of every point on the robot is referred to as a configuration. For our purpose, the vector of joint variables $q$ provide a convenient representation of a configuration

The task of point to point motion is to plan a trajectory from an initial configuration $q t_{o}$ to a final configuration $q t_{f}$. In some cases, there may be constrains on the trajectory (for example, if the robot must start and end with zero velocity). Nevertheless, it's easy to realize that there are infinitely many trajectories that will satisfy a finite number of constrains on the end points.

It's a common practice therefore to choose trajectories from a finitely parameterizable family, for example, polynomials of degree, where ndepends on the number of constrains to be satisfied. This is the approach that we will take in our project.

We will consider tow smooth functions for point to point motion, cubic polynomial and linear segments with parabolic blends, then these functions are substituted on the dynamic equation of the robot to see which function produce less torque and thus less power consumption.

### 4.3.1 Cubic Polynomials

Consider the problem of moving the end-effectors from its initial position to a final position in a particular time. The set of goal joint angles can be calculated using the inverse kinematic for particular values of end effectors position. The initial position of the manipulator is also known on the form of a set of joint angles. To make a smooth motion between the initial and final position of the manipulator, we first have to generate a polynomial joint trajectory between the two configurations, and specify the start and end velocities of the trajectory. This gives four constraints that the trajectory must satisfy, two constrains comes from the selection of initial and final values:

$$
\begin{aligned}
& q t_{0}=q_{0} \\
& q t_{f}=q_{f}
\end{aligned}
$$

For the velocity constraints, if we want to have continuous velocity the final and initial velocities must be zero:

$$
\begin{aligned}
& \dot{q} t_{0}=0 \\
& \dot{q} t_{f}=0
\end{aligned}
$$

Thus we require polynomial with four independent coefficients to satisfy these constraints, so we can consider a cubic polynomial. The cubic polynomial will have this form

$$
\begin{equation*}
q t=a_{0}+a_{1} t+a_{2} t^{2}+a_{3} t^{3} \tag{4.1}
\end{equation*}
$$

For the manipulator we have the initial position $q_{0}$ is known in the form of a set of joint angle, and the final position $q_{f}$ can be determined using inverse kinematic. The velocity and acceleration is given as

$$
\begin{gather*}
\dot{q} t=a_{1}+2 a_{2} t+3 a_{3} t^{2}  \tag{4.2}\\
\ddot{q} t=2 a_{2}+6 a_{3} t \tag{4.3}
\end{gather*}
$$

Combine equations 4.2 and 4.3 with four constraints yields four equation in four unknowns:

$$
\begin{gathered}
q_{0}=a_{0} \\
q_{f}=a_{0}+a_{1} t_{f}+a_{2} t_{f}^{2}+a_{3} t_{f}^{3} \\
\dot{q}_{0}=a_{1} \\
\dot{q}_{f}=a_{1}+a_{2} t_{f}+3 a_{3} t_{f}^{2}
\end{gathered}
$$

Solving these equations for $a_{i}$ we obtain

$$
\begin{gathered}
a_{0}=q_{0} \\
a_{1}=0 \\
a_{2}=3\left(q_{f}-q_{0}\right) \\
a_{3}=2\left(q_{f}-q_{0}\right)
\end{gathered}
$$

Using these parameters we can calculate the cubic polynomial that connects any initial joint angle position with any desired final position. This solution is for the case when the joints starts and finishes at zero velocity. Figure 4.1 shows the cubic polynomial trajectory


Figure 4.1: a) Cubic Polynomial Trajectory b) Velocity profile for Cubic Polynomial Trajectory c) Acceleration profile for cubic polynomial Trajectory

### 4.3.2 Linear segment with parabolic blends (LSPB)

Another way to generate joint space trajectories is by using so-called linear segments with parabolic blends (LSPB). This type of trajectory has trapezoidal velocity profile, and is appropriate when a constant velocity is desired along portion of path.

This is a linear function but we add a parabolic blend region at the beginning and end of the path. These blend regions create a smooth path with continuous position and velocity. Thus, during the blend portion of the trajectory, constant acceleration is used to change velocity smoothly. Figure 4.2 shows a simple path constructed in this way.

In order to construct this single segment we will assume that the parabolic blend both have the same duration, and therefore, they have the same constant acceleration.


Figure 4.2: linear segments with parabolic Blends

For parabolic blends near the path points with the same duration $t_{b}$ (blend time) the whole trajectory is symmetric about the halfway point in time $t_{b}$ and about the halfway point in position $q_{m}$.

The velocity at the end of first blend or at the beginning of second blend must equal to linear segment, thus we have:

$$
\begin{equation*}
\mathrm{q}=\frac{\mathrm{q}_{\mathrm{m}}-\mathrm{q}_{\mathrm{b}}}{\mathrm{t}_{\mathrm{m}}-\mathrm{t}_{\mathrm{b}}} \tag{4.4}
\end{equation*}
$$

Where $q_{b}$ is the value of joint variable at the end of blend segment at time $t_{b}, \ddot{q}$ is the acceleration during the blend segment, and the joint variable $q_{b}$ is given by

$$
\begin{equation*}
q_{b}=q_{0}+\frac{1}{2} \ddot{q} t_{b}^{2} \tag{4.5}
\end{equation*}
$$

Combining equations 3 and 4 and $t=2 t_{h}$

$$
\begin{equation*}
\ddot{q} t_{b}^{2}-t t_{b}+q_{f}-q_{0}=0 \tag{4.6}
\end{equation*}
$$

Where $t$ is the desired duration of the motion. Usually equation 3.10 is solved for a corresponding $t_{b}$, and the acceleration $\ddot{q}$ is chosen. Solving equation (4.5) for $t_{b}$

$$
\begin{equation*}
t_{b}=\frac{t}{2}-\frac{\overline{\ddot{q}^{2} t^{2}-4 \ddot{q}\left(q_{f}-q_{0}\right)}}{2 \ddot{q}} \tag{4.7}
\end{equation*}
$$

The constraint on the choice of acceleration used in blend segment is

$$
\ddot{q} \geq \frac{4 q_{f}-q_{0}}{t^{2}}
$$

The complete LSPB trajectory is given by

$$
q t=\begin{array}{cr}
q_{0}+\frac{\ddot{q} t^{2}}{2}, & 0 \leq t \leq t_{b} \\
q_{0}+\ddot{q} t_{b} t-\frac{t_{b}}{2}, & t_{b}<t \leq t_{f}-t_{b} \\
q_{f}-\frac{1}{2} \ddot{q} t_{f}-t^{2}, & t_{f}-t_{b}<t \leq t \tag{4.8}
\end{array}
$$

So the joint velocity and acceleration is given by

$$
\dot{q} t=\begin{array}{lr}
\ddot{q} t  \tag{4.9}\\
\ddot{q} t_{b} t \\
\ddot{q} t_{f}-t, & 0 \leq t \leq t_{b} \\
t_{b}<t \leq t_{f}-t_{b} \\
t_{f}-t_{b}<t \leq t_{f}
\end{array}
$$

$$
\ddot{q} t=\begin{array}{lr}
\ddot{q}, & 0 \leq t \leq t_{b}  \tag{4.10}\\
\ddot{q} t_{b}, & t_{b}<t \leq t_{f}-t_{b} \\
\ddot{q}, & t_{f}-t_{b}<t \leq t_{f}
\end{array}
$$

Figure 4.3 shows the LSPB Trajectory, velocity profile and acceleration profile.


Figure 4.3: a) LSPB trajectory b) velocity profile for LSPB trajectory c) Acceleration profile for LSPB trajectory.

In Our project we use Cubic Polynomial method since its easier to program and need less user specification than LSPB method

## Chapter Five

## Control Design

### 5.1 Introduction

### 5.2 System Architecture

### 5.2.1 Physical System Description

### 5.2.2 Functional description

### 5.2.3 Hydraulic Description

### 5.2.4 Electrical Description

### 5.3 Closed Loop Control

### 5.3.1 Feedback Sensors

### 5.3.2 Controller and software

### 5.1 Introduction

Robot control concerns with studying how to make a robot manipulator execute the desired task automatically. Typically, a robot takes the form of an equation or an algorithm which is realized via specialized computer program (TwidoSuite in our case). Robots controller's forms the so-called robot control system which is physically constitute of a computer, programmable logic controller, actuator, the robot itself and some extra electronics as illustrated in Figure 5.1.In this chapter we will describe how the overall system works.


