CHAPTER 1

INTRODUCTION

1.1. Introduction

1.2. Objectives

1.3. Literature Review

1.4. Project Plan

1.5. Methodology

INTRODUCTION

1.1. Introduction

This project applies methods of coordinates/datum Transformations using excel work sheets. This requires has the following basic transformation to applied; the first one to transform from Geographic coordinates (λ,φ,h) to Geocentric coordinates (X,Y,Z), where the same ellipsoid is used. The second functionality to transform 3D Geocentric (x,y,z) to 2D projected (Grid) coordinates (Easting, Northing), the used map projection in this transformation tool are: Cassini, Transverse Mercator, Universal Mercator (UTM). The third is to apply the reverse transformation of the first and second functionalities.

In after wards, the datum transformation methods are to be used to calculate the datum transformation parameters using different datum transformation methods, such as similarity 7-parameters, similarity 6-parameters, similarity 3-parameters, Molodensky transformation , Helmert transformations and the 2D methods like 2D(conformal, affine, projective, …….polynomial) transformation.

In Palestine there are four map projections and coordinate systems are used, these are: Palestine_1923 Grid, Palestine_1923 Belt, Palestine_1923 CS Israel Grid and Israel_TM_Grid, where each coordinate system has its own projection parameters, in this project we will use Palestine_1923 Grid .these coordinates system were directly used in the classical surveying. But in the modern form of surveying, GNSS became the used method it uses WGS84 as are reference system to integrate GNSS with the classical survey, the coordinates of GNSS to be transform to the local coordinates in Palestine.

1.2. Objectives

The project has the following objectives:

1. Applying 2D and 3D coordinates transformation between GNSS coordinate (WGS84 coordinate system) to the local coordinates in Palestine specifically Palestine_1923Grid .
2. Different methods are going to applied using 3D considering or 2D consideration.
3. The accuracies and result of the different methods are going to be classified and analyzed .
4. Best method are going to be recommended.

1.3. Literature Review

The transformation of coordinate system had been a case of study from a while, and we are trying to develop this study, a graduate students have researched this before: Somia Zahdeh and Manar Jabari's project made a software that transforms the coordinates from geographic system to geocentric system, and from geocentric to geocentric in different references, and they used 2D-affined, Molodensky, Helmert and others in 2008, Abdallah Radwan (in 2016) developed this software and focused on Molodensky, In 2013 a research has been made by Salem that compared between the Palestinian system and the International Terrestrial Reference Frame (ITRF). By using some of the information of these previous studies, we are going to improve our research to transform from WGS84 to Palestine_1923Grid.

1.4. Project Plan

The project works will be achieved by the following steps:

1. Literature review
2. Research Plan, determining the problem solving scope.
3. The selection of a group of reference point to be used in the calculation.
4. Transform from projected coordinate (E,N) to Geocentric coordinates (X,Y,Z) and in the opposite direction. `
5. There are 4 coordination systems used in Palestine which is (Palestine_1923 Grid),(Palestine_1923 Belt),(Palestine_1923 CS Israel Grid),(Israel_TM_Grid) the (Palestine_1923 Grid) system will be used in this project and transferring coordination's to global coordination's (WGS84) and backwards.
6. The different method of 2D and 3D Coordinate Transformation will programmed on Excel sheet to transform from WGS84 Coordinates to Palestine 1923 Coordinates

Stage Week	1	2	3	4	5	6	7	8
Choosing the project	■							
Problem Definition	■	■						
Literature Review	■	■	■					
Collecting Data			■	■				
Office work					■			
Primary Report of introduction					■	■	■	
Final report of introduction								■

Table (1.1): Time Table

1.5. Methodology

The project has the following scope:

Chapter 1: shows general introduction about the project, its aims and goals, and the working methodology used in it.

Chapter 2: show how geodetic network were built in Palestine .

Chapter 3: : show the coordination's systems used generally and Projection Systems and which one of them used in Palestine.

Chapter 4: shows and explains the transformation ways between systems and the most important out of them is between the Global system(WGS84) and Palestinian system (Palestine_1923Grid), and which is the best way which must be used to transform in what fits well with Palestine's geographical nature.

Chapter 5: Shows result and their analysis

Chapter 6: Conclusion and Recommendation

CHAPTER 2

PALESTINIAN GEODETIC NETWORK

2.1 Introduction

2.2 Historical Background

2.3 Triangulation Survey

2.4 Precise Leveling

2.5 Scale of the Maps of Palestine

2.6 The Geodetic Projection for Palestine

2.7 The Parameter of the Palestine Projected Coordinate

PALESTINIAN GEODETIC NETWORK

2.1 Introduction

Geodetic networks consist of enormous control points that provide the reference frames for positioning determination at all scales.

- Types of Geodetic networks: Horizontal networks
   Vertical networks

Any Geodetic Network in the world has datum, a datum definition is any numerical or geometrical quantity or set of such quantities which work as a reference or base for other quantities. In geodesy we consider two types of datum: a horizontal datum which forms the basis for the computations of horizontal control surveys in which the curvature of the earth is considered, and a vertical datum to which elevations are referred. By way of explanation, the coordinates for points in specific geodetic surveys and triangulation networks are computed from certain initial quantities (datum).

In the past, in areas where direct linear surveys were hard to achieve, the linear survey had to be improve by trigonometrical surveys for determining distances by geometrical calculations. The sitting of points surveyed by the trigonometrical method is determined by calculating the values for sides and angles of triangles. This is the triangulation net. In the past, measuring angles were easier than measuring distances between points in the field, it was done by means of an optical surveying instrument that read angles (Theodolite); the distances between the points could then be calculated trigonometrically. Therefore, when angles are measured in a complex chain of connected triangles, and the distance of one single side is measured in one of these triangles, the length of all the sides of the other triangles in the net can be computed. [3]

2.2 Historical Background

In 1799, French surveyor drown a map, during Napoleon Bonaparte's campaign in Palestine, was the first to be based on original surveys of the country, it was synchronous with the modern mapping projects in other countries. Nearly fifty years before, in France, the first topographic mapping based on a triangulation system had been conducted, by César François Cassini de Thury. From 1747 to 1755, William (later, Major-General) Roy carried out the military survey of Scotland, thereby laying the foundation for the establishment, in 1791 of the Ordnance Survey. [4]

Topographic maps of Britain on a scale of one inch to the mile (1:63,360) began to be published from 1801. In 1806, the Austrian Empire begun the laying out of the first triangulation network; and in 1807 an absolute land measurement project, known as Napoleon's Cadastre, was undertaken in Western Europe—France, Belgium and the Netherlands—for the purpose of reforms in real estate taxation. Cadastral surveys were also conducted in Austria (after 1867, Austria-Hungary), Switzerland, and in several German states. In Great Britain the passing of the

Tithe Commutation Act of 1836, ending tithes on agricultural produce, lead to the attachment of a map to those lands. [1]

Experience in the Dutch colonies. Van de Velde published a map in eight sheets in 1858 to a scale of 1:315,000, covering the country from Tripoli in the north to south of the Dead Sea. It was recognized as the best cartographic work on the country until the appearance of the map of the Palestine Exploration Fund. Van de Velde’s cartographic experience enabled him only to construct his map from earlier ones, mainly the map of Symonds and that of his colleagues, Kiepert’s (1841, 1852), Tobler’s (1845), and de Saulcy’s (1853), and to integrate his own observations. He explained that he did not intend to conduct triangulation measurements, did not have time for this, and lacked the necessary surveying instruments. Van de Velde’s map represents a transition from the compilation stage—drawing up a map from various sources—to mapping based on original surveys. [4]

Figure (2.1): Map Van de Velde 1858 [4]

In Palestine, the measurement of the triangulation net and the control points was begun in 1921, the survey staff numbered eleven senior surveyors and twenty-four junior personnel, and work was carried out with each other at all target sites. Two group were detailed to lay out the major triangulation net. A scouting crew identified the sites for trigonometric stations in the coastal plain, in September their measurement began. The second group set up the stations and constructed beacons—cairns, stakes, pipes with concreted land anchors—that were set up permanently to mark the trigonometric stations, or fixed points. [4]

Between June and October one field group conducted the measurements for the secondary net, a party of apprentices was busy between August and December in measuring the control points, as part of their training as surveyors. In March 1921 the possibility was considered of measuring the baseline in the southern part of the country near Beersheba; but in the end it was measured in October along 4,730.6 metres in the Imara lands, near today's Kibbutz Urim (note the frontispiece).

At the same time, the possibility was investigated of measuring the check baseline of the system in the vicinity of Jenin. In order to calculate the topographic height of the triangulation points, the MSL was measured on the Gaza beach, and the altimetric measurements were connected to the Imara baseline by precise leveling.

In1921, it was still too early for starting a detailed survey for a cadastral program, but the function for this work was prepared by the plane table method. Towards the end of 1922 the net of fixed points was widely spreaded over most of the north of the country, and the measurements were closed to the check line sited in the Haifa Bay, east of Acre, and not in the Jenin region as planned originally.

In December 7th , the map of triangulation points showed that from the start there had been a clear intention to lay out the net only in the areas that were to be subjected to the cadastral survey in the future. No points were measured south of Beersheba; the Judaean Desert and the Judaean mountain region were left out, and so was the Huleh Valley, which at the time had still been excluded definitively within the territory of the Palestine Mandate. [4]

Despite the agreement between Britain and France on the border between Palestine and Lebanon-Syria was signed in Paris on 23 December 1920, it was confirmed only in March 1923, and the transfer of powers to Britain was performance on 1 April 1924. The demarcation of the border from Ras enNaqura (the Ladder of Tyre; Rosh Haniqra) to Samakh was accomplished in the summer of 1925.

In 1923 the major triangulation net of ninety-five fixed points was completed and marked in the field, but the measurements in the Galilee and Mount Carmel were not finished yet. In that year the gaps were closed, and fixed points were measured also in the mountain area north of Ramallah (the Beth-El Mountains) and the Jericho Valley, then the triangulation of Hebron was begun in March 1925.

In April 1924, after the Huleh Valley became a part of Palestine, the northern border was finally demarcated to form the Huleh Salient (the 'Finger of Galilee'), and the Survey Department added five new points to the major triangulation net, and forty-three to the secondary net of third-order triangulation, so as to cover the 'newly acquired territory' by the survey. [4]

In this way the number of points in the major triangulation net reached 100, with point 100 sited, surprisingly in Syrian territory, at what today is known as Mitzpeh Gadot above the old custom house at the Bridge of Jacob's Daughters (Jisr Banat Ya'aqub) across the Jordan. In 1925 the surveyor John Mankin, who surveyed these points in coordination with the Syrians, proposed to improve the major net by establishing an additional fixed triangulation point in Syria, on the plateau northeast of 'Ein Gev, but this was declined.

In 1923 it became clear that the measurement of the check line at Acre was being delayed and was not being carried out properly. In consequence, the Acre line was cancelled and it was decided to establish instead a check baseline south of the Sea of Galilee, near Samakh.

In December 1924, Mankin was ordered to move his surveyors' camp from Athlit to Samakh and to begin measuring a check line. This line was also measured from Afiqim to Deganiya A, a length of 2,901 metres, in the same plain where previously the baseline between points 1200 and 1201 had been measured in connection with the Beisan jiftlik land settlement surveys in 1922. [4]

From the beginning, the two stations of the new line were marked 101M and 102M on the national major triangulation net. Later, however, they were given the numbers 66M and 67M of the points that had been planned but cancelled with the abandonment of the Acre line. In the closing survey that was conducted some time afterwards at the Samakh baseline, there proved to be a discrepancy between the computed trigonometric values and the actual measurement of the check line, and to straighten matters out the Egyptian Survey Department was called in to assist in conducting a professional check. [4]

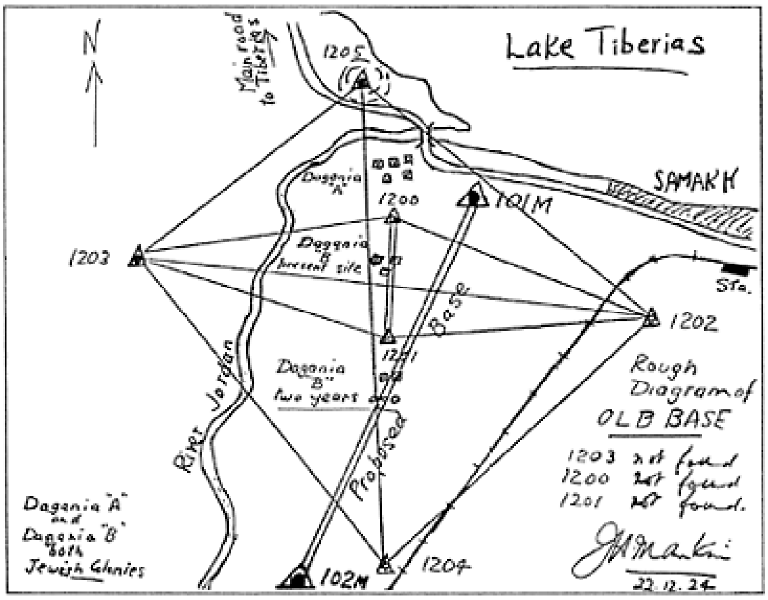

Figure (2.2): Baselines of two triangulation nets in the lands [4]

- This picture only describes how to work and are not the starting point.

2.3 Triangulation Survey

All angles of the triangles formed by the trigonometric points are observed with a theodolite

2.3.1 Categories of Triangulation survey

In February 1921, and after the survey Department moved to its new home in Jaffa, the actual preparations for setting up a triangulation system started; the survey began in May 1921. The first step was for the survey parties to lay out geodetic points throughout whole of the country, to measure their values, and to provide mathematical bases for the survey network. The geodetic points required for mapping are classified in three categories:

1- Fixed points, or trigonometric stations, are determined by trigonometric methods must be in sight of each other for the surveying observations. These virtual lines form the sides of the triangles of the observation net. The data obtained are the position of the points in planimetric coordinates. The reduced level of the points is determined in relation to the mean sea level(MSL).

2- Spot heights are determined by accurate leveling and without the necessity to relate it to the trigonometric net. The topographic heights are measured in relation to the MSL along fixed runs in the field.

3- Gravimetric points, for the determination of the shape of the Earth.

The net of fixed points therefore forms a basic national skeleton system into which link all the survey and mapping projects throughout the country. In order for these separate projects to link into the national net accurately and easily, the density of the measured points must be increased by splitting the major triangulation into secondary nets with triangles having shorter sides: these are the third- or fourth-order triangulation nets, and so on.

Besides the measuring of triangulation nets, the number of triangulation points can be augmented so that in the detailed cadastral survey stage several points linked to the national reference net can be included in every map. [4]

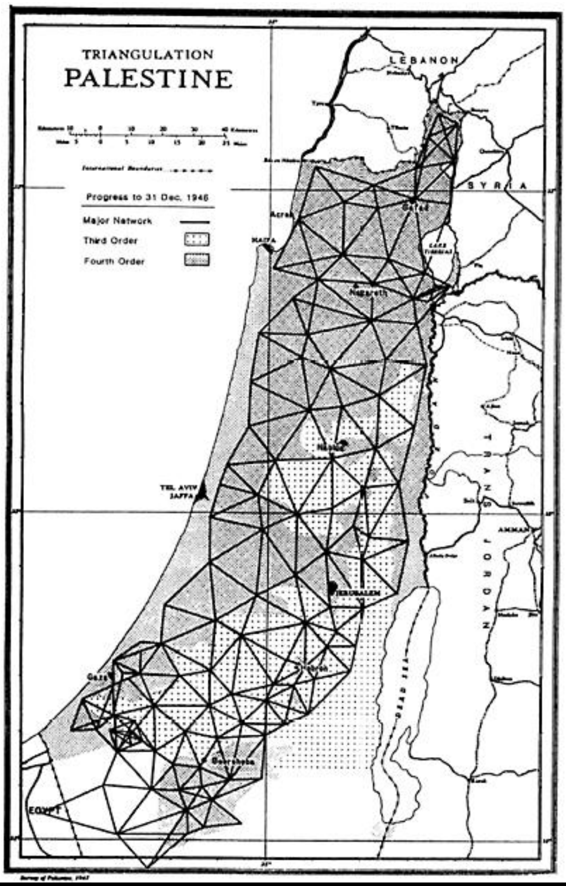

Figure (2.3): Triangulation system in Palestine at the end of the Second World War [4]

2.3.2 Triangulation

Basically, triangulation consists of the measurement of the angles of a series of triangles. The principle of triangulation is based on simple trigonometric procedures. If the distance along one side of a triangle and the angles at each end of the side are accurately measured, the other two sides and the remaining angle can be computed. [2]

Normally, all of the angles of every triangle are measured for the minimization of error and to furnish data for use in computing the precision of the measurements. Also, the latitude and longitude of one end of the measured side along with the length and direction (azimuth) of the side provide sufficient data to compute the latitude and longitude of the other end of the side.

To establish an arc of triangulation between two widely separated locations, a base line may be measured and longitude and latitude determined for the initial point at one end. The locations are then connected by a series of adjoining triangles forming quadrilaterals extending from each end. [7]

With the longitude, latitude, and azimuth of the initial points, similar data is computed for each vertex of the triangles thereby establishing triangulation stations or geodetic control stations.