Figure 5.1: the robot control system

### 5.2 System Architecture

### 5.2.1 Physical System Description

The purpose of the articulated hydraulic manipulator to pick an object and move it to another location as we mentioned before the hydraulic manipulator consist from a three link arm and the end-effectors which is a gripper type. Each link is actuated with a hydraulic rotary actuator with limited rotation. Figure 5.1 shows this type of rotary actuator

For the first link, the rotary actuator provides a rotation of angle from $-67^{\circ}$ to $113^{\circ}$ relative to the base frame. So this link will rotate a full $180^{\circ}$ rotation.

The second link rotation affects the overall angle of the arm, and it permitted to rotate through an angle from $-45^{\circ}$ to $45^{\circ}$ relative to the shoulder joint producing an angle of rotation of $90^{\circ}$.

The third link is rotate by angle of $90^{\circ}$, from $0^{\circ}$ to $90^{\circ}$ relative to the elbow frame.


Figure 5.2: Hydraulic rotary actuator for the joints

### 5.2.2 Functional description

The user is asked to enter the coordinates of the initial position and the final position of the end effectors through the touch screen, the manipulator go the initial position, pick an object and move to the final location. After the final position is reached the object is released, if there is no new coordinates are entered the manipulator may be programmed to move to an assigning suit which is the position where is no motion is executed.

### 5.2.3 Hydraulic Description

The system is operated by a fixed displacement hydraulic pump driven by an electric motor. The hydraulic pump provide a regulated flow to the main directional control valve which control the overall flow of the system by permitting a flow to the other parts of the hydraulic circuit or diverting the flow back to the reservoir. A proportional flow control valve is used to control the flow rate of the hydraulic actuators using a single solenoid.
each rotary actuator is controlled by a directional control valve that allow a clockwise and counterclockwise rotation ,these directional valves are actuated through tow solenoids at the ends of each valve, one solenoid for clockwise rotation and the other for counterclockwise rotation .

The system is protected against overload pressure by using a pressure relief valve and it's placed out of the hydraulic pump. Figure 5.2 shows the hydraulic circuit of the system.


Figure 5.3: Hydraulic circuit for the system

### 5.2.4 Electrical Description

The machine is powered by 220 volt AC power .Incoming AC power is routed in parallel to the hydraulic pump electric motor, main valve and to the DC power supply .the power supply ensures that the system voltage is regulated to 24 volts DC. The system voltage is routed through an emergency power shutoff switch to DC power supply, toggle switches and system electronic unit. A simplifies electrical circuit for 220 volt AC power supply is shown in Figure 5.3


Figure 5.4: Simplifies electric circuit for 220 volt power supply

### 5.3 Closed Loop Control

The term closed-loop control refers to the robot system managing the flow demand by routing the valve system to achieve the desired movement or desired position with smooth motion, and using sensors that give a feedback read for the final position for each link angle. We used the PLC for this operation, this logic controller have the following specification:

1. Four analog inputs to read the position for each link through the potentiometer.
2. Seven digital output; each output is connected to each solenoid to give a signal for desired motion.
3. One analog output to control the flow through proportional valve.

To achieve this specification we choose Schneider PLC with HMI Touch screen to achieve this job. Figure 5.4 shows this type of plc and touch screen.


Figure5.5: Plc and HMI touch screen

As we mentioned before the amount of flow can be controlled by proportional valve through an analog signal ( $0-10 \mathrm{v}$ ), this signal can be changed through an analog output port of the controller.

### 5.3.1 Feedback Sensors

In a closed loop control system, four sensors monitor the system output (joint Angeles) and feed the data to a controller which adjusts the control (joint angle) as necessary to maintain the desired system output (match the desired position which is $X, y$ and $z$ coordinates of the end effectors). This robot uses potentiometers to determines where it and then controls their joints to match the desired position. The output of the potentiometer is an analog voltage that is proportional to the angle of rotation for each joint. These analog signals are connecting to the analog input of the PLC.

## Potentiometer calibration

The calibration of potentiometer aims to find and represent the angles of the joints according to the output voltage of each potentiometer attached to its joint, this voltages inter to the module as analog input, the by some equations we determine the equivalent angle.

For the first link (A), the angle $\theta_{1}$ is between [ $-30{ }_{\sim}$ 160] and the voltage v 1 change between [6.63 _ 1.62], so the relationship between them described as shown in figure 5.6.


Figure 5.6 : relation between voltage and angle for the first joint

So the formula of represent angle to according voltage is

$$
\theta_{1}=-36.28 * v_{1}+214
$$

For the second link (B), the angle $\theta_{2}$ is between [45 _ - 6] and the voltage v2 change between [7.5 _ 2.78] , so the relationship between them described as shown in figure 5.7


Figure 5.7: relation between voltage and angle for the first joint

So the formula of represent angle to according voltage is

$$
\theta_{2}=11.94 * v_{2}-42.37
$$

For the third link (C), the angle $\theta_{3}$ is between [-90 _-135] and the voltage v3 change between [6.05 _ 9.37], so the relationship between them described as shown in figure 5.8.


Figure 5.8 : relation between voltage and angle for the first joint

So the formula of represent angle to according voltage is

$$
\theta_{3}=-11.71 * v_{3}-21.9
$$

### 5.3.2 Controllers and software

We are now interested in solving motion control problem .In motion control problem, the manipulator moves to a position to pick up an object, transport that project to another location, and deposit it. We treat this problem in the joint space.

## Joint space control

The main goal of the joint space control is to design a feedback controller such that the joint coordinates track the desired motion as closely as possible .the control of robot manipulators is naturally achieved in the joint space. Since the control inputs are the joint torques

Figure 5.6 shows the basic outline of the joint space control methods. Firstly the desired motion, which is described in terms of end-effectors coordinates, is converted to a corresponding joint trajectory using the invest kinematics of the manipulator, Then the feedback controller determines the joint torque necessary to move the manipulator along the desired trajectory specified in joint coordinates starting from measurements of the current joint states.


Figure 5.6 Generic concept of joint space control

## Independent joint control

We adapt independent joint control to control the robot manipulator. By independent-joint control (i.e.., decentralized control) we mean that the control inputs of each joint only depends on the measurement of the corresponding joint displacement and velocity. Due to its simple structure, this kind of control schemes offers many advantages. For example by using independent- joint control, communication among different joint is saved. Moreover, since the computational load of controller may be reduced, only low-cost hardware is required in actual implementations. Finally, independent-joint control has the feature of scalability, since the controller on all joints has the same formulation.

The simplest independent-joint control strategy is to control each joint axis as single-input single-output (SISO) system; this type of control appears in figure 5.7. This figure is common for all links.


Figure 5.7: Concept of joint space control
Because of electrically actuated solenoid valves which using for control the actuator is (on /off control). Then we can't control the velocity for each joint independently by this method, as mentioned before, there is one proportional valve to control the over flow and thus the velocity for all links, this leads to seek for a method to control all links velocity with common controller, this controller formulation depends on the hardware in the loop simulation .

In order to find the controller, and since we don't knew the transfer function for the proportional valve, will use the experiments depends on Hardwar-inLoop simulation as mentioned before to find the formulation of the controller.

We will use a PI controller to control the position of each joint and using experiments the best gain values for the controller is:
$K P=50$
$\mathrm{KI}=1200$
$G(s)_{\text {Controller }}=50+\frac{1200}{s}=\frac{50 s+1200}{s}$

## Chapter six

## Hardware and Software Description

6.1 Introduction
6.2 Hardware and Software Components
6.3 Sequence function chart -SFC (State Graph)
6.4 PID functions on Plc.
6.4.1 Introduction
6.4.2 The PID Controller Model
6.4.3 Operating Principles
6.4.4 Principal of the Regulation Loop
6.4.5 Role and Influence of PID Parameters
6.5 Touch Screen and Vijeo Designer

### 6.1 Introductions

In this chapter we will describe the hardware and software components that used in our project, and then we introduce Sequence function chart - SFC (or State graph) and the Concept of PID function, following by an introduction to HMI Software.

### 6.2Hardware and Software Components:

In our project we use these Types of Hardware:

1) TWDLCDE40DRF Twido Controller: which is a compact base controller, 24 v DC, 40 points, 24v DC inputs, 12-2A relay output, 2-1A transistor output, Timer and Calendar and Ethernet 100Base Tx, Removable Battery and Non-removable terminal blocks.
2) TM2AMM6HT Analog Expansion Module: it's an Expansion Module with 4Analog inputs and 2 analog outputs $(0-10 \mathrm{v}, 4-20 \mathrm{~mA}), 12$ bit resolution, removable screw terminals.
3) XBT OT 2210 Touch Screen: its 256 color and has a Supply Voltage Range 19.2 V DC to 28.8 V DC.

In this project we use TwidoSuit V2.31.04 for programming the PlC using sequence function chart (SFC) method.

And we used Vijeo Designer Opti for Programming the Xbt OT 2210 Touch screen.

### 6.3 Sequence function chart -SFC (State Graph)

Sequential Function Charts (SFCs) are a graphical technique for writing concurrent control programs or sequential control algorithms, and they are also known as Grafcet or IEC 848.

A sequence function chart is a pictorial representation of the system's individual operations, which when combined show the complete sequence of events. Once this diagram has been produced, then from it, the corresponding ladder diagram can be more easily designed. Figure 6.1 shows an example of the sequence function chart.

## Description of sequence function chart



Figure 6.1 sequence function chart example

The above figure is an example of sequence function chart and we will describe it in the following points:

1) The sequence function chart consists, basically, of a number of separate sequentially connected states, which are the individual constituents of the complete machine cycle that controls the system. An analogy is that each state is like a piece of a jigsaw puzzle; on its own it does not show very much, but when all the pieces are correctly assembled, then the complete picture is revealed.

Each state has the following:
(a) An input condition.
(b) An output condition.
(c) A transfer condition.

When the input condition into a state is correct, then that state will produce an output condition. That is, an output device or devices will be:
(a) Turned ON and remain ON.
(b) Turned OFF and remain OFF.
2) When the output or outputs are turned ON/OFF, then the system's input conditions will change to produce a transfer condition.
3) The transfer condition is now connected to the input condition of the next sequential state.
4) If the new input condition is correct, then the sequence moves to the next state.
5) From the sequence function chart, it can be seen that when the start pushbutton is operated, this is the input condition for state 0 .
6) The output condition from State 0 is the startup sequence, which will reset both Solenoid A and Solenoid B. With Inputs X2 and X4 now made, the transfer from State 0 can take place.
7) The transfer conditions from State 0 are the correct input conditions for State 1, and hence the process now moves from State 0 to State 1.
8) The process will now continue from one state to the next, until the complete machine cycle is complete.
9) From the sequence function chart, the ladder diagram can now be produced

For the articulated manipulator the sequence function chart is shown in Figure 6.2




Figure 6.2: State graph for the articulated manipulator

This state graph is for three joint controls, the state graph sequence start at state zero and at this state nothing is active. And as we see there is Stop condition input at each state, this input reset all states and take the sequence to state 0 , and any active device will turn OFF.

When the user push on start switch which is the input condition to next state, the state one is active but there is not outputs, it just take to two choices ( two input conditions ) to the next step.

The input conditions now are a comparison of the error (1) it equal zero or not, if the error is equal zero then sequence go to step five which to move joint two, if the error (1) is not equal zero then the sequence go to state two and there is another comparison between the desired and actual angle.

If the desired angle is more than the actual state three will activated and the main valve, PID 0 and $Q 0.11$ is turned on, the result of this is move link A counter clock wise according to the desired angle. If the desired angle is less than the actual state four ( $Q 0.12$ turned on) which move link A clock wise to the desired angle. After link A reach desired angle the sequence move to state five.

The same thing do to the joint two and joint three until the end effectors of the Robot reach to the desired position.

Note: The Ladder Program is shown in Appendix A

### 6.4 PID functions on Plc

### 6.4.1 Introduction

The PID control function onboard all Twido controllers provides an efficient control to simple industrial processes that consist of one system stimulus (referred to as Set point in this Book) and one measurable property of the system (referred to as Measure or Process Variable).

The approach of PID controller used in this project is to achieve responsive and accurate positioning performance of the end effectors of the Robot, for each link (joint) we have a PID controller. By the PID controller the end effectors start moving at somewhat high speed then the speed decreased according to the error which is the difference between actual angle of joint and the desired angle.

This regulation function is particularly adapted to:

1) Answering the needs of the sequential process which need the auxiliary adjustment functions (examples: plastic film packaging machine, finishing treatment machine, presses, etc.)
2) Responding to the needs of the simple adjustment process (examples: metal furnaces, ceramic furnaces, small refrigerating groups, etc.)

It is very easy to install as it is carried out in the:

1. Configuration
2. and Debug

Screens associated with a program line (operation block in Ladder Language or by simply calling the PID in Instruction List) indicating the number of the PID used. The correct syntax when writing a PID instruction is: PID<space>n, when $n$ is the PID number.

Example of a program line in Ladder Language:


NOTE: In any given Twido automation application, the maximum number of configurable PID functions is 14 .

### 6.4.2 The PID Controller Model

The Twido PID controller implements a mixed (serial - parallel) PID correction (see PID Model Diagram below) via an analog measurement and set point in the [0-10000] format and provides an analog command to the controlled process in the same format.
The mixed form of the PID controller model is described in the following diagram:


Figure 6.3 PID controller model

Where:

- I = the integral action (acting independently and parallel to the derivative action),
- $\mathrm{D}=$ the derivative action (acting independently and parallel to the integral action),
- $\mathrm{P}=$ the proportional action (acting serially on the combined output of the integral and derivative actions,
- $\mathrm{U}=$ the PID controller output (later fed as input into the controlled process.)

The PID controller is comprised of the mixed combination (serial - parallel) of the controller gain (Kp), and the integral (Ti) and derivative (Td) time constants. Thus the PID control law that is used by the Twido controller is of the following form:

$$
\begin{equation*}
u i=K_{p} . \varepsilon i+\frac{T_{S}}{T_{i}} \sum_{j=1}^{i} \varepsilon j+\frac{T_{d}}{T_{s}} \varepsilon i-\varepsilon i-1 \tag{6.1}
\end{equation*}
$$

Where

- $\mathrm{Kp}=$ the controller proportional gain,
- $\mathrm{Ti}=$ the integral time constant,
- $\mathrm{Td}=$ the derivative time constant,
- $\mathrm{Ts}=$ the sampling period,
- $\varepsilon(\mathrm{i})=$ the deviation $(\varepsilon(\mathrm{i})=$ set point - process variable. $)$

NOTE: Two different computational algorithms are used, depending on the value of the integral time constant ( Ti ):

- $\mathrm{Ti} \neq 0$ : In this case, an incremental algorithm is used.
- $\mathrm{Ti}=0$ : This is the case for non-integrating processes. In this case, a positional algorithm is used, along with a +5000 offset that is applied to the PID output variable.


### 6.4.3 Operating Principles

The following diagram presents the operating principle of the PID function


Figure 6.4: PID Operating Principals on PLC

Her the setpoint represent the desired angle that the joint should go and the measure point is the actual angle of the respective joint, the error of the summing point enters to the PID controller (PI in our case). According to Controller gains there will be an output, this output can be limited using saturation or limiter as shown in the figure. In our project we use saturation for each joint with min value 3 v and max value 7 v ,since we don't need a very high speed or very low speed.

The PID function has two modes: analog output or PWM output, in our project we use the analog output to control the proportional valve opening which control the flow of the system.

### 6.4.4 Principal of the Regulation Loop

## At a Glance

The working of a regulation loop has three distinct phases:

- The acquisition of data:
$>$ Measurements from the process' sensors (analog, encoders)
$>$ Setpoint(s) generally from the controller's internal variables or from data from a TwidoSuit animation table
- Execution of the PID regulation algorithm.
- The sending of orders adapted to the characteristics of the actuators to be driven via the discrete (PWM) or analog outputs.