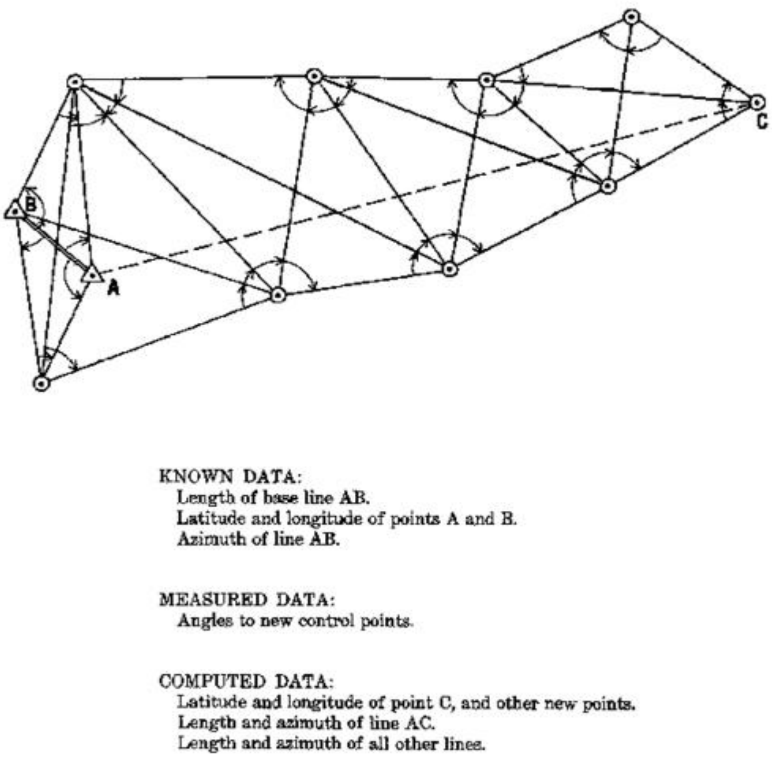

Figure (2.4): A SIMPLE TRIANGULATION NET [7]

2.3.2.1 Principle of Triangulation

Figure 2.5 shows two interconnected triangles ABC and BCD. All the angles in both the triangles, and the length L of the side AB, have been measured. Also the azimuth q of AB has been measured at the triangulation station A, whose coordinates (XA, YA), are known.

The objective is to determine the coordinates of the triangulation stations B, C, and D by the method of triangulation. Let us first calculate the lengths of all the lines.

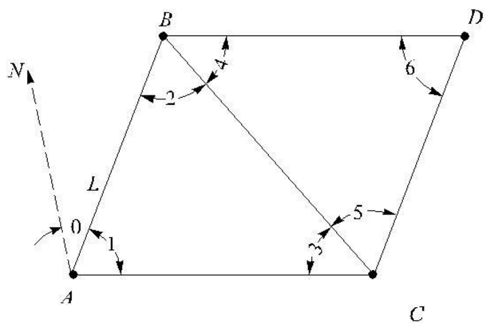

Figure (2.5): Principle of triangulation [11]

By sine rule in ΔABC , we have

$$\frac{AB}{\sin 3} = \frac{BC}{\sin 1} = \frac{CA}{\sin 2}$$

We have

$$AB = L = l_{AB}$$

Or

$$BC = \frac{L \sin 1}{\sin 3} = l_{BC}$$

and

$$CA = \frac{L \sin 2}{\sin 3} = l_{CA}$$

Now the side BC being known in ΔBCD, by sine rule, we have

$$\frac{BC}{\sin 6} = \frac{CD}{\sin 4} = \frac{BD}{\sin 5}$$

We have $BC = \frac{L \sin 1}{\sin 3} = l_{BC}$

or $CD = \frac{L \sin 1}{\sin 3} \frac{\sin 4}{\sin 6} = l_{CD}$

and $BD = \frac{L \sin 1}{\sin 3} \frac{\sin 5}{\sin 6} = l_{BD}$

Let us now calculate the azimuths of all the lines.

Azimuth of $AB = \theta = \theta_{AB}$
Azimuth of $AC = \theta + \angle 1 = \theta_{AC}$
Azimuth of $BC = \theta + 180° - \angle 2 = \theta_{BC}$
Azimuth of $BD = \theta + 180° - (\angle 2 + \angle 4) = \theta_{BD}$
Azimuth of $CD = \theta - \angle 2 + \angle 5 = \theta_{CD}$

From the known length of the sides and the azimuths, the consecutive coordinates can be computed as below.

Latitude of $AB = l_{AB} \cos \theta$
$_{AB} = L_{AB}$ Departure of $AB = l_{AB} \sin \theta$
$_{AB} = D_{AB}$ Latitude of $AC = l_{AC} \cos \theta$
$_{AC} = L_{AC}$ Departure of $AC = l_{AC} \sin \theta$
$_{AC} = D_{AC}$ Latitude of $BD = l_{BD} \cos \theta$
$_{BD} = L_{BD}$ Departure of $BD = l_{BD} \sin \theta_{BD} = L_{BD}$

Latitude of $CD = l_{CD} \cos \theta_{CD} = L_{CD}$
Departure of $CD = l_{CD} \sin \theta_{CD} = D_{CD}$

The desired coordinates of the triangulation stations B, C, and D are as follows :

X-coordinate of *B*, $X_B = X_A + D_{AB}$

Y-coordinate of *B*, $Y_B = Y_B + L_{AB}$

X-coordinate of *C*, $X_C = X_A + D_{AC}$

Y-coordinate of *C*, $Y_C = Y_A + L_{AC}$

X-coordinate of *D*, $X_D = X_B + D_{BD}$

Y-coordinate of *D*, $Y_D = Y_B + L_{BD}$

It would be found that the length of side can be computed more than once following different routes, and therefore, to achieve a better accuracy, the mean of the computed lengths of a side is to be considered. [3]

*** note:** Was used to measure angles in the distribution trig, because the distances between points very long and cannot use a meter to measure which the error would be great, in addition the distribution of points It was on the mountaintops , therefore difficult to measure the lengths between points especially as it is at that time, was not there a tool used to measure great distances accurately

2.3.2.2 Classification of Triangulation System

First-Order (Primary Horizontal Control): is the most accurate triangulation. It is Expensive and time-consuming using the best instruments and rigorous computation methods. First-Order triangulation is usually used to provide the basic framework of horizontal control for a large area such as for a national network. It has also been used in preparation for metropolitan expansion and for scientific studies requiring exact geodetic data. Its accuracy should be at least one part in 100,000.

Second-Order, Class I (Secondary Horizontal Control): includes the area networks between the First-Order arcs and detailed surveys in very high value land areas. Therefore, this class also includes the basic framework for further densification. The internal closures of Second-Order, Class I triangulation should indicate an accuracy of at least one part in 50,000.

The demands for reliable horizontal control surveys in areas which are not in a high state of development or where no such development is anticipated in the near future justifies the need for a triangulation classified as Second-Order, Class II (Supplemental Horizontal Control). This class is used to establish control along the coastline, inland waterways and interstate highways. The control data contributes to the National Network and is published as part of the network. The minimum accuracy allowable in Class II of Second-Order is one part in 20,000.

Third-Order, Class I and Class II (Local Horizontal Control): is used to establish control for local improvements and developments, topographic and hydrographic surveys, or for such other projects for which they provide sufficient accuracy. This triangulation is carefully connected to the National Network. [7]

The work should be performed with sufficient accuracy to satisfy the standards of one part in 10,000 for Class I and one part in 5,000 for Class II. Spires, stacks, standpipes, flag poles and other identifiable objects located to this accuracy also have significant value for many surveying and engineering projects.

The sole accuracy requirement for **Fourth-Order Triangulation** is that the positions be located without any appreciable errors on maps compiled on the basis of the control. [7]

Table (2.1): Triangulation system [11]

S.No.	Characteristics	First-order triangulation	Second-order triangulation	Third-order triangulation
1.	Length of base lines	8 to 12 km	2 to 5 km	100 to 500 m
2.	Lengths of sides	16 to 150 km	10 to 25 km	2 to 10 km
3.	Average triangular error (after correction for spherical excess)	less than 1"	3"	12"
4.	Maximum station closure	not more than 3"	8"	15"
5.	Actual error of base	1 in 50,000	1 in 25,000	1 in 10,000
6.	Probable error of base	1 in 10,00,000	1 in 500,000	1 in 250,000
7.	Discrepancy between two measures (k is distance in kilometer)	5 k mm	10 k mm	25 k mm
8.	Probable error of the computed distances	1 in 50,000 to 1 in 250,000	1 in 20,000 to 1 in 50,000	1 in 5,000 to 1 in 20,000
9.	Probable error in astronomical azimuth	0.5"	5"	10"

A distance accuracy, a, is computed from a minimally constrained, correctly weighted, least squares adjustment by:

$$\mathbf{a = d/s}$$

$$s = \sqrt{\left(\frac{\sum(x-xi)2}{n-1}\right)}$$

where

a = distance accuracy denominator
s = propagated standard deviation of distance between survey points obtained from the least squares adjustment
d =distance between survey points

x = the average

N= the number of observations.

The distance accuracy pertains to all pairs of points (but in practice is computed for a sampling of pairs of points). The worst distance accuracy (smallest denominator) is taken as the provisional accuracy. If this is substantially larger or smaller than the intended accuracy, then the provisional accuracy takes precedence.

Line 1-2 : average distances = **(**17106.83+17107.06+17107.09)/3= **17107.00**

$$\text{Standard division} = \sqrt{\frac{\sum(17107-17106.83)2+(17107-17107.06)2+(17107-17107.09)2}{3-1}} = \mathbf{0.141}$$

$$\text{Distance accuracy} = \frac{d}{s} = \frac{17107}{0.141} = \mathbf{121326}$$

Line 1-3 : average distances = **(**20123.18+20122.95+20122.85)/3= 20123.00

$$\text{Standard division} = \sqrt{\frac{\sum(20123-20123.18)2+(20123-20122.95)2+(20123-20122.85)2}{3-1}} = \mathbf{0.170}$$

$$\text{Distance accuracy} = \frac{d}{s} = \frac{20123.00}{0.170} = \mathbf{118371}$$

Line 1-3 : average distances = **(**15504.87+15505.19+15504.95)/3= 15505.00

$$\text{Standard division} = \sqrt{\frac{\sum(15505-15504.87)2+(15505-15505.19)2+(15505-15504.95)2}{3-1}} = \mathbf{0.164}$$

$$\text{Distance accuracy} = \frac{d}{s} = \frac{15505.00}{0.164} = \mathbf{94543}$$

Table (2.2): Example of distance accuracy [7]

Line	Distance(xi)	average distances(x)	Standard division(S)	Distance accuracy (1:a)
	17106.83			
1-2.	17107.06	17107.00	0.141	1:121326
	17107.09			
	20123.18			
1-3.	20122.95	20123.00	0.170	1:118371
	20122.85			
	15504.87			
2-3.	15505.19	15505.00	0.164	1:94543
	15504.95			

2.3.3 Traverses

The simplest method of extending control is called traverse. The system is similar to dead reckoning navigation where distances and directions are measured. In performing a traverse, the surveyor starts at a known position with a known azimuth (direction) to another point and measures angles and distances between a series of survey points.

With the angular measurements, the direction of each line of the traverse can be computed; and with the measurements of the length of the lines, the position of each control point computed. If the traverse returns to the starting point or some other known position, it is a closed traverse, otherwise the traverse is said to be open. [7]

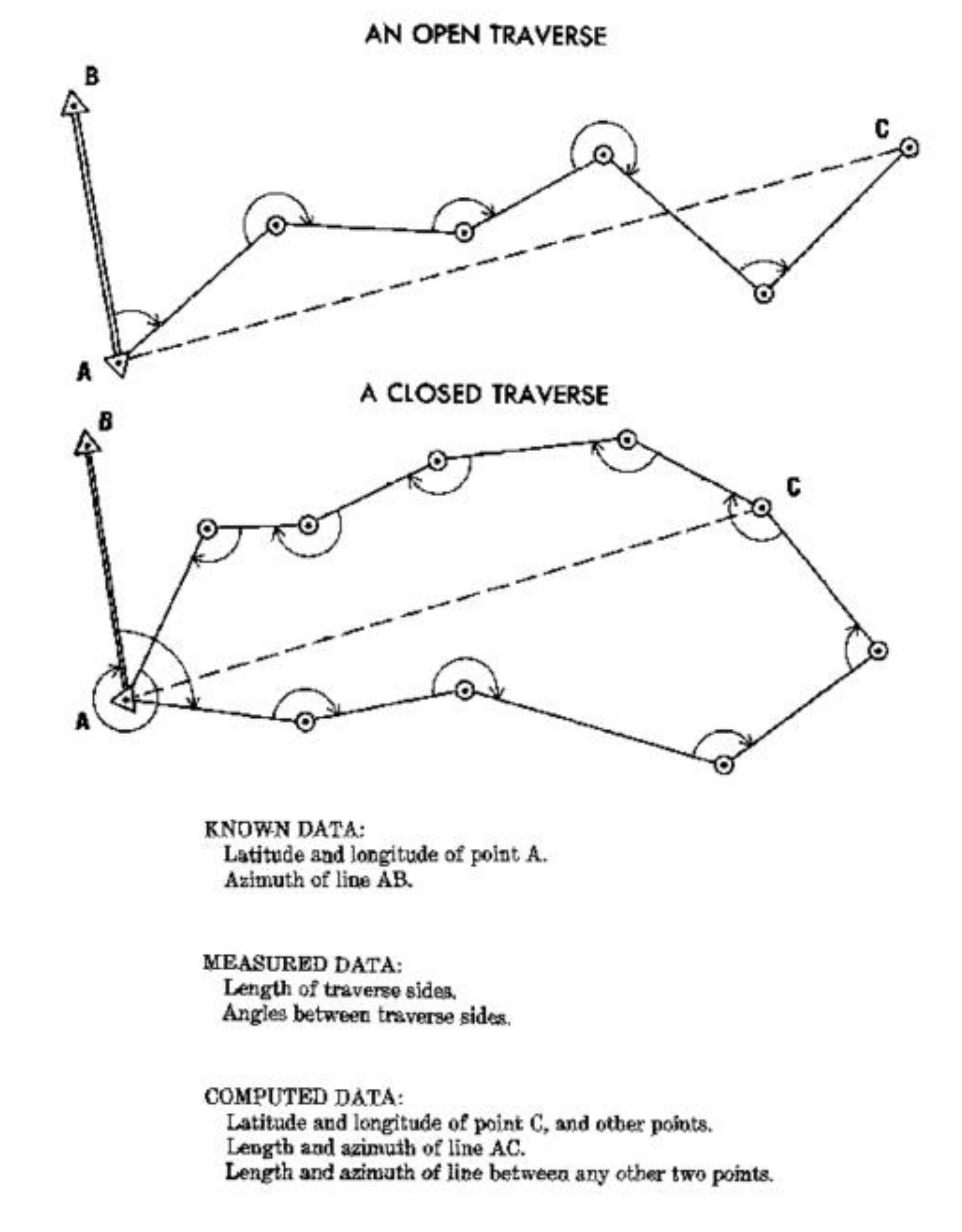

Figure (2.6): Create Traverse to use in measuring [7]

2.3.4 The Survey Check

Triangulation net of Palestine are of so high a level of exactitude to suffice for all purposes of the cadastral survey and the registration of property rights. As to the mathematical discrepancy between the measurements in the field and the calculations in closing the loop at the Samakh line, and adjusted was done.

The computations were conducted in 1926–1927, along with astronomical observations. In 1938 Salmon, the Director of the Survey Department at the time, had occasion to praise the quality of this geodetic work. In a report he submitted to the Royal ('Peel') Commission, he wrote:

"No survey work is perfect and there will always be some discrepancy between the measured length, position and bearing of the check base, and these values as calculated from the triangulation system. The errors thus observed must be distributed throughout the system by a long and precise mathematical process which in the case of Palestine involved 231 differential equations and took 520 man days." [4]

2.3.5 Joining the triangulation net to the neighboring countries

The Survey Department wanted to check the precision of its observations according to the surveys of the French in Syria and the Egyptians in Sinai.

Ley had already proposed setting up a geodetic tie with the French net in the summer of 1923; in August 1924 Dowson discussed at the Colonial Office that before the triangulation system could be accepted as the foundation for a cadastral survey, it was imperative to tie it to, and bring it to the level of, triangulation in the neighboring countries. [4]

2.4 precise leveling

The measuring of topographic spot heights of triangulation points in the field is done in two ways: trigonometrically and by precise leveling. In the trigonometric method the elevations are calculated according to readings of vertical angles in the course of planimetric observations to determine the positions of triangulation points. In the precise leveling method the elevations of points in the field along selected runs are determined by means of a leveling instrument that permits more accurate measurement than the trigonometric method.

In the precise leveling method heights are measured from a base point of established topographic height, by measuring the elevation differentials from point to point and calculating the height of the new point in reference to the measured height of the previous point. These elevation points join to make up measured lines that are resected or measured in circular loops to obtain checks on the accuracy of the measurement and the closing of a series of measurements. Like the triangulation points, the elevation points are also marked in the field as benchmarks cut into the margins of roads, culverts, and the like.

The basic starting point for measuring heights is the mean sea level. In 1921 the MSL was measured for the first time at the Gaza beach and precise leveling conducted to the baseline at Imara. From then until 1927 no further country-wide leveling surveys were conducted in Palestine. [4]

Figure (2.7): Leveling survey [4]

2.5 Scale of the maps of Palestine

The determination of a standard scale for the maps of Palestine constitutes an issue in its own right among all the searching and debates regarding the form of the country's survey maps. With the beginning of work in the field and the production of maps, the Directorate of the Survey Department had yet to formulate guidelines for determining the scale of its maps.