The PID algorithm generates the command signal from:

- The measurement sampled by the input module
- The setpoint value fixed by either the operator or the program
- The values of the different corrector parameters

The signal from the corrector is either directly handled by a controller analog output card linked to the actuator, or handled via a PWM adjustment on a discrete output of the controller.

## Illustration

The following diagram schematizes the principal of a regulation loop


Figure 6.5: principal of a regulation loop

### 6.4.5 Role and Influence of PID Parameters

## Influence of Proportional Action

Proportional action is used to influence the process response speed. The higher the gain, the faster the response and the lower the static error (in direct proportion), though the more stability deteriorates. A suitable compromise between speed and stability must be found. The influence of Proportional on process response to a Scale division is as follows:


Figure 6.6: Influence of Proportional Action

## Influence of Integral Action

Integral action is used to cancel out static error (deviation between the process value and the set point). The higher the level of integral action (low Ti), the faster the response and the more stability deteriorates. It is also necessary to find a suitable compromise between speed and stability. The influence of integral action on process response to a scale division is as follows:


Figure 6.7: Influence of Integral Action

NOTE: A low Ti means a high level of integral action.

Where $\mathrm{Kp}=$ proportional gain, $\mathrm{Ti}=$ integration time and $\mathrm{Td}=$ derivative time .

## Influence of Derivative Action

Derivative action is anticipatory. In practice, it adds a term which takes account of the speed of variation in the deviation, which makes it possible to anticipate changes by accelerating process response times when the deviation increases and by slowing them down when the deviation decreases. The higher the level of derivative action (high Td ), the faster the response. A suitable compromise between speed and stability must be found. The influence of derivative action on process response to scale division is as follows:


Figure 6.7: Influence of Derivative Action

### 6.5 Touch Screen and Vijeo Designer

The touch screen that we use in this project is Magelis XPT OT 2110 Schneider Touch screen; we decide to choose this touch screen because it meet the specification required and hiveless cost and have the following specification:

| Type $\quad$ Advanced touch screen panel |  |
| :--- | :--- |
| Display type | Backlit monochrome STN LCD |
| Supply Voltage Range: | 19.2 V DC to 28.8V DC |
| Display color | 16 levels of grey Blue and white |
| Display resolution | $320 \times 240$ pixels QVGA |
| Display | size 5.7 inch |
| Software type | Configuration software |
| Software designation | Vijeo Designer |
| Operating system | Magelis |
| Processor name | CPU RISC |
| Processor frequency | 133 MHz |
| Memory description | Back up of data SRAM 128 kB lithium battery |
|  | $\quad$ Application memory flash EPROM 16 MB |

We use Vijeo Designer Opti software to program the touch screen, and by touch screen the user can inter the desired coordinate of the end effectors (X, Y,Z).This coordinates are transferred to PLC via Modbus communication protocol which is a master/slave protocol that's allow for one, and only one, master to request responses from slaves or to act based on the request.

The touch screen will be like this:


The user is asked to enter the desired position then these coordinates are transferred to ple to move to the desired position

## Chapter Seven

## Experiments and results

### 7.1 Introduction

7.2 Experimental results

### 7.3 Conclusion

7.4 Future Work

### 7.1 Introduction

This chapter contains the results that are obtained from the experiments which are done to verify the theoretical results that disused on the previous chapters. Then these results are discussed and explained from the experimental side.

### 7.2 Experimental results

In this section we mention the experimental results that we performed, here we show the step response of each joint as shown below:

## First Joint



Figure7.1:step response of first joint

## second Joint



Figure7.2:step response of second joint

Third Joint


Figure7.3:step response of third joint

Each time we perform the experiments there is an error on each angle and this error appear due to several Reasons:

1) The hydraulic system is a non linear system so we can't model the system to find the appropriate controller, however we use tuning process to find the most suitable controller based on experimental computation.
2) The feedback sensors are potiometers so there must be an error associated with the mechanical part of the sensor, and as a result the robot will reach the desired position with some certain errors.

### 7.3 Conclusion

Robotics has become recently an interesting area of research. In our project we apply robotic theories, control techniques to accomplish the main goal of our project which is to move the robot to the desired position.

Moving the robot requires a control system, we used the Twido controller for this task, and based on some control algorithms we program our system. Moving all the joint together can't be accomplished using PLC so we design control technique to move the first joint followed by the second joint and then the third joint.

### 7.4 Future Work

The results of this work can be a basic point for the future studies .we can design different control algorithms with different hardware and servo system to produce a high performance robot arm. Another subject is that we could use another method of control; it may be microcontroller or DAQ system.

## Appendices A

TwidoSuit Programs

# TwidoSuite Articulated Manipulator 



## TwidoSuite Version 2.31.04

## Project Information

Print date
7/31/2013
Author
Department
Index
Industrial
Property
Comment

## History

## Contents

First Page ..... 1
History ..... 3
Contents ..... 4
Graphical Description ..... 5
Properties ..... 7
Bill of material ..... 8
Hardware ..... 9
Memory objects ..... 10
Memory report ..... 15
Behavior configuration ..... 16
Content ..... 17
symbols ..... 47
Cross references ..... 50
Animation table ..... 51
Preferences ..... 53
About ..... 55
Total Page Number ..... 55


Page 2


Page 1

## Properties



## Bill of material

| Family | Reference number | Quantity |
| :--- | :--- | :--- |
| Twido | TWDLCDE40DRF | 1 |
| Twido | TM2AMM6HT | 1 |
| Modbus elements | Magelis | 1 |

## Hardware configuration

## Base

TWDLCDE40DRF

## Expansion bus modules

1 : TM2AMM6HT

## Memory objects configuration

Timer configuration (\%TM)
Counter configuration (\%C)
Register configuration (\%R)
Drum configuration (\%DR)
Scheduler block configuration (\%SCH)
Fast counters configuration (\%FC)
Very fast counters configuration (\%VFC)
Memory words (\%MD)
Memory words (\%MW)

| Used | \%MWW | Symbol | Allocated |
| :--- | :--- | :--- | :--- |
| Yes | \%MWO |  | Yes |
| Yes | \%MW1 |  | Yes |
| Yes | \%MW20 | Z1_ACT | Yes |
| Yes | \%MW22 | Z1_DIS | Yes |
| Yes | \%MW30 | Z2_DIS | Yes |
| Yes | \%MW39 | Z3_ACT | Yes |
| Yes | \%MW76 |  | Yes |
| Yes | \%MW88 |  | Yes |
| Yes | \%MW90 | Z2_ACT | Yes |
| Yes | \%MW100 |  | Yes |
| Yes | \%MW156 |  | Yes |
| Yes | \%MW160 |  | Yes |
| Yes | \%MW193 |  | Yes |
| Yes | \%MW333 |  | Yes |
| Yes | \%MW350 | Yes |  |
| Yes | \%MW702 | Z3_DIS | Yes |
| Memory words (\%MF) |  |  |  |


| Used | \%MF | Symbol | Allocated |
| :--- | :--- | :--- | :--- |
| Yes | \%MF127 |  | Yes |
| Yes | MMF130 | X0 | Yes |
| Yes | \%MF132 | Y0 | Yes |
| Yes | \%MF134 | Z0 | Yes |
| Yes | \%MF200 | Z_DIS1 | Yes |
| Yes | \%MF202 | Z_ACT1 | Yes |
| Yes | \%MF203 | E2 | Yes |
| Yes | \%MF204 | E1 | Yes |
| Yes | \%MF206 |  | Yes |
| Yes | \%MF208 | VOLTAGE_IN_CHANAL_1 | Yes |
| Yes | \%MF212 | V2 | Yes |
| Yes | \%MF214 | V3 | Yes |
| Yes | \%MF252 |  | Yes |
| Yes | \%MF290 | E2_OO | Yes |
| Yes | \%MF300 |  | Yes |