At first, the system that prevailed in Egypt was applied in Palestine, along with the basic scale of the maps. As work progressed, the differences between Egypt and Palestine became evident, and in the Survey of Palestine discussions were held regarding a suitable scale hierarchy for the landscape of the country, and the size of the field sheets to be mounted on the plane table for topographic and administrative maps. [4]

The main question hinged on the choice between a cadastral and a topographic scale. The decision was made only in1928 in the wake of the land settlement reform. It was determined then that there would be one basic scale of 1:10,000, from which would be derived cadastral scales in one direction and topographic scales in the other, usually in even multiples:
1:100,000←1:50,000←1:20,000←1:10,000→1:5,000→1:2,500→1:625

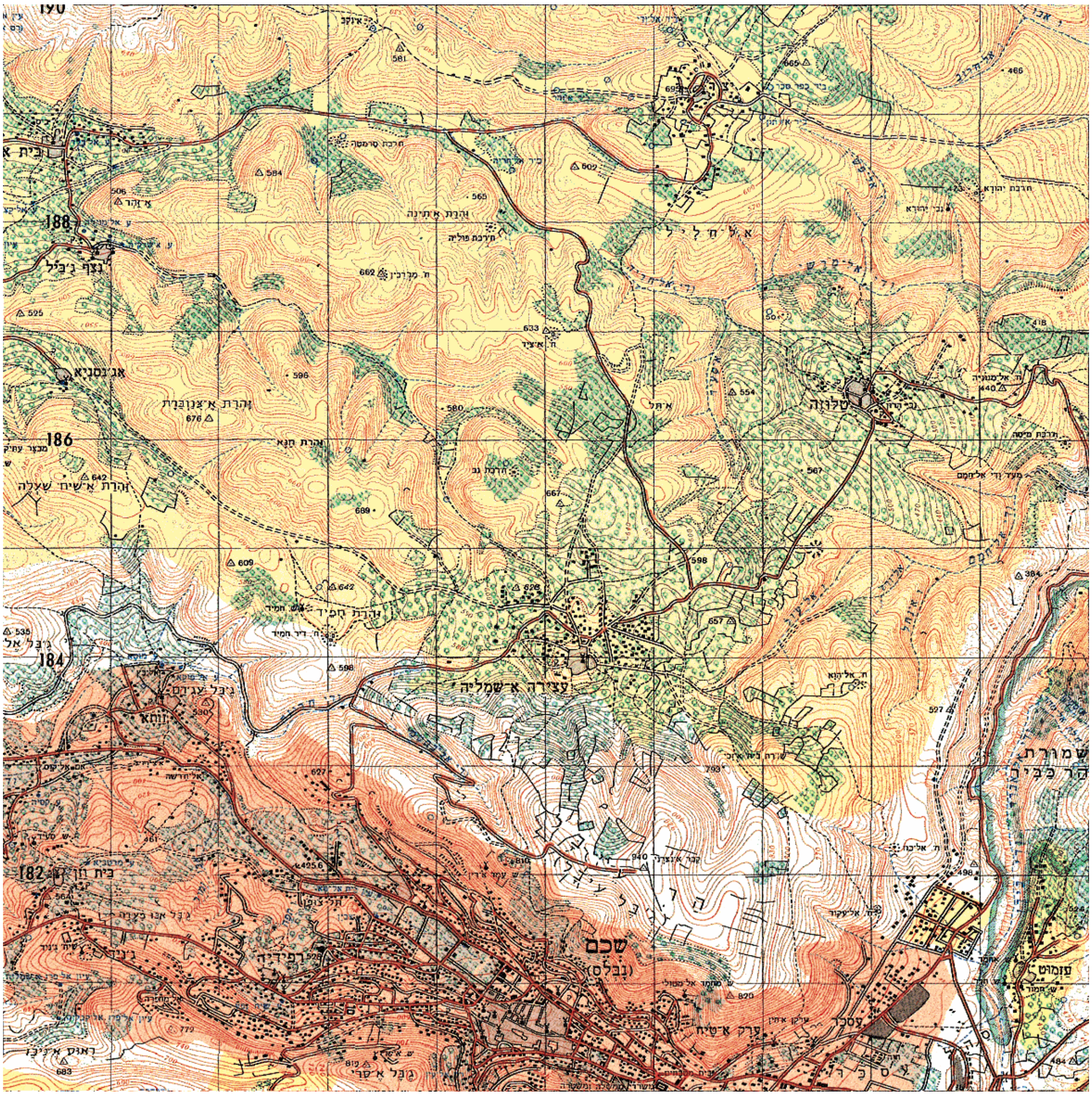

Figure (2.8): A Map of Nablus Scale 1:50000

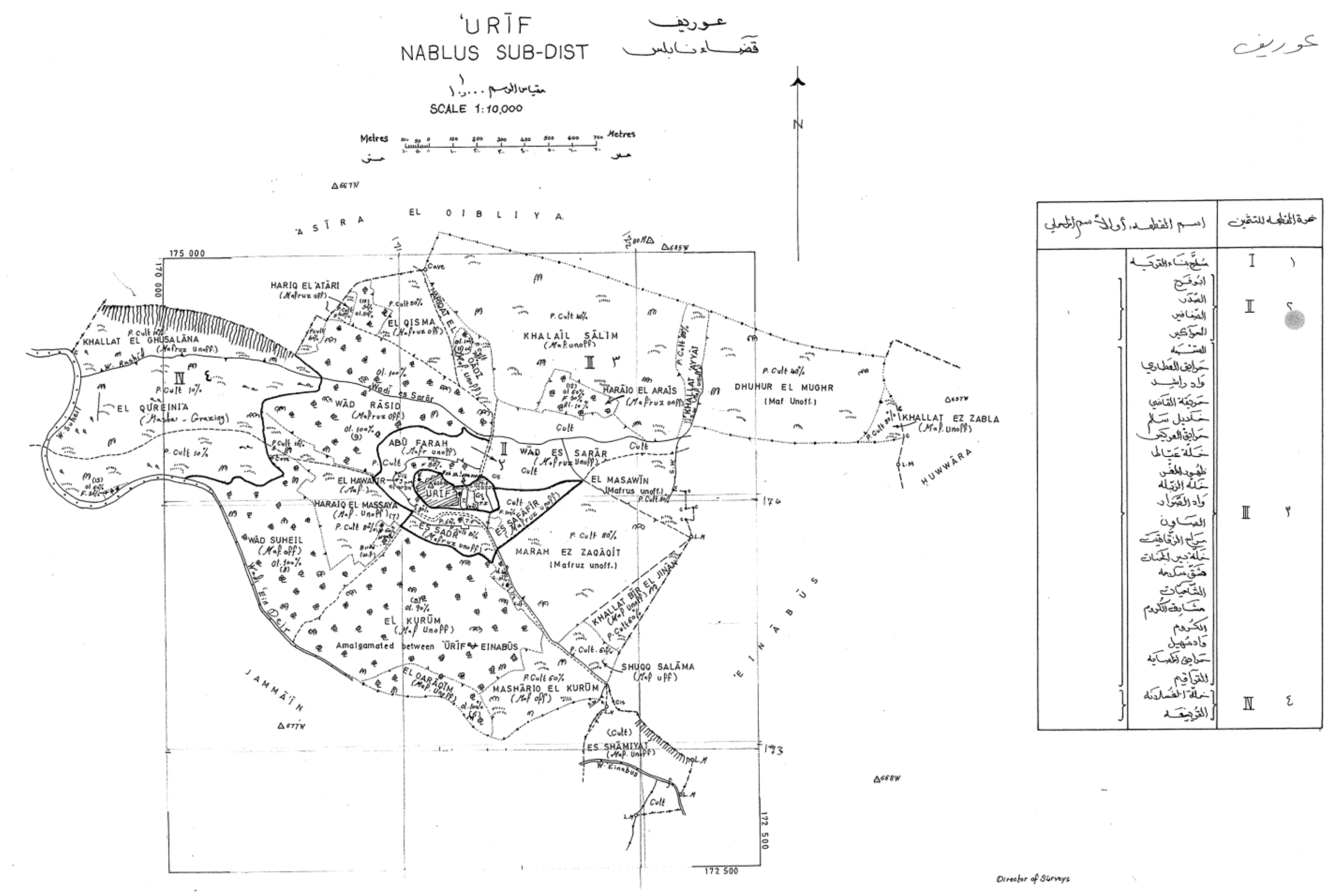

Figure (2.9): A Map of URIF Scale 1:10000

2.6 The geodetic projection for Palestine

The land of Israel occupies a very small area on the globe. A single country, groups of countries, or the entire surface of the globe can be represented by means of different methods of cartographic and geodetic projections.

A projection is the transfer of a point from one plane to another. Mapping theory entails ways of projecting parallels and meridians from the global surface of the earth upon the flat map. Cartographic projections enable large parts of the globe to be represented on small-scale maps, as in atlases, so that a general idea can be obtained of the parallels and meridians on the map. By means of geodetic projection the geographic graticule is exchanged for a rectangular coordinates grid, so that triangulation points can be defined and elements in the field located by values of the national grid. More than it influences the outline of the country's map, the choice of the projection dictates the essential geodetic attributes for precise work. Hence, the choice of a suitable projection for Palestine depended on the geometrical characteristics of the projection, the size of the country, its elongated, narrow north-south form, and the purposes of mapping—in this case cadastral. [4]

The mapping of Palestine was also influenced by the cartographic traditions in the colonies and by consideration of the available mathematical tables compiled and calculated beforehand in Britain and other countries.

We do not know what prior considerations led the British to select any particular geodetic projection for Palestine. The decision narrowed down between two projections: Gauss-Conformal, known as Transverse Mercator Projection, and Cassini—Soldner, since these were accepted as convenient projections for both cadastral and topographic mapping.

In 1922 the survey experts in Palestine fixed upon the Cassini geodetic projection with rectangular coordinates as calculated by Soldner as the projection for Palestine, based on the Jerusalem central meridian. The Cassini projection had been used by the British since 1745, and it was commended by the leading British survey

This projection was considered easy for computation and suitable for areas of restricted size. From its geometrical attributes and its transverse construction, the Cassini projection answers the geodetic needs of Palestine within a strip 50–80 kilometres wide on both sides of a central meridian, usually passing through the centre of the area to be mapped. [6]

The British bestowed this honour on Jerusalem, so that the meridian became the central longitudinal line, even though it did not divide the country down the middle. The meridian of Jerusalem goes through the Jaffa Gate, and the main triangulation point82’M, which became the reference point of the system, was fixed higher up, on top of the Mar Elias monastery hill south of Jerusalem

Figure (2.10): Mar Elias Monastery south of Jerusalem; triangulation point 82’M was positioned on top of the hill [4]

In the geodetic projection, importance is given not to the transfer of the elliptic geographic graticule of meridians and parallels, but to the replacement with a rectangular national grid system. The Surveys Directorate decided that the grid would encompass all the parts of the country to be mapped—which did not include the Negev south of Beersheba. Therefore, its staff established a trigonometrical station at the top of the ‘Ali el-Muntar hill, which dominates the town of Gaza, in the heart of the area that was the first to be mapped in detail, and gave it values of 100–100 in the national grid. This point became the true origin of the Palestine grid. [4]

In this way the zero point, or the false origin, of the Palestine axial system was 100 kilometres west and 100 kilometres south in north Sinai, near Jebel Maghara. The choice of the true point of origin was not a good one because it left the southern Negev with negative values south of the zero line. Thus, for example, Elat would have been given a negative northern coordinate of −116. In order to avoid negative values, the British set the value of the zero line at 1,000, so that anyplace south of the line would have positive values; Elat would thus be at 884 of the northern coordinate.

When Richards conducted the check of the surveys in Palestine in 1925, he argued against this peculiar layout of the national grid. He remarked that the zero point of the main axes ought to have been at the intersection of the geographical coordinates 34° longitude and 29° latitude, which fall in south Sinai, so that all of Palestine would be within the positive values of the national grid. [4]

Richards also commented on the determination of the central meridian of the projection at Jerusalem, which it would have been better to move eastwards, for example to the Jordan Valley, so that in due course it would be possible to extend the grid system to Transjordan. These comments had no practical connotations, since the entire system was already in operation.

The episode is mentioned here only to illustrate the absolute professional independence of the Directors of the Palestine Survey Department, despite the prestige of the Survey of Egypt, which assisted the local department in its first steps

2.7 The Parameter of the Palestine Projected Coordinate

Cassini_Soldner Projection

The name Cassini-Soldner refers to the more accurate ellipsoidal version, developed in the 19th century. This transverse cylindrical projection maintains scale along the central meridian and all lines parallel to it and is neither equal area nor conformal. It is most suited for large scale mapping of areas predominantly north-south in extent. [5]

To define a coordinate system using Cassini projection the following parameters are to be considered reference ellipsoid:

- False Easting
- False Northing
- Central Meridian
- Scale Factor = 1
- Latitude of Origin

The Palestinian grid named Palestine_1923_Grid is built using Cassini projection, which normally used in land surveying and engineering projects with the following parameters:

• False Easting • False Northing • Central Meridian • Scale Factor • Latitude of Origin • Spheroid • Semi major axis • Semi minor axis • Inverse flattening	170251.555000 126867.909000 35.212081 1.000000 31.734.097 Clarke_1880_Benoit 6378300.790000000 6356566.430000036 293.46623457099997

Israel Old Grid is the same of Palestine grid (Paestine-1923-Grid), but 1 million is added to the northing value, because the coordinates of the south of Palestine (Al- Naqab) are negative, so it has been added 1 million to become All coordinates positive .

• False Easting	170251.555000
• False Northing	1126867.909000
• Central Meridian	35.212081
• Scale Factor	1.000000
• Latitude of Origin	31.734.097
• Spheroid	Clarke_1880_Benoit
• Semi major axis	6378300.790000000
• Semi minor axis	6356566.430000036
• Inverse flattening	293.46623457099997

CHAPTER 3

COORDINATE SYSTEMS

COORDINATE SYSTEMS

3.1 Introduction

Coordinate system is a system to determine location on the surface of the earth, different units and length to the angular distance have been used. The mathematical figure of the earth is applied to the classical definition of the geoid defined as equipotent surface of the earth gravitation field that nearly coincides with the mean sea level (MSL).

A reference surface is chosen so that reductions are applied to the surface. At the beginning this surface was defined as a sphere with a radius R (approximately R=3678km), later it was defined as a rotational ellipsoid, the circle on the equator with radius (a) and the distance from the center to the north or south pole is (b). (a) is called the semi major axis and (b) is called the semi minor axis, while always (a) is larger than (b).

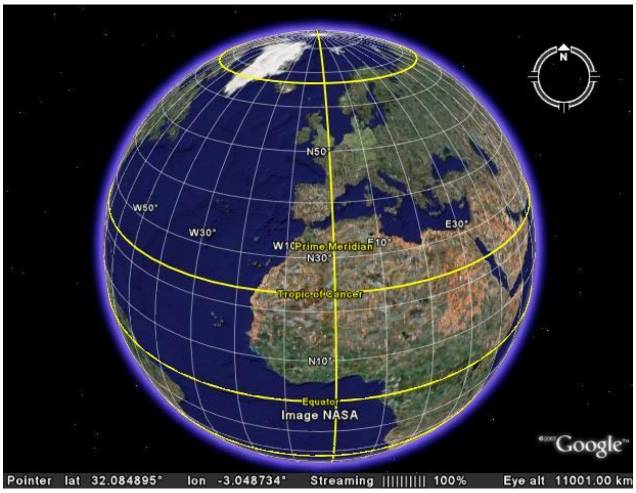

Figure (3.1): The Earth [9]

3.2 Spherical Coordinate

Considering the earth as a sphere with radius R, the position of the point is defined by polar coordinate (λ,φ,h), or by Geocentric coordinates (X,Y,Z) with origin at the earth center.
Where;

λ = longitude; defined as the angular distance on the equator from Greenwich meridian to the local meridian.

φ= latitude; defined as the angular distance long the meridian from the equator to the point.

h = ellipsoid height; defined as the distance along the normal from the ellipsoid surface to the point.

The conversion between the different coordinates can be calculated as follows:

$$X = (R + h)\cos\varphi \sin\lambda \quad (3.1)$$

$$Y = (R + h)\cos\varphi \sin\lambda \quad (3.2)$$

$$Z = (R + h)\sin\varphi \quad (3.3)$$

The reverse formulas are:

$$r = \sqrt{X^2 + Y^2 + Z^2} \quad (3.4)$$

$$h = r - R \quad (3.5)$$

$$\varphi = \tan^{-1}\frac{Z}{\sqrt{(X^2+Y^2)}} \quad (3.6)$$

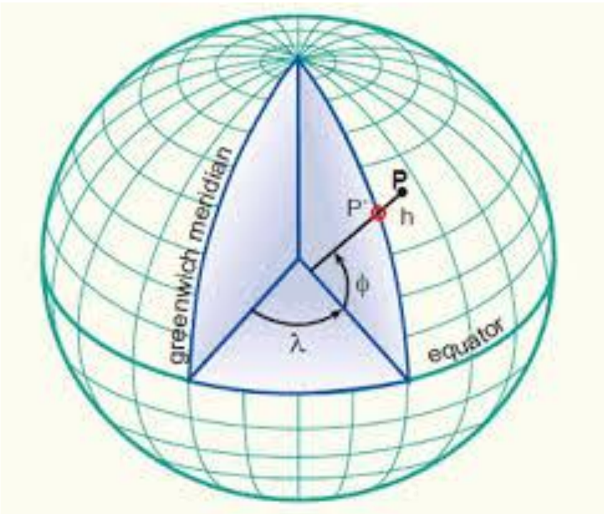

Figure (3.2): Sphere as the earth figure [7]

3.3 Ellipsoidal Coordinates

There are a three-dimensional orthogonal coordinate system (λ,φ,h) that generalizes the two-dimensional elliptic coordinate system. Unlike most three-dimensional orthogonal coordinate systems that feature quadratic coordinate surfaces, the ellipsoidal coordinate system is not produced by rotating or projecting any two-dimensional orthogonal coordinate system.

3.3.1 Ellipsoidal earth figure

In classic definitions; earth is considered to be an ellipsoid with semi major axis (a) & semi minor axis (b), the other basic parameters can be calculated using the basic axis, table (3.1) show selected reference ellipsoids. Table (3.2), show the basic defining parameters of an ellipsoid.