| Used | \%MF | Symbol | Allocated |
| :---: | :---: | :---: | :---: |
| Yes | \%MF302 | Z_ACT2 | Yes |
| Yes | \%MF306 | Z_DIS2 | Yes |
| Yes | \%MF315 |  | Yes |
| Yes | \%MF317 | VOLTAGE_IN_CH2 | Yes |
| Yes | \%MF320 |  | Yes |
| Yes | \%MF402 | Z_ACT3 | Yes |
| Yes | \%MF407 | Z_DIS3 | Yes |
| Yes | \%MF412 |  | Yes |
| Yes | \%MF420 | X1 | Yes |
| Yes | \%MF422 | Y1 | Yes |
| Yes | \%MF424 | Z1 | Yes |
| Yes | \%MF430 |  | Yes |
| Yes | \%MF432 |  | Yes |
| Yes | \%MF434 |  | Yes |
| Yes | \%MF436 |  | Yes |
| Yes | \%MF563 | E3 | Yes |
| Yes | \%MF566 | E3_00 | Yes |
| Yes | \%MF600 |  | Yes |
| Yes | \%MF623 |  | Yes |
| Yes | \%MF650 | VOLTAGE_IN_CHO | Yes |
| Yes | \%MFF777 | VOLTAGE_OUT_CHO | Yes |
| Yes | \%MF1006 |  | Yes |
| Yes | \%MF1008 |  | Yes |
| Yes | \%MF1010 |  | Yes |
| Yes | \%MF1012 |  | Yes |
| Yes | \%MF1014 |  | Yes |
| Yes | \%MF1016 |  | Yes |
| Yes | \%MF1018 |  | Yes |
| Yes | \%MF1042 |  | Yes |
| Yes | \%MF1044 |  | Yes |
| Yes | \%MF1046 |  | Yes |
| Yes | \%MF1048 |  | Yes |
| Yes | \%MF1050 |  | Yes |
| Yes | \%MF1052 |  | Yes |
| Yes | \%MF1054 |  | Yes |
| Yes | $\% M F 1056$ |  | Yes |
| Yes | \%MF1058 |  | Yes |
| Yes | \%MF1060 |  | Yes |
| Yes | \%MF1062 |  | Yes |
| Yes | \%MF1066 |  | Yes |
| Yes | \%MF1068 |  | Yes |
| Yes | \%MF1070 |  | Yes |
| Yes | \%MF1088 |  | Yes |
| Yes | \%MF1090 |  | Yes |
| Yes | \%MF1092 |  | Yes |
| Yes | \%MF1094 |  | Yes |
| Yes | \%MF1096 |  | Yes |
| Yes | \%MF1098 |  | Yes |
| Yes | \%MF1102 |  | Yes |
| Yes | \%MF1104 |  | Yes |
| Yes | \%MF1106 |  | Yes |
| Yes | \%MF1200 |  | Yes |
| Yes | \%MF1202 |  | Yes |
| Yes | \%MF1204 |  | Yes |
| Yes | \%MF1208 |  | Yes |
| Yes | \%MF1210 |  | Yes |
| Yes | \%MF1212 |  | Yes |
| Yes | \%MF1214 |  | Yes |
| Yes | \%MF1216 |  | Yes |
| Yes | \%MF1218 |  | Yes |


| Used | \%MF | Symbol | Allocated |
| :--- | :--- | :--- | :--- |
| Yes | \%MF1800 | XC | Yes |
| Yes | \%MF1802 | YC | Yes |
| Yes | \%MF1804 | ZC | Yes |
| Yes | \%MF2008 |  | Yes |
| Yes | \%MF2013 | THETA2 | Yes |
| Yes | \%MF2025 | THETA3 | Yes |
| Yes | \%MF2156 | Yes |  |
| Memory bits $(\% M)$ |  |  |  |


| Used | \%M | Symbol | Allocated |
| :---: | :---: | :---: | :---: |
| Yes | \%MO | T0_1 | Yes |
| Yes | \%M1 | T1_2 | Yes |
| Yes | \%M2 | T2_3 | Yes |
| Yes | \% M 3 | T3_4 | Yes |
| Yes | \%M4 | T4_5 | Yes |
| yes | \%M5 | T4_6 | Yes |
| Yes | \%M6 | T1-0 | Yes |
| Yes | \%M14 | T5_7 | Yes |
| yes | \%M15 | T6_7 | Yes |
| Yes | \%M16 | T7-8 | Yes |
| Yes | \%M17 | T8_9 | Yes |
| Yes | \%M18 | T9_10 | Yes |
| Yes | \%M19 | T10_11 | Yes |
| Yes | \%m20 | T11_12 | Yes |
| Yes | \%M21 | T11_13 | Yes |
| Yes | \%M22 | T12_14 | Yes |
| Yes | \%M23 | T13_14 | Yes |
| Yes | \%M31 | T14_15 | Yes |
| Yes | \%M32 | T15_16 | Yes |
| Yes | \%9M40 | T21.0 | Yes |
| Yes | \%M41 | T9_8 | Yes |
| Yes | \%M42 | T16_15 | Yes |
| Yes | \%M43 | T16_17 | Yes |
| Yes | \%M44 | T17_18 | Yes |
| Yes | \%M45 | T18_19 | Yes |
| Yes | \%M46 | T18_20 | Yes |
| Yes | \%M47 | T19_21 | Yes |
| Yes | \%M48 | T20_21 | Yes |
| Yes | \%M57 | StART | Yes |
| Yes | \%M59 | STOP | Yes |
| Yes | \%M77 |  | Yes |
| Yes | \% 6888 |  | Yes |
| Yes | \%M90 |  | Yes |
| Yes | \%M91 |  | Yes |
| Yes | \%M92 |  | Yes |
| Yes | \%M99 |  | Yes |
| Yes | \%M100 | ANGLE1_REACHED | Yes |
| Yes | \%M101 | ANGLE2_REACHED | Yes |
| Yes | \%M233 |  | Yes |
| Yes | \%M244 |  | Yes |


| O 0 : configured |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| General |  |  |  |  |  |  |
| Operating mode | : | Word ad |  |  |  |  |
| PID Status | : | Allow |  |  |  |  |
| Input |  |  |  |  |  |  |
| Measure | : | \%MW20 |  |  |  |  |
| Conversion | : | Inhibit | Min | : | Max | : |

TwidoSuite Version 2.31.04
Articulated Manipulator
7/31/2013


## Constant configuration (\%KD)

## Constant configuration (\%KW)

## Constant configuration (\%KF)

| Used | \%KF | Symbol | Float |
| :--- | :--- | :--- | :--- |
| Yes | \%KF50 | ZERO | 0.0 |
| Yes | \%KF60 | C1 | 0.0024414 |
| Yes | \%KFF90 | GLL | 16.5 |
| Yes | \%KF233 | D1 | 0.27235 |
| Yes | \%KF240 | A2 | 0.2604 |
| Yes | \%KF244 | A3 | 0.25 |

PLS/PWM configuration (\%PLS/\%PWM)
Configuration of external objects Comm
Configuration of external objects Drive
Configuration of external objects Tesys
Configuration of external objects Advantys OTB

## Memory

## Memory usage statistic

| Memory bits | : 245 Bits | 0.5\% |
| :---: | :---: | :---: |
| Memory words | : 2158 Words | 62.6\% |
| Backed up | : 715 words |  |
| RAM = EEPROM | : ??? |  |
| Constants | : 246 words | 7.2\% |
| Configuration | : 873 words | 25.3\% |
| Avail. mem. data | : 25 words | 0.6\% |
| User program |  |  |
| Executable code | : 2560 words | 16.6\% |
| Prog. data | : 4 Words | 0.1\% |
| Online modif. | : 0 words | 0.0\% |
| Avail. code mem. | : 12161 words | 78.7\% |
| other |  |  |
| Execution data | : 127 Words | 3.7\% |
| : | $\begin{aligned} & 123.7 \% \\ & 7 \\ & \text { Wo } \\ & \text { rd } \\ & \mathrm{s} \end{aligned}$ |  |

## Configure the behavior

## Functional levels

Functional levels management

```
Management :
Level :
Automatic
The highest possible
```


## Scan mode

Scan mode

```
Mode : Periodic
Duration (ms) :
150
```

Watchdog

```
Duration (ms) :20
```


## Periodic event

```
Not used :
    Yes
```


## Startup

## Parameters

```
Automatic start in Run :
Run/Stop Input:
No
None
```


## Autosave

## Parameters

```
Autosave RAM=>EEPROM :
    Yes
```


## Program lists and diagrams




## (2) LD Transitions - Sequential Function Chart [SFC]

## YC/XC

Rung 0


Rung 1


THETA 1 FOR C1 : : : : : : : : : :: : :: : :: : : :: : : : :: : : :: : : :: :
::::::::: :
Rung 2