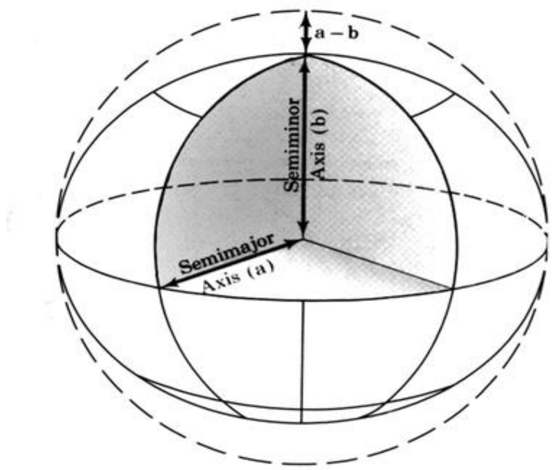

Figure (3.3): Ellipsoid as the figure of the earth [13]

Table (3.1): Selected reference Ellipsoid [8]

Ellipse	Semi-Major axis (meter)	1/Flattening
WGS 60	6378165.0	298.3
WGS 66	6378145.0	298.25
WGS72	6378135.0	298.26
WGS 84	6378137.0	298.257223563
South American 1969	6378160.0	298.25
Krassovky	6378245.0	298.3
International	6378388.0	297.0
Clarke 1866	6378206.4	294.978698
Clarke 1880	6378249.145	293.465
GRS 1975	6378140.0	298.257
GRS 1980	6378137.0	298.2572221

Table (3.2): The relation between the different parameters [2]

Notation	a/b	e^2	e'^2	f	n
a					$\frac{c(1-n)}{1+n}$
b		$a(1-e^2)^{1/2}$	$\frac{a}{(1+e'^2)^{1/2}}$	$a(1-f)$	
$\frac{a}{b}$				$\frac{1}{1-f}$	
$\frac{b}{a}$		$(1-e^2)^{1/2}$	$\frac{1}{(1+e'^2)^{1/2}}$	$(1-f)$	$\frac{1-n}{1+n}$
c	$\frac{a^2}{b}$	$\frac{a}{(1-e^2)^{1/2}}$	$a(1+e'^2)^{1/2}$	$\frac{a}{1-f}$	
e^2	$\frac{a^2-b^2}{a^2}$		$\frac{e'^2}{1+e'^2}$	$f(2-f)$	$\frac{4n}{(1+n)^2}$
e'^2	$\frac{a^2-b^2}{b^2}$		$\frac{f(2-f)}{(1-f)^2}$		$\frac{4n}{(1-n)^2}$
f	$\frac{a-b}{a}$	$1-(1-e^2)^{1/2}$	$1-(1+e'^2)^{-1/2}$		$\frac{2n}{1+n}$
n	$\frac{a-b}{a-b}$	$\frac{1-(1-e^2)^{1/2}}{1+(1-e^2)^{1/2}}$	$\frac{(1+e'^2)^{1/2}-1}{(1+e'^2)^{1/2}+1}$	$\frac{f}{2-f}$	

In modern definition the ellipsoid is defined as an equipotential surface. The normal potential (U) on the reference ellipsoid is equal to the geopotential (W) on the geoid. The total mass of the reference ellipsoid is equal to that of the earth, and reference ellipsoid is rotating around its minor axis at the same angular velocity as the earth rotation.

Many ellipsoids were defined by physical definition on the principle of normal gravity. Examples of physically defined ellipsoids are GRS 67, GRS 80 and WGS 84. The defining parameters for GRS 80 ellipsoid are shown in table (3.3).

Table (3.3): GRS 80 ellipsoid parameters [2]

Notation	Constant	Unit	Numerical value
a	Semi-major axis	m	6378137.000
GM	Product of G and total mass M	$m^3 s^{-2}$	0.3986005 E15
J2	Dynamic from factor $\frac{C-A}{Ma^2}$		0.00108263
ω	Angular velocity	s^{-1}	0.72921151 E-4
b	Semi-minor axis	m	6356752.3141
f	Geometrical flattening		1/298.257222101 = 0.003352810681
e^2	First eccentricity squared		0.006694380023
$e^{`2}$	Second eccentricity squared	s^{-1}	0.006739496775
U₀	Normal potential on the ellipsoid	$m^2 s^2$	62636860.850
ɤp	Normal gravity on the poles	Gal	983.21863685
ɤe	Normal gravity on the equator	Gal	978.03267715
f*	Gravity flattening		1/188.592417552 = 0.005302440112
k	(b.ɤp –a.ɤe)/(a.ɤe)		0.001931851353
m	$\omega^2 a^2 b/(GM)$		0.003449786003
ɤ45	Normal gravity at altitude 45°	Gal	980.6199203
ɤ		Gal	979.7644656

The four parameters defining for WGS are shown in table (3.4).

Table (3.4) WGS ellipsoid parameters [8]

Parameter	Notation	Value
Semi-major axis	a	6378137.0 m
Reciprocal of Flattening	1/f	298.257223563
Angular Velocity of the Earth	ω	7292115.0*10 rad/s
Earth's Gravitational constant	GM	3986004.418*10 m^2/s^2

3.3.2 Geographic Coordinate

The most commonly used coordinate system today is the latitude, longitude & height.
Equator & the prime meridian (Greenwich) are the reference planes used to define latitude and longitude.
The geodetic latitude (φ) of a point is the angle from the equatorial plane to the vertical direction of a line normal to the reference ellipsoid passing the point, the geodetic longitude (λ) of a point is the angle between a reference meridian (Greenwich) and the vertical plane passing through the point measure along the equator, both plans being perpendicular to the equatorial plane.

The geodetic (ellipsoid or normal) height (h) at a point is the distance from the reference ellipsoid to the point in the direction normal to the ellipsoid. [11]

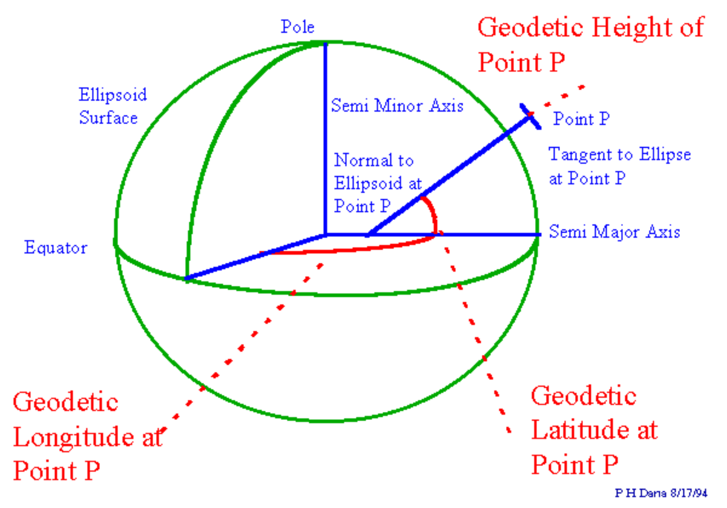

Figure (3.4): Geographic coordinate system [1]

3.3.3 Geocentric coordinates

It's a system of three dimensional earths centered reference system in which location are identified by their: X, Y and Z value.

The X axis is in the equatorial plane of intersects the prime meridian (Greenwich).

The Y axis is in the equatorial plane of intersects the +90° meridian.

The Z axis coincides with the polar axis and is positive toward the North Pole.

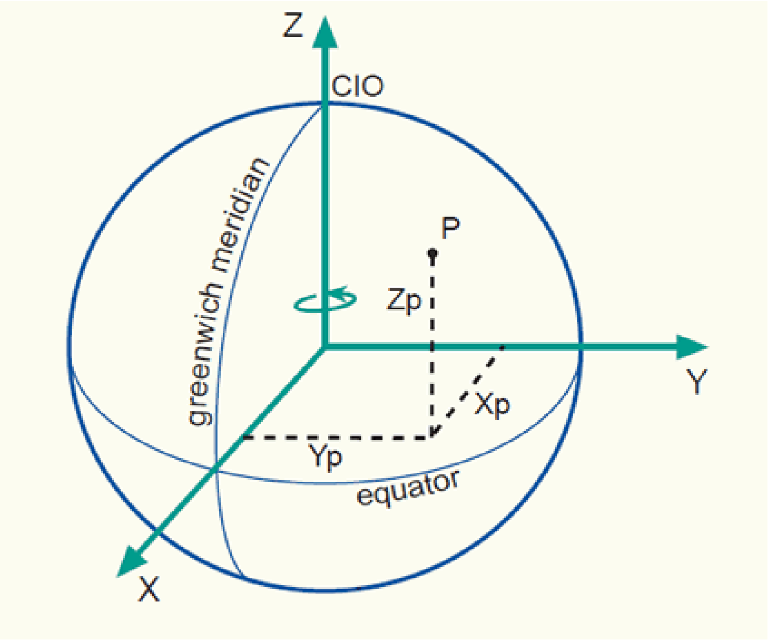

Figure (3.5): Geocentric system [12]

This system is not a grid system. The earth is modeled as a sphere or spheroid in a right handed X, Y, Z system

$$X = (N + h)\cos\varphi\cos\lambda \qquad (3.7)$$
$$Y = (N + h).\cos\varphi.\sin\lambda \qquad (3.8)$$
$$Z = ((1 - e).N + h).\sin\varphi \qquad (3.9)$$

The inverse problem is solved in an iterative solution (Torge method): [9]

$$\lambda = \tan^{-1}\frac{Y}{X} \quad \text{(Does not need iteration).} \qquad (3.10)$$

$$h = \frac{\sqrt{X^2+Y^2}}{\cos\varphi} - N \qquad (3.11)$$

$$\varphi = \tan^{-1}\left(\frac{Z}{\sqrt{X^2+Y^2}}\left(1 - e^2\frac{N}{N+h}\right)^{-1}\right) \qquad (3.12)$$

φ As initial value to start the iterative solution.

$$\varphi = \tan^{-1}\left(\frac{Z}{\sqrt{X^2+Y^2}}(1 - e^2)^{-1}\right) \qquad (3.13)$$

3.3.4 Topocentric Coordinates

In the topocentric coordinates, we use the point of origin with known geographic coordinate P_o (λ,φ,h) or (X,Y,Z), the (X) direction in to the north, the (Y) direction is to the east and the (Z) direction is perpendicular on the (X, Y) plane. [1]

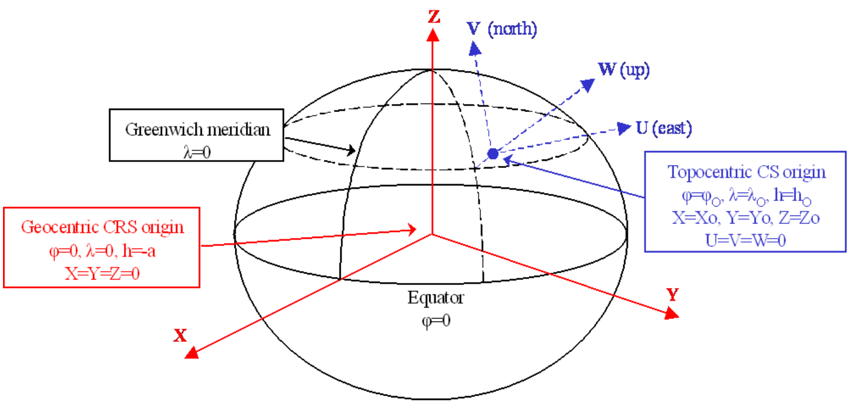

Figure (3.6): Topocentric coordinate system [1]

The position of the point is defined by the zenith (z_e), distance (s) and Azimuth (Az) measured clockwise from the north,

- Topocentric coordinates are used for analysis the results of observations in astronomy, astrometry, geodesy ,stars, galaxies, and satellite geodesy. Depending on the choice of the coordinate reference plane, topocentric coordinates may be equatorial, horizontal, or orbital.

Where;

$$X = S \cos Az \sin z_e \quad (3.14)$$
$$Y = S \sin Az \sin z_e \quad (3.15)$$
$$Z = S \cos z_e \quad (3.16)$$

If geocentric coordinates are used

$$\mathrm{X} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}, \quad \mathrm{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad (3.17)$$

To convert from topocentric to geocentric coordinate, the following can be applied in matrix form.

$$\Delta X = \mathrm{Ax} \quad (3.18)$$

$$\begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} = \begin{bmatrix} -\sin\varphi_o \cos\lambda_o & -\sin\lambda_o & \cos\varphi_o \cos\lambda_o \\ -\sin\varphi_o \sin\lambda_o & \cos\lambda_o & \cos\varphi_o \sin\lambda_o \\ \cos\varphi_o & 0 & \sin\varphi_o \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad (3.19)$$

$$X = X_{p0} + \Delta X \quad (3.20)$$
$$X = A^{-1}\Delta X = A^T \Delta X \quad (3.21)$$

3.4 Map Projection (Grid Coordinates System)

The basic idea at map projection is to convert from geographic (∅, λ) to local (E, N) Grid system and in the opposite direction, map projections are attempts to portray the surface of the earth or a portion of the earth on a flat surface. Some distortions of conformality, distance, direction, scale and area always result from this process. Some projections minimize distortions in some of these properties at the expense of maximizing errors in others. Some projections are attempts to only moderately distort all of these properties, the properties are:

1. Conformalety: when the scale of a map at any point on the map is the same in any direction, the projection is conformal. Meridians (Lines of longitude) and parallels (lines of latitude) intersect at right angles. Shape is preserved locally on conformal maps.
2. Distance: A map is equidistant when it portrays distances from the center of the projection to any other place on the map.
3. Direction: A map preserves direction when azimuths (angles from a point on a line to another point) are portrayed correctly in all directions.
4. Scale: is the relationship between a distance portrayed on a map and the same distance on the earth.

5. Area: When a map portrays areas over the entire map so that all mapped areas have the same proportional relationship to the areas on the earth that they represent, the map is an equal-area map. [1]

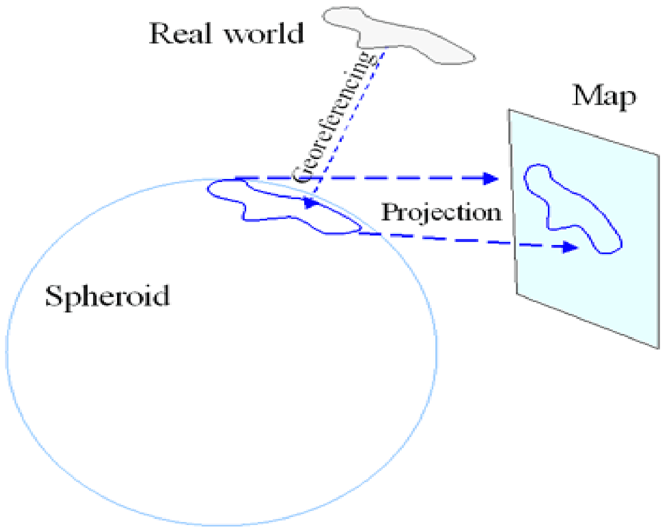

Figure (3.7): Map projection (from 3D to grid system) [1]

3.4.1 Map projections General Classes

Map projections fall into four general classes depend on the methods of projection, and they are:

1. Cylindrical projections: result from projecting a spherical surface onto a cylinder.
    a. When the cylinder is **<u>tangent</u>** to the sphere the contact is along a great circle (the circle formed on the surface of the earth by a plane passing through the center of the earth).

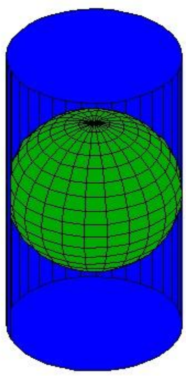

Figure (3.8): Projection of a sphere onto a cylinder (Tangent case). [5]

In the **secant** case, the cylinder touches the sphere along two lines

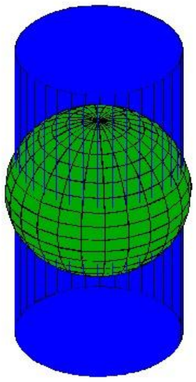

Figure (3.9): Projection of a sphere onto a cylinder (Secant case). [5]

b. When the cylinder upon which the sphere is projected is at right angles to the poles, the cylinder and resulting projection are **transverse**.

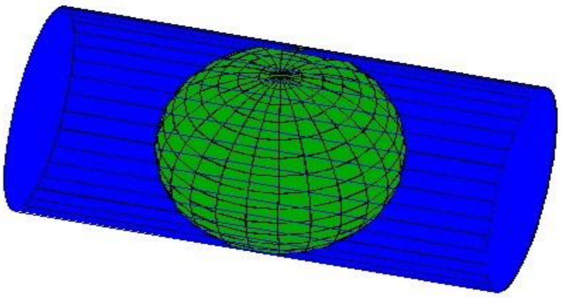

Figure (3.10): Transverse Projection of a sphere onto a cylinder (Tangent case). [5]

c. When the cylinder is at some other, non-orthogonal angle with respect to the poles, the cylinder and resulting projection is **oblique**.

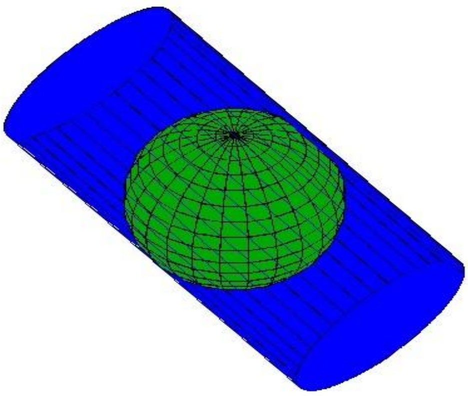

Figure (3.11): Oblique Projection of a sphere onto a cylinder (Tangent case). [5]

2. Conic Projections: result from projecting a spherical surface onto a cone.
    a. When the cone is **tangent** to the sphere the contact is along small circle.

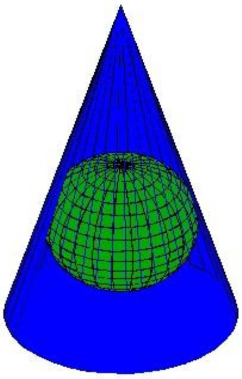

Figure (3.12): Projection of a sphere onto a cone (Tangent case). [5]

    b. In the **secant** case, the cone touches the sphere along two lines; the lower line makes a greater circle than the upper line.