Rung 3


Rung 4


Rung 5


Rung 6

$X^{\wedge}{ }^{\wedge} 2$
Rung 7



## S1 $=$ ZC - D1



## $\mathrm{R}^{\wedge} 2+\mathrm{S} 1^{\wedge} \mathbf{2}=\left(\mathrm{XC}^{\wedge} 2+\mathrm{YC}{ }^{\wedge} 2\right)+(Z C-D 1)^{\wedge} \mathbf{2}$



## A2^2



## A3^2

Rung 15


## 

Rung 16

$\left(R 1^{\wedge} 2+S 1^{\wedge} 2\right)-A 2^{\wedge} 2-A 3^{\wedge} 2$



## 2*A2*A3

Rung 19


Rung 22


Rung 24


## THETA 3 FOR D $>0$

Rung 25


THETA 3 FOR D < 0
Rung 26


THETA 3 FOR D $=0$



| Rung 29 | SHORT | \%MF1068: $=$ ATAN(\%MF1066) |
| :---: | :---: | :---: |
|  |  | \%MF1068 := ATAN( \%MF1066) |
|  |  |  |

## SECONED PART OF THETA 2 FOR ZC >= D1 CASE 1



ZC $<$ D1
Rung 31


S2^2


## S2^2+R1^2



## S2^2+R1^2+A2^2

| Rung 37 | SHORT |
| :--- | :--- |
|  |  |



A2 * (SQRT (R1^2+S2^2))

2.0 * A2 * (SQRT (R1^2+S2^2))

$\left(S 2^{\wedge} \mathbf{2 H R}^{+} 1^{\wedge} 2+A 2^{\wedge} 2-A 3^{\wedge} 2\right) /\left[2.0^{*} A 2^{*}\left(S Q R T\left(R 1^{\wedge} 2+S 2^{\wedge} 2\right)\right)\right]$
Rung 42


Rung 43


## FIRST PART OF THETA 2

Rung 44


THETA 2 FOR C1 :: : : :: : : : : :: : :: : :: : : :: : :: : : :: : :: : : :: : :
Rung 45


THETA 2 FOR C2 :: : : :: : : : : : : : : : : :: : : : : : : : : : : :: : :: : : : : : : :


Rung 47



## SECONED PART OF THETA 2 FOR ZC < D1 CASE 2



## S2^2



## S2^2+R1^2



## S2^2+R1^2+A2^2



## S2^2+R1^2+A2^2-A3^2



## SQRT(R1^2+S2^2)



## A2 * (SQRT (R1^2+S2^2))

Rung 57

\%MF1208 : = \%MF1204 * A2 \%MF1208:= \%MF1204 * \%KF240
2.0 * A2 * (SQRT (R1^2+S2^2))


Rung 60


## FIRST PART OF THETA 2



THETA 2 FOR C1 :: : : :: : : : : :: : :: : :: : : :: : :: : : :: : :: : : :: : :
Rung 62


THETA 2 FOR C2 :: : : :: : : : : :: : :: : :: : : :: : :: : : :: : :: : : :: : :


Rung 64





Rung 85

| STATE13 | E2 $<2.0$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| \%MWO:X1 | \%MF203 $<2.0$ |  |  |  |
| 3 |  |  | T13 14 |  |
| 3 |  |  |  |  |
|  |  |  |  |  |



| Rung 87 | STATE15 | T15 16 |
| :---: | :---: | :---: |
|  | \%MWO:X1 | \%M32 |
|  |  |  |
|  |  | () |


| Rung 88 | STATE16 | E3 $=0.0$ |
| :--- | :---: | :---: |
| $\% M W 1: X 0$ | \%MF563 $=0.0$ |  |
|  |  | $<$ |

T16 15 \%M42 ( )


| Rung 90 | STATE17 | T17 18 |
| :---: | :---: | :---: |
|  | \%MW1:X1 | \%M44 |
|  |  | () |

Rung 91


Rung 94

(3) LD States - Sequential Function Chart [SFC]



## STATE5

Rung 10

| T4 5$\%$ M 4 |
| :---: |
|  |  |
|  |  |



## STATE6

## Rung 12



Rung 13



STATE9
Rung 18



STATE10
Rung 20



STATE11
[CHECK THIS] 2
Rung 22


Rung 23


STATE12

Rung $24 |$| T11 | 12 |
| :--- | :--- | :--- |
| $\% M 20$ |  |

STATE12 \%MWO:X1


## STATE13

## Rung 26

| T11 13 |
| :--- |
| $\%$ M21 |
| -1 |

STATE13 \%MWO:X1 3
(s) -

STATE14
Rung 28

| T12 | 14 |  |
| :---: | :---: | :---: |
| $\% M 22$ |  |  |
|  |  |  |
|  |  |  |
| T13 | 14 |  |
| $\% M 23$ |  |  |
|  |  |  |
|  |  |  |

Rung 29

STATE15

| Rung 30 | $\begin{gathered} \text { T14_15 } \\ \% M 31 \end{gathered}$ |
| :---: | :---: |

Rung 31

STATE16

## Rung 32

| T15 16 |
| :--- |
| $\% M 32$ |

STATE16 \%MW1:X0
(s)-


## STATE17




| STATE |  | [CHECK THIS] 3 |
| :---: | :---: | :---: |
| Rung 36 | T17_18 | STATE18 |
| Rung 36 | \%M44 | \%MW1:X2 |
|  |  | $-(s)$ |

## STATE18

Rung 37


## STATE19



STATE19
Rung 39

| T19 21 | STATE19 |
| :---: | :---: |
| \%M47 | \%MW1:X3 |
|  | $(R)$ |
| STOP |  |
| \%M59 |  |
|  |  |




| (5) LD | Calculation of the first Joint |  |
| :---: | :--- | :--- |

## VOLTAGE IN CHANAL 0

Rung 0


Rung 1


VOLTAGE IN CHANAL 0 [FINAL VALUE]
VOLTAGE IN FOE CHANEL 0
Rung 2


## ANGLE 1 [Z_ACT 1] CALCULATION



Rung 4


## ANGLE 1 [FINAL VALUE]

Rung 5


Rung 6


## DISERED ANGLE OF JOINT 1

Rung 7


## ERORE OF THE ANGLE

Rung 8


TO THE ABSOLUTE VALUE OF THE ERRORE

| Rung 9 |  |  |
| :--- | :--- | :--- |
|  | EM90 $:=A B S(\% M F 252)$ |  |



## (6) LD Seconed Joint Calculations

VOLATGE IN CALCULATION :
Rung 0


Rung 1


VOLTAGE IN CHANAL 1 :[ FINAL VALUE]
Rung 2


ANGLE 2 [Z_ACT2] CALCULATION FROM VOLTAGE IN 1
Rung 3


## Z_ACT2 [ FINAL VALUE]

Rung 4


Z2_ACT [ FINAL VALUE]
Rung 5


## Z2_DIS [FINAL VALUE]



## ERRORE OF THE SECONED ANGLE

## Rung 7



Rung 8


## (7) LD Thired Joint Calculations

VOLTAGE IN CHANAL 2 CALCLATION
Rung 0


Rung 1


VOLTAGE IN CHANAL 2 [FINAL VALUE]
Rung 2


## ACTUAL ANGLE 3 CALUALATION

Rung 3


ACTUAL ANGLE 3 [ FINAL VALUE]
Rung 4


Rung 5


Rung 6



## DISERED ANGLE 3 [FINAL VALUE]

Rung 8


ERROR OF THE THIRED ANGLE


## ABSOLUTE VALUE OF THE ERROR



| 8 LD | PID Output Selection |  |
| :--- | :--- | :--- |

Rung 0

| STATE3 | $\mathrm{AO}:=\% \mathrm{MW88}$ |
| :---: | :---: |
| \%MVV0:X3 | \%QW0.1.0 := \%MW88 |
|  |  |
| STATE2 |  |
| STATE2 |  |
| \%MWV: X2 |  |
|  |  |
| STATE1 |  |
| \%MW0: X 1 |  |
|  |  |
| STATE5 |  |
| \%MWO:X5 |  |
|  |  |
| STATE4 |  |
| \%MWO:X4 |  |
|  |  |
| STATE7 |  |
| \%MWV: X7 |  |
|  |  |
| STATE6 |  |
| \%MVN0:X6 |  |
|  |  |