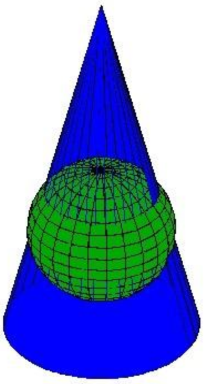

Figure (3.13): Projection of a sphere onto a cone (Secant case). [5]

3. Azimuthal Projections: result from projecting a spherical surface onto a plane.
    a. When a plane is **tangent** to the sphere the contact is at a single point on the surface of the earth.

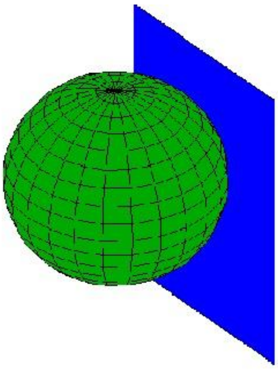

Figure (3.14): Projection of a sphere onto a plane (Tangent case). [5]

b. In the **secant** case, the plane touches the sphere along a small circle if the plane does not pass through the center of the earth, when it will touch along a great circle.

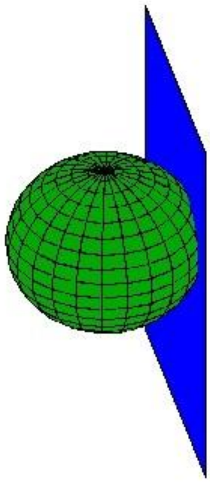

Figure (3.15): Projection of a sphere onto a plane (Secant case). [5]

4. Miscellaneous Projections: include unprojected ones such as rectangular latitude and longitude grids and other examples of that do not fall into the cylindrical, conic or azimuthal categories.

3.4.2 Transverse Mercator

It is known as Gauss-Kruger and similar to the Mercator, except that the cylinder is longitudinal along a meridian instead of the equator. The result is a conformal projection that does not maintain true directions exclude small areas. The central meridian is placed in the center of the region of interest. This centering minimizes distortion of all properties in that region. This projection is best suited for north-south shaped areas.

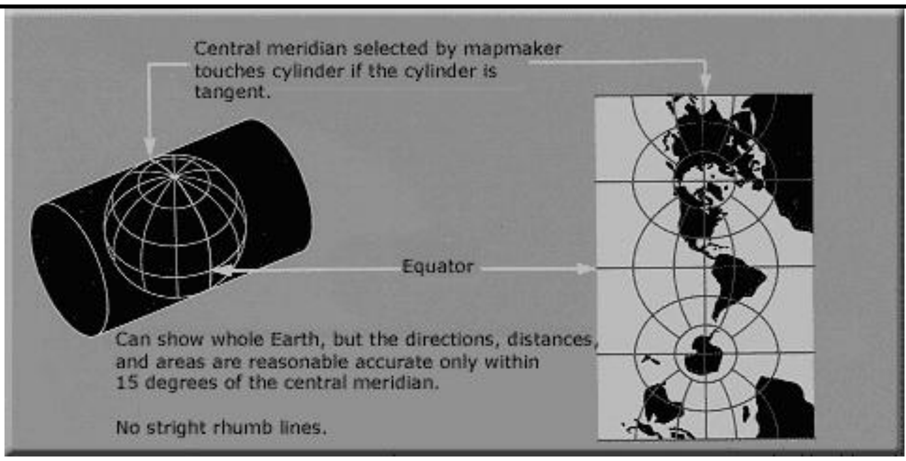

Figure (3.16): Transverse Mercator [5]

To define coordinates system using Transverse Mercator, the following parameters have to be defined reference ellipsoid

- False Easting
- False Northing
- Central Meridian
- Scale Factor
- Latitude of Origin
- Scale Factor at Central Meridian

In Palestine, there is a coordinates system named Palestinian Transverse Mercator or Palestine-1923-Belt with the following parameters:

• False Easting	170251.555000
• False Northing	1126867.909000
• Central Meridian	35.212081
• Scale Factor	1.000000
• Latitude of Origin	31.734097
• Spheroid	Clarke_1880_Benoit
• Semi major axis	6378300.790000000
• Semi minor axis	6356566.430000036
• Inverse flattening	293.46623457099997

The other common system is the Israel Transverse Mercator (Israel-TM-Grid) with the following parameters:

• False Easting	219529.584000
• False Northing	626907.39000
• Central Meridian	35.204517
• Scale Factor	1.000007
• Latitude of Origin	31.734394
• Spheroid	GRS1980
• Semi major axis	6378137.000000
• Semi minor axis	6356752.314000
• Inverse flattening	298.2572221

3.4.3 Cassini_Soldner Projection

The name Cassini-Soldner refers to the more accurate ellipsoidal version, developed in the 19th century. This transverse cylindrical projection maintains scale along the central meridian and all lines parallel to it and is neither equal area nor conformal. It is most suited for large scale mapping of areas predominantly north-south in extent. [2]

- The British Mandate using this system in Palestine, because it was used in the British and thus calculation for this system was ready and do not have to make new calculation, in addition the shape of Palestine was most accurate in Cassini Soldner.

To define a coordinate system using Cassini projection the following parameters are to be considered reference ellipsoid:

- False Easting
- False Northing
- Central Meridian
- Scale Factor = 1
- Latitude of Origin

The Palestinian grid named Palestine_1923_Grid is built using Cassini projection, which normally used in land surveying and engineering projects with the following parameters:

• False Easting • False Northing • Central Meridian • Scale Factor • Latitude of Origin • Spheroid • Semi major axis • Semi minor axis • Inverse flattening	170251.555000 126867.909000 35.212081 1.000000 31.734.097 Clarke_1880_Benoit 6378300.790000000 6356566.430000036 293.46623457099997

Israel Old Grid is the same of Palestine grid (Paestine-1923-Grid), but 1 million is added to the northing value, because the coordinates of the south of Palestine (Al- Naqab) are negative, so it has been added 1 million to become All coordinates positive

• False Easting	170251.555000
• False Northing	1126867.909000
• Central Meridian	35.212081
• Scale Factor	1.000000
• Latitude of Origin	31.734.097
• Spheroid	Clarke_1880_Benoit
• Semi major axis	6378300.790000000
• Semi minor axis	6356566.430000036
• Inverse flattening	293.46623457099997

3.4.5 Universal Transverse Mercator (UTM)

UTM coordinate system is used in survey navigation and in GIS and it's the most commonly used in Transverse Mercator. From the figure (3.17) below, we can see that UTM zone numbers designate 6 degrees longitudinal strips extending from 80 degree south latitude to 84 degree north longitude , do not extend to 84 south, because there is no people live there. [5]

To find the central meridian of a UTM zone: [1]

$$\text{Central Meridian} = (\text{zone\#} * 6 - 3) - 180 \qquad (3.22)$$

To find which zone you belong to at a given longitude: [2]

$$\text{Zone} = \text{int}\left\{\frac{\lambda+180}{6}\right\} + 1 \qquad (3.23)$$

Example : the longitude(λ) of Palestine approximately = 32, then you need to calculate number of zone of Palestine and the central meridian

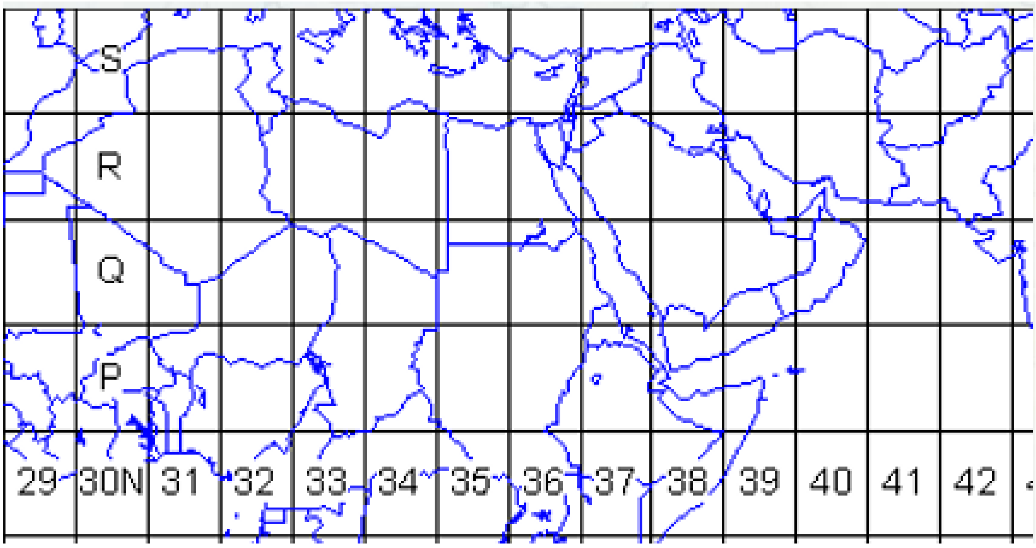

Figure (3.17): Universal Transverse Mercator [5]

$\text{Zone} = \text{int}\left\{\frac{\lambda+180}{6}\right\} + 1 \rightarrow \text{int}\left\{\frac{32+180}{6}\right\} + 1$

zone= int{35.33333} + 1 ➔ zone = 35+1

then Zone of Palestine = 36 S

Central Meridian = (36*6-3)-180= 33 degree

- The Value cannot be accurate because the value of longitude was proximate .

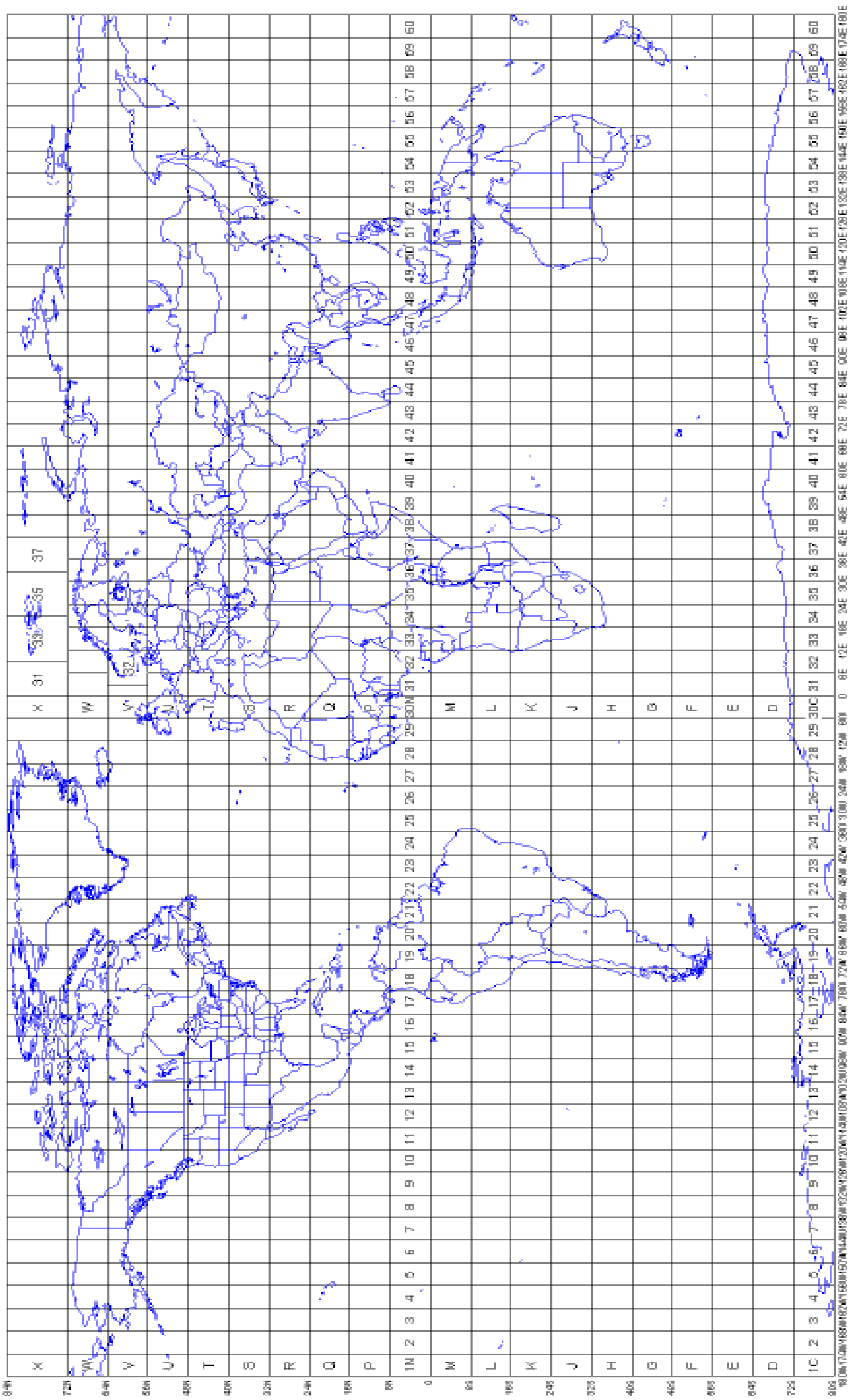

Figure (3.17): Universal Transverse Mercator [5]

CHAPTER 4

DATUM TRANSFORMATION

4.1 Introduction

The coordinates of all locations on the earth are defined referring to a datum. While a spheroid nearly represents the shape of the earth, a datum defines the position of a spheroid relative to the center of the earth. A point on the surface of the earth is matched to a particular position on the surface of the ellipsoid. This point is known as the origin point of the coordinates system on the datum. The coordinates of the origin point coordinates system are fixed, and all other points are calculated referring to it. The coordinate system origin of a local datum is not at the center of the earth. The center of the spheroid local datum is offset from the earth's center, depending on a global datum like WGS 84.

A datum provides a frame or reference for measuring locations on the surface of the earth. It defines the origin and orientation of latitude and longitude lines. Whenever change the datum, or more correctly, the geographic coordinate system, the coordinate values of a point will change. [1]

4.2 Datum and Ellipsoids

The shape of the earth is ellipsoid because the distance from the center of the earth to the equator is larger than the distance from the center to the poles by about 23km.

To make an ellipsoid model of the earth, rotate the ellipse about the shorter polar axis (semi-minor axis b) to form a solid surface, see figure (4.1) a datum is defined by choosing an ellipsoid and then a primary reference point.

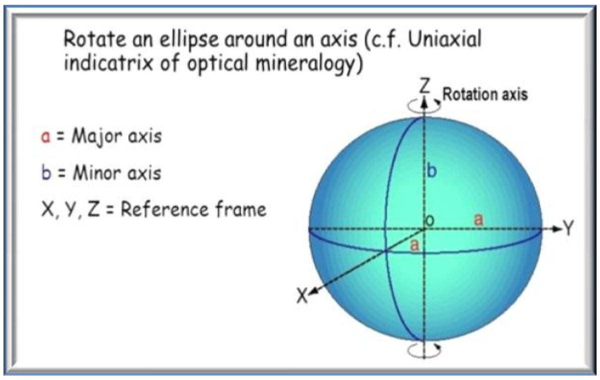

Figure (4.1): Ellipse Rotation [6]

The reference ellipsoid of the Palestine_1923_Grid, Palestine_1923_Israel_CS_Grid and Palestine_1923_Belt is the Clarke_1880_Benoit. The reference ellipsoid of the Israel_TM_Grid is the Geodetic Reference System of 1980 (GRS80).

4.3 Datum Transformation

4.3.1 Introduction

Equation-based transformation methods can classified into the following basic four methods. Having data in one datum and needing the coordinates in other is a common task in geodesy, surveying and GPS. A transformation must be used to display coordinates from a GPS receiver in any other datum than WGS84. Over any small area (150x150 km), the transformation will be a constant shift in latitude, longitude and height, neglecting scaling and rotation. But for countries with large areas seven parameter are preferred; scale, three rotations around the three axis and three translations.

There are three common methods of making these transformations from one datum to another. These methods are 3D similarity, Helmert and Molodensky method, Figure (4.2) shows the basic parameters for datum transformation.

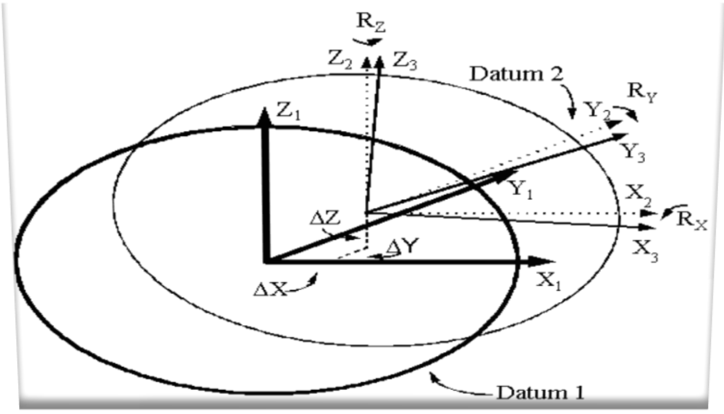

Figure (4.2): Datum Transformation [3]

4.3.2 Least square solution

To solve the datum transformation problem, control points with known positions in both datums have to be available. To solve the parameters, nonlinear least square solution method is used with basic matrix equation. For (m) number of observation and (n) number of unknowns.

$$AX = L+V \tag{4.1}$$

Where A is the matrix of coefficients multiplied by the unknown parameters:

$$A = \begin{bmatrix} a_{11} & a_{12} & \Lambda & a_{1n} \\ a_{21} & a_{22} & \Lambda & a_{2n} \\ M & M & M & M \\ a_{m1} & a_{m2} & \Lambda & a_{mn} \end{bmatrix} \tag{4.2}$$

And X is the matrix of unknowns:

$$X = \begin{bmatrix} dx_1 \\ dx_2 \\ M \\ dx_n \end{bmatrix} \tag{4.3}$$

And L is observations matrix, which is the difference between the measured values and the computed ones using the initial values of the unknown parameters:

$$L = \begin{bmatrix} l_1 - l_{1o} \\ l_2 - l_{2o} \\ M \\ l_m - l_{mo} \end{bmatrix} \tag{4.4}$$

V is the residuals matrix:

$$V = \begin{bmatrix} v_1 \\ v_2 \\ M \\ vm \end{bmatrix} \tag{4.5}$$

Multiply (AX = L + V) on left by (A^T) to get:

$$A^T AX = A^T L \tag{4.6}$$

Where;

$$A^T A = \begin{bmatrix} a_{11} & a_{21} & \Lambda & a_{m1} \\ a_{12} & a_{22} & \Lambda & a_{m2} \\ M & M & M & M \\ a_{1n} & a_{2n} & \Lambda & a_{mn} \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & \Lambda & a_{1n} \\ a_{21} & a_{22} & \Lambda & a_{2n} \\ M & M & M & M \\ a_{m1} & a_{m2} & \Lambda & a_{mn} \end{bmatrix} = N \tag{4.7}$$

$$\begin{array}{llll} n_{11} = \sum_{i=1}^{m} a_{i1}^2 & n_{12} = \sum_{i=1}^{m} a_{i1}a_{i2} & \Lambda & n_{1n} = \sum_{i=1}^{m} a_{i1}a_{in} \\ n_{21} = \sum_{i=1}^{m} a_{i2}a_{i1} & n_{22} = \sum_{i=1}^{m} a_{i2}^2 & \Lambda & n_{2n} = \sum_{i=1}^{m} a_{i2}a_{in} \\ M & M & M & M \\ n_{n1} = \sum_{i=1}^{m} a_{in}a_{i1} & n_{n2} = \sum_{i=1}^{m} a_{in}a_{i2} & \Lambda & n_{nn} = \sum_{i=1}^{m} a_{in}^2 \end{array} \tag{4.8}$$

When observations are not weighted, then:

$$X = (A^T A)^{-1} A^T L \tag{4.9}$$

When weighted observation are used, then:

$$X = (A^T W A)^{-1} A^T W L \tag{4.10}$$

4.3.5 3D similarity 3-Parameter

The simplest datum transformation method is three-parameter transformation or a geocentric translation. When the minor axes of the ellipsoids are assumed to be parallel, to transform assuming that rotations are zeroes, then shifts (T_x, T_y, T_z) are defined from source geocentric coordinate system to target geocentric coordinate system or from local datum to WGS 1984 or another geocentric datum as common in GPS. The three parameters are linear shifts and are always in distance units, and usually called datum shifts. [4]

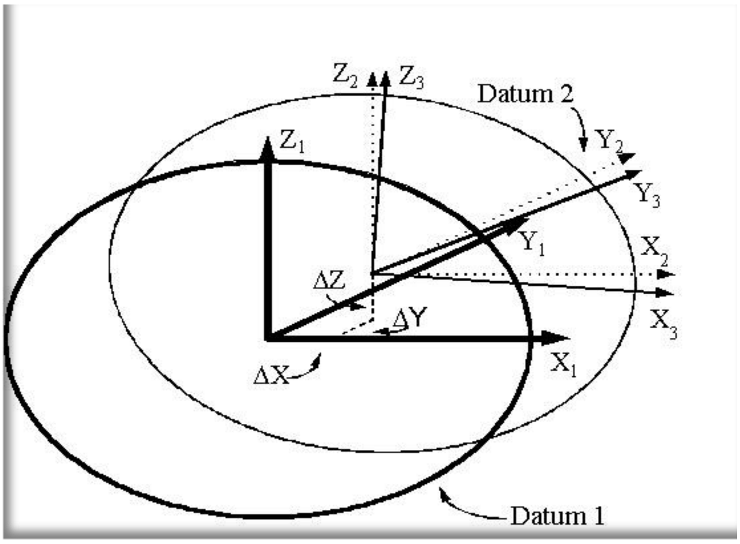

Figure (4.3): 3-Parameter Transformation [3]

The mathematical model for the 3D similarity three-parameter transformation is:

$$X = x + T_x \quad (4.11)$$

$$Y = y + T_y \quad (4.12)$$

$$Z = z + T_z \quad (4.13)$$

In matrix form;

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix} \quad (4.14)$$

This equation can be obtained from 3D similarity seven-parameter transformation

$$X = S(m_{11}x + m_{21}y + m_{31}z) + T_x \quad (4.15)$$

$$Y = S(m_{12}x + m_{22}y + m_{32}z) + T_y \quad (4.16)$$

$$Z = S(m_{13}x + m_{23}y + m_{33}z) + T_z \quad (4.17)$$

When;

$$S = 1 \quad (4.18)$$

$$\omega = \emptyset = \kappa = 0 \tag{4.19}$$

The least squares solution as linear solution:

$$AX = L + V \tag{4.20}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} \tag{4.21}$$

The final solution of the unknowns i:

$$X = (A^T A)^{-1} A^T L \tag{4.22}$$

$$X = \begin{bmatrix} dT_x \\ dT_y \\ dT_z \end{bmatrix} \tag{4.23}$$

The 3-parameters are calculated in equation (4.10), after first iteration, initial values of second iterations are calculated using the final results of the first iteration:

$$T_x = T_{x0} + dT_x \tag{4.24}$$

$$T_y = T_{y0} + dT_y \tag{4.25}$$

$$T_z = T_{z0} + dT_z \tag{4.26}$$

In Palestine_1923_Grid, the transformation is defined by three translations. The values of the translation are taken from Trimble geometric office software. According to the following equations:

$$X_{Palestine\_1923} = X_{WGS84} + \triangle X \tag{4.27}$$

$$Y_{Palestine\_1923} = Y_{WGS84} + \triangle Y \tag{4.28}$$

$$Z_{Palestine\_1923} = Z_{WGS84} + \triangle Z \tag{4.29}$$

Where;

$$\triangle X = 230.00m, \quad \triangle Y = 71.00m, \quad \triangle Z = -273.00m \tag{4.30}$$

4.3.6 Helmert Transformations

When dealing with map, we have maps with different scales, orientations and coordinates origins. Then coordinates transformations are used to move the coordinates from one map system to the other. These transformations are; 2D Conformal, 2D Affine, 2D Projective ,3D Conformal and 3D Linearized.

4.3.6.1. 2D Conformal

In this type of coordinate's transformations as shown in the figure below, we have three steps; scale change, rotation and two translations:

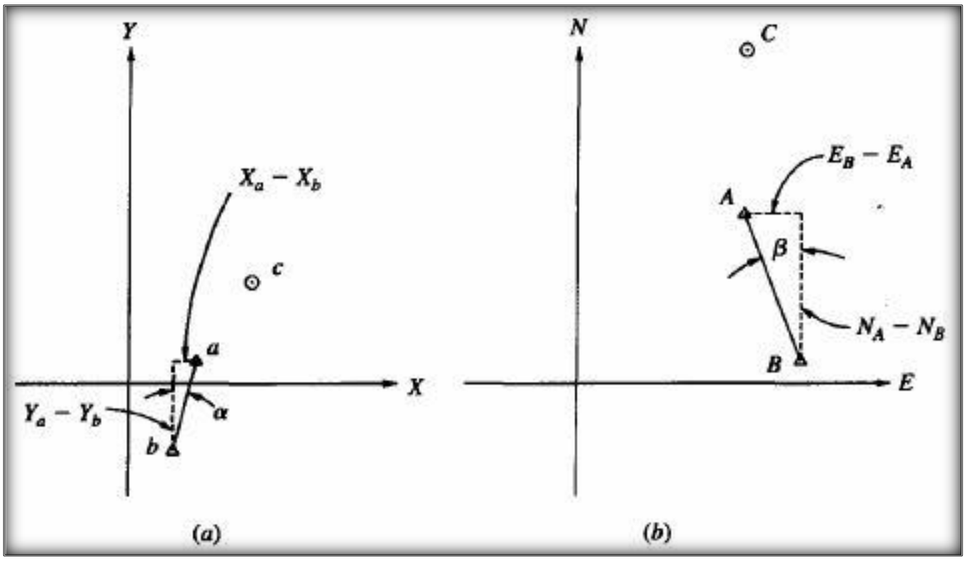

Figure (4.4): 2D Conformal Helmert Transformation [2]

Step 1: Scale Change

- The lengths of lines ab and AB are unequal; hence the scales of the two coordinate systems are unequal.
- The scale of the XY system is made equal to that of the EN system by multiplying each X and Y coordinate by a scale factor s. The scaled coordinates are designated as X' and Y'.
- By use of the two control points, the scale factor is calculated in relation to the two lengths AB and ab as:

$$s = \frac{AB}{ab} = \frac{\sqrt{(E_B - E_A)^2 + (N_B - N_A)^2}}{\sqrt{(X_b - X_a)^2 + (Y_b - Y_a)^2}} \quad (4.31)$$

Step 2: Rotation

- If the scaled X'Y' coordinate system is superimposed over the EN system , so that line ***AB*** in both systems coincide, the result is as shown in Figure below.
- An auxiliary axis system E'N' is constructed through the origin of the X'Y' axis system parallel to the EN axes.
- It is necessary to rotate from the X' Y' system to the E'N' system, or in other words, to calculate E'N' coordinates for the unknown points from their X' Y' coordinates.

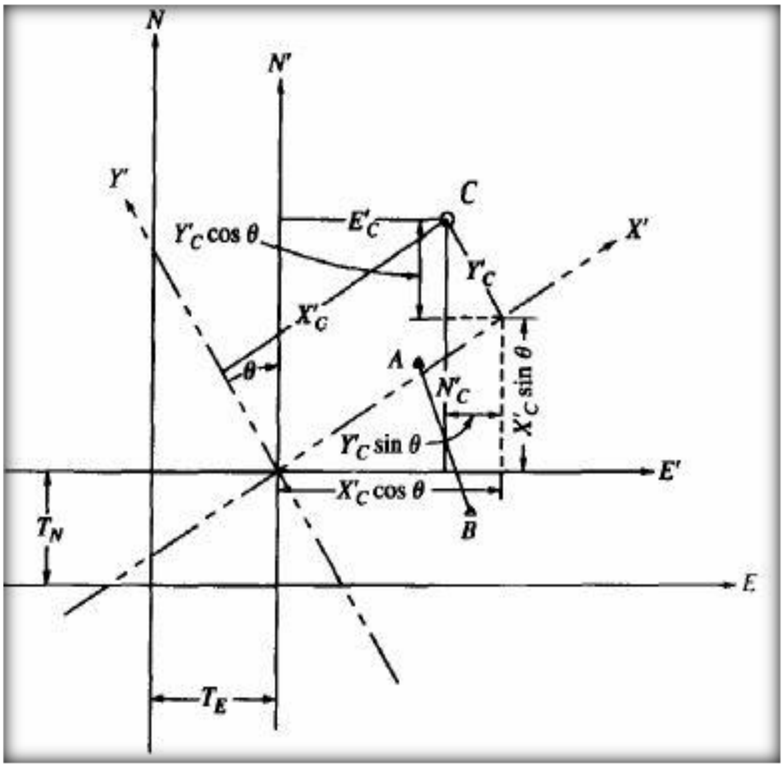

Figure (4.5): Rotation calculation in 2D Conformal [7]

- The E'N' coordinates of point C may be calculated in term of the clockwise angle θ by using the following equations:

$$E'_C = X'_C . \cos\theta - Y'_C . \sin\theta \quad (4.32)$$

$$N'_C = X'_C . \sin\theta + Y'_C . \cos\theta \quad (4.33)$$

Where,

$$\theta = \alpha + \beta \quad (4.34)$$

$$\alpha = tan^{-1}\left(\frac{X_b - X_a}{Y_b - Y_a}\right) \quad (4.35)$$

$$\beta = tan^{-1}\left(\frac{E_B - E_A}{N_B - N_A}\right) \quad (4.36)$$

Or more general:

$$\theta = AZ_{ab} + AZ_{AB} \quad (4.37)$$

- The final step in the coordinate transformation is a translation of the origin of the E'N' system to the origin of the EN system.

- The translation factors required are T_E and $T_{N,}$ which are illustrated in the above figure. Final E and N ground coordinates for points C then are:

$$E_C = E'_C + T_E \qquad (4.38)$$

$$N_C = N'_C + T_N \qquad (4.39)$$

$$T_E = E_A - E'_A = E_B - E'_B \qquad (4.38)$$

$$T_N = N_A - N'_A = N_B - N'_B \qquad (4.38)$$

Other method to write the equations:

For the following coordinates transformations equation:

$$E_a = sX_a \cos\theta - sY_a \sin\theta + T_e \qquad (4.39)$$

$$N_a = sX_a \sin\theta + sY_a \cos\theta + T_n \qquad (4.40)$$

$$E_b = sX_b \cos\theta - sY_b \sin\theta + T_e \qquad (4.41)$$

$$N_b = sX_b \sin\theta + sY_a \cos\theta + T_n \qquad (4.42)$$

If we suppose $a = s\cos\ \theta$ and $= s\sin\ \theta$, then:

$$\mathrm{E_A = aX_a - bY_a + T_e} \qquad (4.43)$$

$$N_A = aY_a + bX_a + T_n \qquad (4.44)$$

$$\mathrm{E_B = aX_b - bY_b + T_e} \qquad (4.45)$$

$$N_B = aY_b + bX_b + T_n \qquad (4.46)$$

$$s = \sqrt{a^2 + b^2} \qquad (4.47)$$

$$\theta = tan^{-1}\left(\frac{b}{a}\right) \qquad (4.48)$$

In matrix form:

$$\begin{bmatrix} y \\ x \end{bmatrix}_{Target} = S.R(\theta).\begin{bmatrix} y \\ x \end{bmatrix}_{Source} + \begin{bmatrix} T_y \\ T_x \end{bmatrix} = S.\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}.\begin{bmatrix} y \\ x \end{bmatrix}_{Source} + \begin{bmatrix} T_y \\ T_x \end{bmatrix}$$

(4.49)

Where,

$\mathrm{x, y_{Target}}$: the target coordinate system.

$\mathrm{x, y_{Source}}$: the source coordinate system.

S: scale

R(θ): rotation matrix around x,y axis

T_x, T_y: translations

Example 4.1 : A survey conducted in an arbitrary X,Y coordinate system produced station coordinates for A and B , as shown in table :

Point	E	N	X	Y
A	1,049,422.40	51,089.20	121.622	−128.066
B	1,049,413.95	49,659.30	141.228	187.718

Calculate the Scale factor and rotation angel (θ).

1- $s = \frac{AB}{ab} = \frac{\sqrt{(E_B-E_A)^2+(N_B-N_A)^2}}{\sqrt{(X_b-X_a)^2+(Y_b-Y_a)^2}}$ ➔ $= \frac{\sqrt{(1049413.95-1049422.40)^2+(49659.30-51089.20)^2}}{\sqrt{(141.228-121.622)^2+(187.718—128.066)^2}}$

➔ S= 4.519471849

2- $\alpha = tan^{-1}\left(\frac{X_a-X_b}{Y_a-Y_b}\right)$ ➔ $= tan^{-1}\left(\frac{141.228-121.622}{187.718—128.066}\right)$ ➔ α=0.008758741degree

3- $\beta = tan^{-1}\left(\frac{E_B-E_A}{N_B-N_A}\right)$ ➔$tan^{-1}\left(\frac{1049413.95-1049422.40}{49659.30-51089.20}\right)$ +180 ➔ β = 180.33858570609

4- $\theta = \alpha + \beta$ ➔ $\theta = 180.3473444$ degree

4.3.6.2. 2D Affine

In the affine coordinates transformations, to transform from xy-coordinates to XY-coordinates. As shown in the figure below, we have:

• Different scales in the x direction and y-direction.
• Rotation angle.
• The x-axis and y-axis are not orthogonal.
• Two translations in the x and y directions.

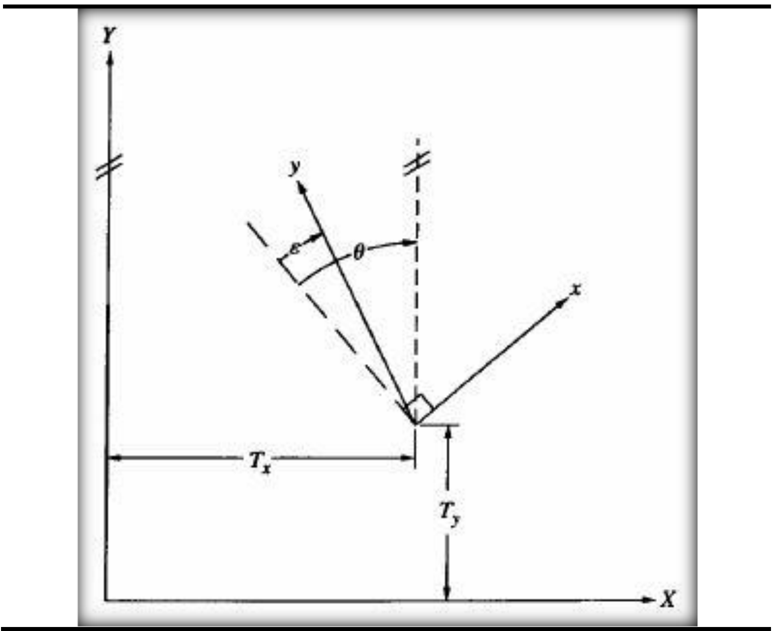

Figure (4.6): 2D Affine Helmert Transformation [7]

Step 1: scaling in the x and y direction

If we scale the xy-coordinates we get new coordinates system y'y'-coordinates:

$$x' = s_x x \qquad (4.50)$$

$$y' = s_y y \qquad (4.51)$$

Step 2: Non-Orthogonality Correction

this step we find new coordinates system x"y". so that x"-axis and y"-axis are orthogonal. In the figure below, ε is the non-orthogonality angle.