(9) LD Actions - Sequential Function Chart [SFC] $\quad$ I




## START THE PI CONTROLLER _OPEN PROPOTIONAL VALV



RESET OUTPUT \%Q0.0.12 IF ERROR <3.0
Rung 8

| STATE5 | E1 < 2.0 |
| :---: | :---: |
| $\% M W 0: X 5$ | $\%$ MF204 < 2.0 |
|  | $<$ |



RESET OUTPUT \%Q0.0.11 IF ERROR <3.0



Rung 13


FOR DIRECT AND REVERSE OF PID 1



| Rung 16 | $\begin{array}{\|c\|} \hline \text { STATE12 } \\ \% M W 0: X 1 \end{array}$ | $\begin{gathered} \text { OPEN_CD } \\ \text { W_DCV_J } \\ \text { OINT2 } \\ \% \text { \%Q.6 } \end{gathered}$ |
| :---: | :---: | :---: |

RESET OUTPUT \%Q0.0.6 IF ERRORE < 3.0
Rung 17


OPEN CC

| Rung 18 | STATE13 <br> \%MWO:X1 | $\begin{array}{\|l\|} \hline \text { OPEN_C } \\ \text { W_DCV_J } \\ \text { OINT2 } \\ \hline \% Q 0.7 \end{array}$ |
| :---: | :---: | :---: |

RESET OUTPUT \%Q0.0.7 IF ERROR <3.0
Rung 19


Rung 20


Rung 21


## [\%MF412] IS THE ABSOLUTE VALUE OF Z_DIS3

Rung 22


FOR DIRECT AND REVERSE OF PID2
Rung 23



## RESET OUTPUT \%Q0.0.0 IF ERRPR <3.0



Rung 27


RESET OUTPUT \%Q0.0.9 IF ERROR < 3.0


## Symbols

| Used | Address | Symbol | Comment |
| :---: | :---: | :---: | :---: |
| Yes | \%KF240 | A2 |  |
| Yes | \%KF244 | A3 |  |
| Yes | \%M100 | ANGLE1_REACHED |  |
| Yes | \%M101 | ANGLE2_REACHED |  |
| Yes | \%QW1.0 | AO |  |
| Yes | \%KF60 | C1 |  |
| No | \%KF70 | C2 |  |
| No | \%KF80 | C3 |  |
| Yes | \%KF233 | D1 |  |
| Yes | \%MF204 | E1 |  |
| No | \%MW95 | E1_ABS |  |
| Yes | \%6MF203 | E2 |  |
| No | \%Mw320 | E2_ABSS |  |
| Yes | \%MMF290 | E2_00 |  |
| Yes | \%MMF563 | E3 |  |
| Yes | \%MF5666 | E3_00 |  |
| Yes | \%KF90 | G_L |  |
| No | \%MW371 | KIO |  |
| No | \%MW311 | KI1 |  |
| No | \%MW18 | KI2 |  |
| No | \%Mw370 | KPO |  |
| No | \%MW310 | KP1 |  |
| No | \%MW17 | KP2 |  |
| Yes | \%Q0.11 | OPEN_CCW_DCV_JOINT1 |  |
| Yes | \% 00.6 | OPEN_CCW_DCV_JOINT2 |  |
| Yes | \%Q0. 9 | OPEN_CCW_DCV_JOINT3 |  |
| Yes | \%Q0. 12 | OPEN_CW_DCV |  |
| Yes | \%Q0.7 | OPEN_CW_DCV_JOINT2 |  |
| Yes | \%Q0.8 | OPEN_CW_DCV_JOINT3 |  |
| Yes | \%Q0. 10 | OPEN_MAIN_VALVE |  |
| Yes | \%IW1.1 | POT2 |  |
| Yes | \%IW1.2 | POT3 |  |
| No | \%IW1. 3 | POT4 |  |
| Yes | \%M57 | START |  |
| Yes | \%MW0: X0 | STATEO |  |
| Yes | \%MWO: X1 | STATE1 |  |
| Yes | \%MW0: $\times 2$ | STATE2 |  |
| Yes | \%MWW0: ${ }^{3} 3$ | STATE3 |  |
| Yes | \%MW0: X 4 | STATE4 |  |
| Yes | \%MWO: X5 | STATE5 |  |
| Yes | \%MWW0: $\mathrm{X6}$ | STATE6 |  |
| Yes | \%MWW0: $\times 7$ | STATE7 |  |
| Yes | \%MW0: $\times 8$ | STATE8 |  |
| Yes | \%MWW0: $\mathrm{X9}$ | STATE9 |  |
| Yes | \%MW0: $\times 10$ | STATE10 |  |
| Yes | \%MW0: $\times 11$ | STATE11 |  |
| Yes | \%WMW0: $\times 12$ | STATE12 |  |
| Yes | \%MW0: $\times 13$ | STATE13 |  |
| Yes | \%MW0: $\times 14$ | STATE14 |  |
| Yes | \%MW0: $\times 15$ | STATE15 |  |
| Yes | \%MW1: X0 | STATE16 |  |
| Yes | \%MW1: X1 | STATE17 |  |
| Yes | \%MW1: X2 | STATE18 |  |
| Yes | \%MW1: $\times 3$ | STATE19 |  |
| Yes | \%MW1: $\times 4$ | STATE20 |  |


| Used | Address | Symbol | Comment |
| :---: | :---: | :---: | :---: |
| Yes | \%MW1: X5 | STATE21 |  |
| Yes | \%M59 | STOP |  |
| Yes | \%MO | T0_1 |  |
| No | \%M11 | T1 |  |
| Yes | \%M6 | T1_0 |  |
| Yes | \%M1 | T1_2 |  |
| No | \%9M7 | T2_0 |  |
| Yes | \%M2 | T2_3 |  |
| No | \%M8 | T3_0 |  |
| Yes | \%M3 | T3_4 |  |
| No | \%M9 | T4_0 |  |
| Yes | \%M4 | T4_5 |  |
| Yes | \%M5 | T4_6 |  |
| No | \%M10 | T5_0 |  |
| No | \%M12 | T5_4 |  |
| Yes | \%M14 | T5_7 |  |
| No | \%M13 | T6_4 |  |
| Yes | \%M15 | T6_7 |  |
| No | \%M24 | T7_0 |  |
| Yes | \%6M16 | T7_8 |  |
| No | \%M25 | T8_0 |  |
| Yes | \%M17 | T8_9 |  |
| No | \%M26 | T9_0 |  |
| Yes | \%M41 | T9_8 |  |
| Yes | \%M18 | T9_10 |  |
| No | \%M27 | T10_0 |  |
| Yes | \%M19 | T10_11 |  |
| No | \%M28 | T11-0 |  |
| Yes | \%M20 | T11. 12 |  |
| Yes | \%M21 | T11_13 |  |
| No | \%M29 | T12_0 |  |
| No | \%M50 | T12_11 |  |
| Yes | \%M22 | T12_14 |  |
| No | \%M30 | T13_0 |  |
| No | \%M49 | T13_11 |  |
| Yes | \%M23 | T13_14 |  |
| No | \%M33 | T14_0 |  |
| Yes | \%M31 | T14_15 |  |
| No | \%M34 | T15_0 |  |
| Yes | \%M32 | T15_16 |  |
| No | \%M35 | T16_0 |  |
| Yes | \%M42 | T16_15 |  |
| Yes | \%M43 | T16_17 |  |
| No | \%M36 | T17_0 |  |
| Yes | \%M44 | T17_18 |  |
| No | \%M37 | T18_0 |  |
| Yes | \%M45 | T18_19 |  |
| Yes | \%M46 | T18_20 |  |
| No | \%M38 | T19_0 |  |
| No | \%M52 | T19_18 |  |
| Yes | \%M47 | T19_21 |  |
| No | \%M39 | T20_0 |  |
| No | \%M51 | T20_18 |  |
| Yes | \%M48 | T20_21 |  |
| Yes | \%M40 | T21-0 |  |
| Yes | \%MF2013 | THETA2 |  |
| Yes | \%MF2025 | THETA3 |  |
| No | \%MMF210 | V1 |  |
| Yes | \%MF212 | V2 |  |
| Yes | \%MF214 | V3 |  |