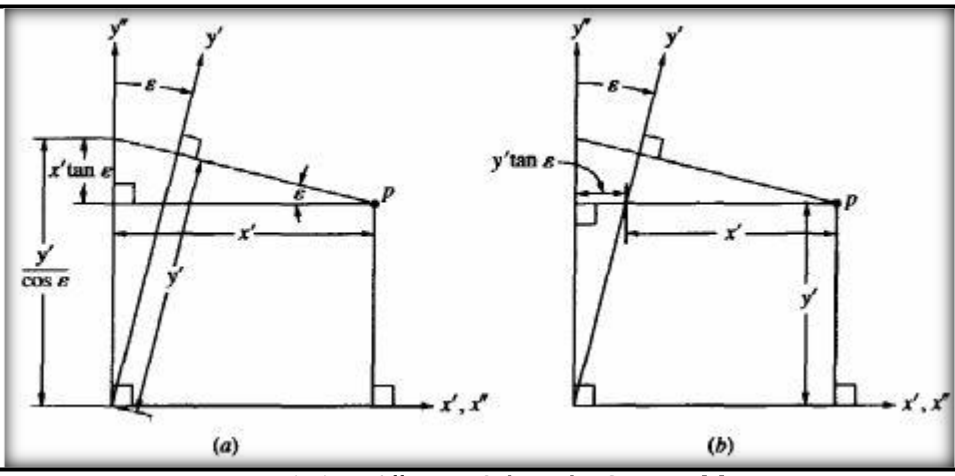

Figure (4.7): 2D Affine Non-Orthogonality Correction [7]

$$x'' = x' \qquad (4.52)$$

$$y'' = y'/\cos\,\varepsilon\; - x'\tan\,\varepsilon \qquad (4.53)$$

Step 3: Rotation

In this step we rotate the x"y"-coordinates X'Y'-coordinates, where X'Y'- coordinate are parallel to XY-coordinates. θ is the rotation angle between the y"-
axis and the Y'-axis:

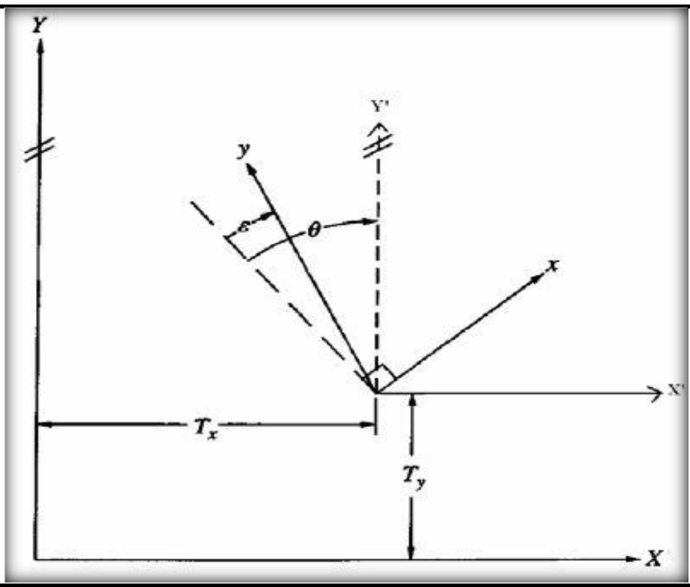

Figure (4.8): 2D Affine Rotation Angle [7]

$$X' = x\cos\theta - y\sin\theta \qquad (4.54)$$
$$Y' = x\sin\theta + y\cos\theta \qquad (4.55)$$

Step 4: Translation

In this step we add shift values to X'Y'-coordinates to get XY required coordinates:

$$X = X' + T_x \qquad (4.56)$$
$$Y = Y' + T_y \qquad (4.57)$$

Using the values of X'Y', we get:

$$X = s_x x\cos\theta - \left(\frac{s_y y}{\cos\varepsilon} - s_x x\sin\varepsilon\right)\sin\theta + T_x \qquad (4.58)$$
$$Y = s_x x\sin\theta - \left(\frac{s_y y}{\cos\varepsilon} - s_x x\tan\varepsilon\right)\tan\theta + T_y \qquad (4.59)$$

This can be arranged as follows:

$$X = s_x x(\cos\theta + \tan\varepsilon\sin\theta) - s_y y\frac{\sin\theta}{\cos\varepsilon} + T_X \qquad (4.60)$$
$$Y = s_x x(\sin\theta + \tan\varepsilon\cos\theta) - s_y y\frac{\cos\theta}{\cos\varepsilon} + T_Y \qquad (4.61)$$

In other way, we arrange the above equations as follows:

$$X = T_X + s_x x\frac{\cos\varepsilon\cos\theta + \sin\varepsilon\sin\theta}{\cos\varepsilon} - s_y y\frac{\sin\theta}{\cos\varepsilon} \qquad (4.62)$$
$$Y = T_Y + s_x x\frac{\cos\varepsilon\sin\theta + \sin\varepsilon\cos\theta}{\cos\varepsilon} + s_y y\frac{\cos\theta}{\cos\varepsilon} \qquad (4.63)$$

Using the trigonometric function relation :

$$X = T_X + s_x x\frac{\cos(\varepsilon-\theta)}{\cos\varepsilon} - s_y y\frac{\sin\theta}{\cos\varepsilon} \qquad (4.64)$$
$$Y = T_Y + s_x x\frac{\sin(\varepsilon-\theta)}{\cos\varepsilon} - s_y y\frac{\cos\theta}{\cos\varepsilon} \qquad (4.65)$$

Other form of the above equation is:

$$X = a_0 + a_1 x + a_2 y \qquad (4.66)$$
$$Y = b_0 + b_1 x + b_2 y \qquad (4.67)$$

Where,

$$a_0 = T_X \qquad (4.68)$$
$$a_1 = s_x x\frac{\cos(\varepsilon-\theta)}{\cos\varepsilon} \qquad (4.69)$$
$$a_2 = s_y y\frac{\sin\theta}{\cos\varepsilon} \qquad (4.70)$$
$$b_0 = T_Y \qquad (4.71)$$
$$b_1 = s_x x\frac{\sin(\varepsilon-\theta)}{\cos\varepsilon} \qquad (4.72)$$
$$b_2 = s_y y\frac{\cos\theta}{\cos\varepsilon} \qquad (4.73)$$

In the opposite direction we get:

$$\theta = \tan^{-1}(\frac{-a_2}{b_2}) \tag{4.74}$$

$$\varepsilon - \theta = \tan^{-1}(\frac{b_1}{a_1}) \tag{4.75}$$

$$s_x = a_1 \frac{\cos\varepsilon}{\cos(\epsilon-\theta)} \tag{4.76}$$

$$s_y = b_2 \frac{\cos\varepsilon}{\cos\theta} \tag{4.77}$$

$$T_X = a_0 \tag{4.78}$$

$$T_Y = b_0 \tag{4.79}$$

In matrix form:

$$\begin{bmatrix} y \\ x \end{bmatrix}_{Target} = A.\begin{bmatrix} y \\ x \end{bmatrix}_{Source} + \begin{bmatrix} T_y \\ T_x \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}.\begin{bmatrix} y \\ x \end{bmatrix}_{Source} + \begin{bmatrix} T_y \\ T_x \end{bmatrix} \tag{4.81}$$

$$\begin{bmatrix} y \\ x \end{bmatrix}_{Target} = R.V.\begin{bmatrix} y \\ x \end{bmatrix}_{Source} + \begin{bmatrix} T_y \\ T_x \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}.\begin{bmatrix} E_{yy} & E_{yx} \\ E_{xy} & E_{xx} \end{bmatrix}.\begin{bmatrix} y \\ x \end{bmatrix}_{Source} + \begin{bmatrix} T_y \\ T_x \end{bmatrix} \tag{4.82}$$

Where,

x, y_{Target} : the target coordinate system.

x, y_{Source} : the source coordinate system.

R: rotation matrix around x,y axis.

V: distortion matrix.

T_x, T_y: translations.

4.3.6.3 2D Projective Coordinate Transformation

The two-dimensional projective coordinate transformation is also known as the eight-parameter transformation. It is appropriate to use when one two dimensional coordinate system is projected onto another nonparallel system.
This transformation is commonly used in photogrammetry and can also be used to transform NAD 27 coordinates into the NAD 83 system. In their final form, the two-dimensional projective coordinate observation equations are: [2]

$$X = \frac{a1x+b1y+c}{a3x+b3y+1} \tag{4.83}$$

$$Y = \frac{a2x+b2y+c}{a3x+b3y+1} \tag{4.84}$$

Upon inspection it can be seen that these equations are similar to the affine transformation. In fact, if a3 and b3 are equal to zero, these equations are the affine transformation. With eight unknowns, this transformation requires a minimum of four control points. If there are more than four control points, the least squares solution may be used. Since these are nonlinear equations, they must be linearized and solved using Equation (3.83) or (3.84). The linearized form of these equations is:

$$\begin{bmatrix} \left(\frac{\partial X}{\partial a_1}\right)_0 & \left(\frac{\partial X}{\partial b_1}\right)_0 & \left(\frac{\partial X}{\partial c_1}\right)_0 & 0 & 0 & 0 & \left(\frac{\partial X}{\partial a_3}\right)_0 & \left(\frac{\partial X}{\partial b_3}\right)_0 \\ 0 & 0 & 0 & \left(\frac{\partial Y}{\partial a_2}\right)_0 & \left(\frac{\partial Y}{\partial b_2}\right)_0 0 & \left(\frac{\partial Y}{\partial c_2}\right)_0 & \left(\frac{\partial Y}{\partial a_3}\right)_0 & \left(\frac{\partial Y}{\partial b_3}\right)_0 \end{bmatrix} \begin{bmatrix} da_1 \\ db_1 \\ dc_1 \\ da_2 \\ db_2 \\ dc_2 \\ da_3 \\ db_3 \end{bmatrix}$$

$$= \begin{bmatrix} X - X_0 \\ Y - Y_0 \end{bmatrix} + \begin{bmatrix} v_x \\ v_y \end{bmatrix}$$

Where :

$$\frac{\partial X}{\partial a_1} = \frac{x}{a_3x + b_3 + 1} \quad \frac{\partial X}{\partial b_1} = \frac{y}{a_3x + b_3 + 1} \quad \frac{\partial X}{\partial c_1} = \frac{1}{a_3x + b_3 + 1}$$

$$\frac{\partial Y}{\partial a_2} = \frac{x}{a_3x + b_3 + 1} \quad \frac{\partial Y}{\partial b_2} = \frac{y}{a_3x + b_3 + 1} \quad \frac{\partial Y}{\partial c_2} = \frac{1}{a_3x + b_3 + 1}$$

$$\frac{\partial X}{\partial a_3} = -\frac{(a_1x + b_1y + c_1)}{(a_3x + b_3 + 1)^2}\, x \quad \frac{\partial X}{\partial b_3} = -\frac{(a_1x + b_1y + c_1)}{(a_3x + b_3 + 1)^2}\, y$$

$$\frac{\partial Y}{\partial a_3} = -\frac{(a_2x + b_2y + c_2)}{(a_3x + b_3 + 1)^2}\, x \quad \frac{\partial Y}{\partial b_3} = -\frac{(a_2x + b_2y + c_2)}{(a_3x + b_3 + 1)^2}\, y$$

For each control point, a set of equations of the form of Equation (4.84) can be written. A redundant system of equations can be solved by least squares to yield the eight unknown parameters. With these values, the remaining points in the xy coordinate system are transformed into the XY system using Equation (4.83).

4.3.6.4. 3D Conformal

The 3D Conformal Helmert Transformation has seven parameter transformations that include the three translation parameters, three rotation parameters and a scale parameter

Parameters are: S, ω, ∅, k, T_x, T_y and T_z the equations for the 3D Conformal Helmert transformation are:

$$X = S(m_{11}x + m_{21}y + m_{31}z) + T_x \qquad (4.85)$$
$$Y = S(m_{12}x + m_{22}y + m_{32}z) + T_y \qquad (4.86)$$
$$Z = S(m_{13}x + m_{23}y + m_{33}z) + T_z \qquad (4.87)$$

In matrix form:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = S \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix} \tag{4.88}$$

$$X = S.M.x + T_x \tag{4.89}$$
$$Y = S.M.y + T_y \tag{4.90}$$
$$Z = S.M.z + T_z \tag{4.91}$$

Where;

X, Y, Z: Coordinate system in the first datum

x, y, z: Coordinate system in the second datum

S: Scale

M: Rotation matrix

T_x, T_y, T_z: Translation matrix

$$m_{11} = \cos \emptyset \cos \kappa \tag{4.92}$$
$$m_{12} = \sin \omega \sin \emptyset \cos \kappa + \cos \omega \sin \kappa \tag{4.93}$$
$$m_{13} = -\cos \omega \sin \emptyset \cos \kappa + \sin \omega \sin \kappa \tag{4.94}$$
$$m_{21} = -\cos \emptyset \sin \kappa \tag{4.95}$$
$$m_{22} = - \sin \omega \sin \emptyset \sin \kappa + \cos \omega \cos \kappa \tag{4.96}$$
$$m_{23} = \cos \omega \sin \emptyset \sin \kappa + \sin \omega \cos \kappa \tag{4.97}$$
$$m_{31} = \sin \emptyset \tag{4.98}$$
$$m_{32} = -\sin \omega \cos \emptyset \tag{4.99}$$
$$m_{33} = \cos \omega \cos \emptyset \tag{4.100}$$

In the system of the equations, seven-parameter require a minimum number of two horizontal control stations with known X-Y and x-y-z coordinates, in addition to three stations with known Z and x-y-z coordinates. If there is more than the minimum number of observation, a least-square solution can be applied. The equations are nonlinear with respect to the unknowns and must be linearized for a solution. The following linearized equations for a full control point (Horizontal and Vertical) [2]

$$\begin{bmatrix} (\frac{\partial X}{\partial S})_0 & 0 & (\frac{\partial X}{\partial \emptyset})_0 & (\frac{\partial X}{\partial \kappa})_0 & 1 & 0 & 0 \\ (\frac{\partial Y}{\partial S})_0 & (\frac{\partial Y}{\partial \omega})_0 & (\frac{\partial Y}{\partial \emptyset})_0 & (\frac{\partial Y}{\partial \kappa})_0 & 0 & 1 & 0 \\ (\frac{\partial Z}{\partial S})_0 & (\frac{\partial Z}{\partial \omega})_0 & (\frac{\partial Z}{\partial \emptyset})_0 & (\frac{\partial Z}{\partial \kappa})_0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} dS \\ d\omega \\ d\emptyset \\ d\kappa \\ dT_x \\ dT_y \\ dT_z \end{bmatrix} = \begin{bmatrix} X - X_0 \\ Y - Y_0 \\ Z - Z_0 \end{bmatrix} + \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} \tag{4.101}$$

Where;

$$\frac{\partial X}{\partial S} = m_{11}x + m_{21}y + m_{31}z \quad (4.102)$$

$$\frac{\partial Y}{\partial S} = m_{12}x + m_{22}y + m_{32}z \quad (4.103)$$

$$\frac{\partial Z}{\partial S} = m_{13}x + m_{23}y + m_{33}z \quad (4.104)$$

$$\frac{\partial Y}{\partial \omega} = -S(m_{13}x + m_{23}y + m_{33}z) \quad (4.105)$$

$$\frac{\partial Z}{\partial \omega} = S(m_{12}x + m_{22}y + m_{32}z) \quad (4.106)$$

$$\frac{\partial X}{\partial \emptyset} = S[-\sin(\emptyset)\cos(\kappa)\,x + \sin(\emptyset)\sin(\kappa)\,y + \cos\emptyset\, z] \quad (4.107)$$

$$\frac{\partial Y}{\partial \emptyset} = S[\sin(\omega)\cos(\emptyset)\cos(\kappa)\,x - \sin(\omega)\cos(\emptyset)\sin(\kappa)\,y + \sin(\omega)\sin(\emptyset)\,z]$$

(4.108)

$$\frac{\partial Z}{\partial \emptyset} = S[-\cos(\omega)\cos(\emptyset)\cos(\kappa)\,x + \cos(\omega)\cos(\emptyset)\sin(\kappa)\,y - \cos(\omega)\sin(\emptyset)\,z]$$

(4.109)

$$\frac{\partial X}{\partial \kappa} = S(m_{21}x - m_{11}y) \quad (4.110)$$

$$\frac{\partial Y}{\partial \kappa} = S(m_{22}x - m_{12}y) \quad (4.111)$$

$$\frac{\partial Z}{\partial \kappa} = S(m_{23}x - m_{13}y) \quad (4.112)$$

The matrix solution is:

$$X = (A^T A)^{-1} A^T L \quad (4.113)$$

$$X = \begin{bmatrix} dS \\ d\omega \\ d\emptyset \\ d\kappa \\ dT_x \\ dT_y \\ dT_z \end{bmatrix} \quad (4.114)$$

The seven-parameter are calculated after first iteration, an assigned as the initial value of second iteration, where:

$$S = S_0 + dS \quad (4.115)$$

$$\omega = \omega_0 + d\omega \quad (4.116)$$

$$\emptyset = \emptyset_0 + d\emptyset \quad (4.117)$$

$$\kappa = \kappa_0 + d\kappa \quad (4.118)$$

$$T_x = T_{x0} + dT_x \quad (4.119)$$

$$T_y = T_{y0} + dT_y \quad (4.120)$$

$$T_z = T_{z0} + dT_z \quad (4.121)$$

The rotation matrix M for the transformation is:

$$M = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} \quad (4.122)$$

To calculate the elements of last matrix we must give initial values for the parameters, using one, normally, the following are useful.

$$S_0 = 1 \quad (4.123)$$
$$\omega_0 = \emptyset_0 = \kappa_0 = 0 \quad (4.124)$$
$$T_{x0} = (X - x), \quad T_{y0} = (Y - y), \quad T_{z0} = (Z - z) \quad (4.125)$$