| TwidoSuite Version 2.31.04 |  | Articulated Manipulator |  |
| :---: | :---: | :---: | :---: |
| Used | Address | Symbol | Comment |
| Yes | \%MF650 | VOLTAGE_IN_CHO |  |
| Yes | \%MF317 | VOLTAGE_IN_CH2 |  |
| Yes | \%MF208 | VOLTAGE_IN_CHANAL_1 |  |
| Yes | \%MF777 | VOLTAGE_OUT_CHO |  |
| Yes | \%MF130 | X0 |  |
| Yes | \%MF420 | x1 |  |
| Yes | \%MF1800 | XC |  |
| Yes | \%MF132 | Yo |  |
| Yes | \%MF422 | Y1 |  |
| Yes | \%MF1802 | YC |  |
| Yes | \%MF134 | z0 |  |
| Yes | \%MF424 | z1 |  |
| Yes | \%MW20 | Z1_ACT |  |
| Yes | \%MW22 | Z1_DIS |  |
| No | \%MW2 | Z2 |  |
| Yes | \%Mw90 | Z2_ACT |  |
| Yes | \%MW30 | Z2_DIS |  |
| Yes | \%MW39 | Z3_ACT |  |
| Yes | \%MWW702 | Z3_DIS |  |
| Yes | \%MF1804 | ZC |  |
| Yes | \%KF50 | ZERO |  |
| Yes | \%MF202 | Z_ACT1 |  |
| Yes | \%MF302 | Z_ACT2 |  |
| Yes | \%MF402 | Z_ACT3 |  |
| Yes | \%MF200 | Z_DIS1 |  |
| Yes | \%MF306 | Z_DIS2 |  |
| Yes | \%6MF407 | Z_DIS3 |  |

## Cross references

| Address | Symbol | Section | Lines/Networks | Operator |
| :---: | :---: | :---: | :---: | :---: |
| \%KF90 | G_L | 1 | 5 | [ : = * ] |
|  | G_L | 1 | 6 | [ $:=* *]$ |
| \%MF132 | YO | 1 | 1 | $[:=+]$ |
| \%MF134 | z0 | 1 | 2 | [ $:=1$ |
|  | z0 | 1 | 9 | [ : $=$ ] |
| \%MF420 | X1 | 1 | 7 | $[:=+]$ |
| \%MF422 | Y1 | 1 | 1 | $[:=+]$ |
|  | Y1 | 1 | 8 | [ : = - ] |
| \%MF424 | Z1 | 1 | 2 | [ $:=]$ |
| \%MF430 |  | 1 | 3 | $[:=\cos ]$ |
|  |  | 1 | 5 | [ : $=$ * ] |
| \%MF432 |  | 1 | 4 | [ $:=$ SIN $]$ |
|  |  | 1 | 6 | $[:=*]$ |
| \%MF434 |  | 1 | 5 | $[:=*]$ |
|  |  | 1 | 7 | $[:=+]$ |
| \%MF436 |  | 1 | 6 | [ $:=*$ ] |
|  |  | 1 | 8 | $[:=-]$ |
| \%MF1800 | XC | 1 | 7 | $[:=+]$ |
| \%MF1802 | YC | 1 | 8 | $[:=-]$ |
| \%MF1804 | ZC | 1 | 9 | [ : $=$ ] |
| \%MF2008 |  | 1 | 3 | $[:=\cos ]$ |
|  |  | 1 | 4 | $[:=$ SIN $]$ |
| 1 |  | 1 | 1 | LD |
|  |  | 1 | 2 | LD |
|  |  | 1 | 3 | LD |
|  |  | 1 | 4 | LD |
|  |  | 1 | 5 | LD |
|  |  | 1 | 6 | LD |
|  |  | 1 | 7 | LD |
|  |  | 1 | 8 | LD |
|  |  | 1 | 9 | LD |
| 9.3 |  | 1 | 1 | [ : $=+$ ] |

## Animation table



| Yes | \%MF202 | Z_ACT1 | Floating Point |
| :---: | :---: | :---: | :---: |
| Yes | SOMW20 | 21.ACT | Decimal |
| Yes | \%QWO.1.0 | AO | Decimal |
| Yes | \%MMF777 | VOLTAGE__OUT_CHO | Floating Point |
| Yes | \%MF204 | E1 | Floating Point |
| Yes | \%MF650 | VOLTAGE_IN_CHO | Floating Point |
| Yes | \%M77 |  | Decimal |
| PID 1 |  |  |  |
| Used | Address | Symbol | Units |
| Yes | \%MMF306 | Z_DIS2 | Floating Point |
| Yes | \%MW30 | Z2_DIS | Decimal |
| Yes | \%MF302 | Z_ACT2 | Floating Point |
| Yes | \%OMW90 | Z2_ACT | Decimal |
| Yes | \%QWO.1.0 | AO | Decimal |
| Yes | \%MF777 | VOLTAGE__OUT_CHO | Floating Point |
| Yes | \%MF290 | E2_00 | Floating Point |
| Yes | \%MF208 | VOLTAGE_IN_CHANAL_1 | Floating Point |
| Yes | \%M88 |  | Decimal |
| Yes | \%MF203 | E2 | Floating Point |
| PId 2 |  |  |  |
| Used | Address | Symbol | Units |
| Yes | \%MF407 | Z_DIS3 | Floating Point |
| Yes | \%MW702 | Z3_DIS | Decimal |
| Yes | \%QW0.1.0 | AO | Decimal |
| Yes | \%QW0.1.1 |  | Decimal |
| Yes | \%MF777 | VOLTAGE_OUT_CHO | Floating Point |
| Yes | \%MMF317 | VOLTAGE_IN_CH2 | Floating Point |
| Yes | \%M999 |  | Decimal |
| Yes | \%MF402 | Z_ACT3 | Floating Point |
| Yes | \%MW39 | Z3_ACT | Decimal |
| Yes | \%MF563 | E3 | Floating Point |

## List of preferences to print

## Directory:

## Parameters

$$
\text { Path: } \quad \begin{aligned}
& \text { C: } \backslash \text { Program Files } \backslash \text { Schneider Electric\Twidosuite } \backslash M y \\
& \text { projects }
\end{aligned}
$$

## Image:

## Parameters

Image: $\quad$ Default image
Path:

## Functional levels:

Parameters

```
Type:
Level:
Automatic
The very highest
```


## Connections management:

## Connection

| Name | COM1 |
| :--- | :--- |
| Connection type | COM |
| IP / Phone | COM1 |
| Punit / Address | Punit |
| Baud rate <br> Parity |  |
| Stop bits <br> Timeout <br> Break timeout | None |

## Connection

| Name | COM2 |
| :--- | :--- |
| Connection type | COM |
| IP / Phone | COM2 |
| Punit / Address | Punit |
| Baud rate <br> Parity <br> Stop bits <br> Timeout | None |
|  | 5000 |

## About

## License:

```
Company: hroosh co.
User First Name -
User Last Name -
State: Test version
Number of test days: 79
```


## References

1) M. W. Song, S.Hutchinsons, and M. Vidyasagar, "Robot Modeling and control", John Wiley \& Sons, Inc, 2006
2) B. Siciliana, L. Sciavicoo, L. Villain, and G. Oriolo, "Robotics modeling, planning and control", Springer- Verlag London Limited, 2009
3) J.J Craig, "Introduction to Robotics Mechanics and Control", second edition, Silma. Inc, 1989
4) R. Kelly, V. Santibanes and A. Loria "Control of Robot Manipulator in Joint Space", Spring- Verlag London Limited, 2005
5) Andrew J. smith and Van Batavia, "Electronic Control for Hydraulic Applications", Eaton Corporation, 2008