To calculate the initial values of the observations:

$$X_0 = S(m_{11}x + m_{21}y + m_{31}z) + T_{x0} \quad (4.126)$$
$$Y_0 = S(m_{12}x + m_{22}y + m_{32}z) + T_{y0} \quad (4.127)$$
$$Z_0 = S(m_{13}x + m_{23}y + m_{33}z) + T_{z0} \quad (4.128)$$

$$m_{11} = \cos\emptyset \cos\kappa \quad (4.129)$$
$$m_{11} = \cos 0 \cos 0 = 1 \quad (4.130)$$

$$m_{12} = \sin\omega \sin\emptyset \cos\kappa + \cos\omega \sin\kappa \quad (4.131)$$
$$m_{12} = \sin 0 \sin 0 \cos 0 + \cos 0 \sin 0 = 0 \quad (4.132)$$

$$m_{13} = -\cos\omega \sin\emptyset \cos\kappa + \sin\omega \sin\kappa \quad (4.133)$$
$$m_{13} = -\cos 0 \sin 0 \cos 0 + \sin 0 \sin 0 = 0 \quad (4.134)$$

$$m_{21} = -\cos\emptyset \sin\kappa \quad (4.135)$$
$$m_{21} = -\cos 0 \sin 0 = 0 \quad (4.136)$$

$$m_{22} = -\sin\omega \sin\emptyset \sin\kappa + \cos\omega \cos\kappa \quad (4.137)$$
$$m_{22} = -\sin 0 \sin 0 \sin 0 + \cos 0 \cos 0 = 1 \quad (4.138)$$

$$m_{23} = \cos\omega \sin\emptyset \sin\kappa + \sin\omega \cos\kappa \quad (4.139)$$
$$m_{23} = \cos 0 \sin 0 \sin 0 + \sin 0 \cos 0 = 0 \quad (4.140)$$

$$m_{31} = \sin\emptyset \quad (4.141)$$
$$m_{31} = \sin 0 = 0 \quad (4.142)$$

$$m_{32} = -\sin\omega \cos\emptyset \quad (4.143)$$
$$m_{32} = -\sin 0 \cos 0 = 0 \quad (4.144)$$

$$m_{33} = \cos\omega \cos\emptyset \quad (4.145)$$
$$m_{33} = \cos 0 \cos 0 = 1 \quad (4.146)$$

And;

$$\frac{\partial X}{\partial S} = m_{11}x + m_{21}y + m_{31}z = x \qquad (4.147)$$

$$\frac{\partial Y}{\partial S} = m_{12}x + m_{22}y + m_{32}z = y \qquad (4.148)$$

$$\frac{\partial z}{\partial S} = m_{13}x + m_{23}y + m_{33}z = z \qquad (4.149)$$

$$\frac{\partial Y}{\partial \omega} = -S(m_{13}x + m_{23}y + m_{33}z) = -z \qquad (4.150)$$

$$\frac{\partial Z}{\partial \omega} = S(m_{12}x + m_{22}y + m_{32}z) = y \qquad (4.151)$$

$$\frac{\partial X}{\partial \emptyset} = S[-\sin(\emptyset)\cos(\kappa)\,\mathrm{x} + \sin(\emptyset)\sin(\kappa)\,y + \cos(\emptyset)\,z] = z \qquad (4.152)$$

$$\frac{\partial Y}{\partial \emptyset} = S[\sin(\omega)\cos(\emptyset)\cos(\kappa)\,\mathrm{x} - \sin(\omega)\cos(\emptyset)\sin(\kappa)\,y + \sin(\omega)\sin(\emptyset)\,z] = 0 \qquad (4.153)$$

$$\frac{\partial Z}{\partial \emptyset} = S[-\cos(\omega)\cos(\emptyset)\cos(\kappa)\,\mathrm{x} + \cos(\omega)\cos(\emptyset)\sin(\kappa)\,y - \cos(\omega)\sin(\emptyset)\,z] = -x \qquad (4.154)$$

$$\frac{\partial X}{\partial \kappa} = S(m_{21}x - m_{11}y) = -y \qquad (4.155)$$

$$\frac{\partial Y}{\partial \kappa} = S(m_{22}x - m_{12}y) = x \qquad (4.156)$$

$$\frac{\partial Z}{\partial \kappa} = -S(m_{23}x - m_{13}y) = 0 \qquad (4.157)$$

The linearized equations of each control point, at first iteration one:

$$\begin{bmatrix} (\frac{\partial X}{\partial S})_0 & 0 & (\frac{\partial X}{\partial \emptyset})_0 & (\frac{\partial X}{\partial \kappa})_0 & 1 & 0 & 0 \\ (\frac{\partial Y}{\partial S})_0 & (\frac{\partial Y}{\partial \omega})_0 & (\frac{\partial Y}{\partial \emptyset})_0 & (\frac{\partial Y}{\partial \kappa})_0 & 0 & 1 & 0 \\ (\frac{\partial Z}{\partial S})_0 & (\frac{\partial Z}{\partial \omega})_0 & (\frac{\partial Z}{\partial \emptyset})_0 & (\frac{\partial Z}{\partial \kappa})_0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} dS \\ d\omega \\ d\emptyset \\ d\kappa \\ dT_x \\ dT_y \\ dT_z \end{bmatrix} = \begin{bmatrix} X - X_0 \\ Y - Y_0 \\ Z - Z_0 \end{bmatrix} + \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} \qquad (4.158)$$

$$\begin{bmatrix} x & 0 & z & -y & 1 & 0 & 0 \\ y & -z & 0 & x & 0 & 1 & 0 \\ z & y & -x & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} dS \\ d\omega \\ d\emptyset \\ d\kappa \\ dT_x \\ dT_y \\ dT_z \end{bmatrix} = \begin{bmatrix} X - X_0 \\ Y - Y_0 \\ Z - Z_0 \end{bmatrix} + \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} \qquad (4.159)$$

The calculate matrix X and calculate the seven parameter S,ω, ∅, κ, T_x, T_y, T_z

$$S = S_0 + dS \tag{4.160}$$
$$\omega = \omega_0 + d\omega \tag{4.161}$$
$$\emptyset = \emptyset_0 + d\emptyset \tag{4.162}$$
$$\kappa = \kappa_0 + d\kappa \tag{4.163}$$
$$T_x = T_{x0} + dT_x \tag{4.164}$$
$$T_y = T_{y0} + dT_y \tag{4.165}$$
$$T_z = T_{z0} + dT_z \tag{4.166}$$

4.3.6.5. 3D Linearized

The 3D Linearized Helmert transformations parameters are three linear shifts (T_x, T_y, T_z), three angular rotations around each axis (r_x, r_y, r_z), and scale factor (S). the rotation values are given in decimal seconds.

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{new} = \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix} + S.\begin{bmatrix} 1 & r_z & -r_y \\ -r_z & 1 & r_x \\ r_y & -r_x & 1 \end{bmatrix}\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{original} \tag{4.167}$$

$$X_{new} = S(X + r_z Y - r_y Z) + T_x \tag{4.168}$$
$$Y_{new} = S(-r_z X + Y + r_x Z) + T_y \tag{4.169}$$
$$Z_{new} = S(r_y X - r_x Y + Z) + T_z \tag{4.170}$$

Where;

X, Y, Z: point coordinate in the target system

x, y, z: point coordinate in the source

S: scale factor

r_x, r_y, r_z: angular rotations

T_x, T_y, T_z: linear shifts

A least-squares solution can be applied. The equations are nonlinear with respect to the unknowns and must be linearized for a solution. The following linearized equations for a full control point (Horizontal and Vertical).

$$\begin{bmatrix} (\frac{\partial X}{\partial S})_0 & (\frac{\partial X}{\partial r_x})_0 & (\frac{\partial X}{\partial r_y})_0 & (\frac{\partial X}{\partial r_z})_0 & 1 & 0 & 0 \\ (\frac{\partial Y}{\partial S})_0 & (\frac{\partial Y}{\partial r_x})_0 & (\frac{\partial Y}{\partial r_y})_0 & (\frac{\partial Y}{\partial r_z})_0 & 0 & 1 & 0 \\ (\frac{\partial Z}{\partial S})_0 & (\frac{\partial Z}{\partial r_x})_0 & (\frac{\partial Z}{\partial r_y})_0 & (\frac{\partial Z}{\partial r_z})_0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} dS \\ dr_x \\ dr_y \\ dr_z \\ dT_x \\ dT_y \\ dT_z \end{bmatrix} = \begin{bmatrix} X - X_0 \\ Y - Y_0 \\ Z - Z_0 \end{bmatrix} + \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} \tag{4.171}$$

Where;

$$\frac{\partial X}{\partial S} = X + r_x Y - r_y Z \qquad (4.172)$$

$$\frac{\partial Y}{\partial S} = -r_z X + Y + r_x Z \qquad (4.173)$$

$$\frac{\partial Z}{\partial S} = r_y X - r_x Y + Z \qquad (4.174)$$

$$\frac{\partial X}{\partial r_x} = 0 \qquad (4.175)$$

$$\frac{\partial X}{\partial r_y} = -SZ \qquad (4.176)$$

$$\frac{\partial X}{\partial r_z} = SY \qquad (4.177)$$

$$\frac{\partial Y}{\partial r_x} = SZ \qquad (4.178)$$

$$\frac{\partial Y}{\partial r_y} = 0 \qquad (4.179)$$

$$\frac{\partial Y}{\partial r_z} = -SX \qquad (4.180)$$

$$\frac{\partial Z}{\partial r_x} = -SY \qquad (4.181)$$

$$\frac{\partial Z}{\partial r_y} = SX \qquad (4.182)$$

$$\frac{\partial Z}{\partial z} = 0 \qquad (4.183)$$

The matrix solution is:

$$X = (A^T A)^{-1} A^T L \qquad (4.184)$$

$$X = \begin{bmatrix} dS \\ dr_x \\ dr_y \\ dr_z \\ dT_x \\ dT_y \\ dT_z \end{bmatrix} \qquad (4.185)$$

The 3D Linearized Helmert transformation are calculated after first iteration, and assigned as the initial value of the second iteration, where;

$$S = S_0 + dS \quad (4.186)$$
$$r_x = r_{x0} + dr_x \quad (4.187)$$
$$r_y = r_{y0} + dr_y \quad (4.188)$$
$$r_z = r_{z0} + dr_z \quad (4.189)$$
$$T_x = T_{x0} + dT_x \quad (4.190)$$
$$T_y = T_{y0} + dT_y \quad (4.191)$$
$$T_z = T_{z0} + dT_z \quad (4.192)$$

4.3.7 Molodensky Method

The Molodensky method is a complex formula for the shift in latitude, longitude and height. The Molodensky Transformation has seven parameter transformations that include the three translation parameters, three rotation parameters and a scale parameter.

Parameters are: S, ω, ∅, k, T_x, T_y and T_z .

It converts directly between two geographic coordinate systems without converting to an X, Y, Z system. This method has three translations (∆ λ, ∆φ, ∆h) and the differences between the semi-major axes (∆a) and the flattening (∆f) of the two spheroids. The system automatically calculates the spheroid differences according to the datums, solve for ∆ λ and ∆φ. The amounts are added automatically by the system. [1]

The Molodensky transformation is based of all differences due to:

1- A shift in origin by a vector with components (dX, dY, dZ).
2- A difference in ellipsoids of size, (∆a) and flattening (∆f)

The Molodensky transformation gives directly the shifts in latitude, longitude and height. The angular values are in arc-seconds.

Starting from the general relationship between the geocentric coordinates (X, Y, Z) and to a specific ellipsoid with half-axes (a & b) geographic coordinates (φ, λ, h).

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} (N+h).\cos\varphi.\cos\lambda \\ (N+h).\cos\varphi.\sin\lambda \\ \left(\frac{b^2}{a^2}.N+h\right).\sin\varphi \end{bmatrix} \quad (4.193)$$

The matrix obtained by differentiation of (4.193)

$$\begin{bmatrix} dX \\ dY \\ dZ \end{bmatrix} = \begin{bmatrix} \frac{\partial X}{\partial\varphi} & \frac{\partial X}{\partial\lambda} & \frac{\partial X}{\partial h} \\ \frac{\partial Y}{\partial\varphi} & \frac{\partial Y}{\partial\lambda} & \frac{\partial Y}{\partial h} \\ \frac{\partial Z}{\partial\varphi} & \frac{\partial Z}{\partial\lambda} & \frac{\partial Z}{\partial h} \end{bmatrix} . \begin{bmatrix} d\varphi \\ d\lambda \\ dh \end{bmatrix}$$

$$\begin{bmatrix} -(M+h).\sin\varphi\cos\lambda & -(N+h).\cos\varphi\cos\lambda & \cos\varphi\cos\lambda \\ -(M+h).\sin\varphi\cos\lambda & (N+h).\cos\varphi\cos\lambda & \cos\varphi\sin\lambda \\ (M+h).\cos\varphi & 0 & \sin\varphi \end{bmatrix} . \begin{bmatrix} d\varphi \\ d\lambda \\ dh \end{bmatrix} \quad (4.194)$$

$$= \begin{bmatrix} A(dX\_dGeo) \end{bmatrix} . \begin{bmatrix} d\varphi \\ d\lambda \\ dh \end{bmatrix}$$

For two differentially adjacent points $(X,Y,Z)_1$ and $(X,Y,Z)_2$ therefore, with (4.195) can be written:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_2 - \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_1 = \begin{bmatrix} A(dX\_dGeo) \end{bmatrix}_1 . (\begin{bmatrix} \varphi \\ \lambda \\ h \end{bmatrix}_2 - \begin{bmatrix} \varphi \\ \lambda \\ h \end{bmatrix}_1) \quad (4.196)$$

The index 1 indicates that calculated under linearization at the point $(\varphi, \lambda, h)_1$ matrix [A (dX_dGeo)]. The inversion of (3.193) provides:

$$\begin{bmatrix} \varphi \\ \lambda \\ h \end{bmatrix}_2 - \begin{bmatrix} \varphi \\ \lambda \\ h \end{bmatrix}_1 = \begin{bmatrix} A(dX_{dGeo}) \end{bmatrix}_1^{-1} . (\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_2 - \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_1) \quad (4.197)$$

Furthermore it is assumed that the position differences of $(X,Y,Z)_1$ and $(X,Y,Z)_2$ by a datum transformation with the datum parameter $d = (\varepsilon_x, \varepsilon_y, \varepsilon_z, s, t_x, t_y, t_z)$ for 3 rotations $d_{rotation}$ $= (\varepsilon_x, \varepsilon_y, \varepsilon_z)$ a scale $d_s = s$ and three translations d = (t_x, t_y, t_z) will be achieved. The mater relation is obtained using classical relations for a nonlinear seven-parameter similarity transformation, so initially:

$$\begin{bmatrix} \varphi \\ \lambda \\ h \end{bmatrix}_2 = \begin{bmatrix} \varphi \\ \lambda \\ h \end{bmatrix}_1 + \begin{bmatrix} A(dX_{dGeo}) \end{bmatrix}_1^{-1} . (s.R(\varepsilon_x, \varepsilon_y, \varepsilon_z). \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_1 + t - \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_2) \quad (4.198)$$

It is assumed by (4.198), that this Datum parameters are a non-linear, that (4.198) can be linearized at the point $d_0 = (\varepsilon_x = 0, \varepsilon_y = 0, \varepsilon_z = 0, s = 1, t_x = 0, t_y = 0, t_z = 0)$. Thus the obtained matrix (4.198):

$$\begin{bmatrix} \varphi \\ \lambda \\ h \end{bmatrix}_2 = \begin{bmatrix} \varphi \\ \lambda \\ h \end{bmatrix}_1 - \begin{bmatrix} A(dX_{dGeo}) \end{bmatrix}_1^{-1} . \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_1 +$$

$$\begin{bmatrix} A(dX_{dGeo}) \end{bmatrix}_1^{-1} (\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_1 + \begin{bmatrix} 0 & -Z_1 & Y_1 & |X_1| & 1 & 0 & 0 \\ Z_1 & 0 & -X_1 & |Y_1| & 0 & 1 & 0 \\ -Y_1 & X_1 & 0 & |Z_1| & 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \Delta s \\ t_x \\ t_y \\ t_z \end{bmatrix} \quad (4.199)$$

An alternative for (4.198) is given in equation (4.199) :

$$\begin{bmatrix}\varphi\\ \lambda\\ h\end{bmatrix}_2 = \begin{bmatrix}\varphi\\ \lambda\\ h\end{bmatrix}_1 - \left[A(dX_{dGeo})\right]_1^{-1} .\left(\begin{bmatrix}0 & -z_1 & y_1 & |X_1| & 1 & 0 & 0\\ z_1 & 0 & -x_1 & |Y_1| & 0 & 1 & 0\\ -y_1 & x_1 & 0 & |Z_1| & 0 & 0 & 1\end{bmatrix}.\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\\ \Delta s\\ t_x\\ t_y\\ t_z\end{bmatrix}\right.$$

(4.200)

In (4.197) the $(X,Y,Z)_1$ inserted for in turn of (4.191) between the Cartesian coordinates $(X,Y,Z)_1$ and the geographical coordinates $(\varphi,\lambda,h)_1$, and the relation is obtained by multiplying them both ,so that final relation:

$$\begin{bmatrix}\varphi\\ \lambda\\ h\end{bmatrix}_2 = \begin{bmatrix}\varphi\\ \lambda\\ h\end{bmatrix}_1 + [Molodensky]_{(\varphi,\lambda,h)_1}.\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\\ \Delta s\\ t_x\\ t_y\\ t_z\end{bmatrix} \quad (4.201)$$

The position $(\varphi,\lambda,h)_1$ not only referenced to a different datum but at the same time there is another applicable reference ellipsoid, so to $(\varphi,\lambda,h)_1$ and $(\varphi,\lambda,h)_2$ belonging reference ellipsoid (a_1,b_1) and (a_2,b_2) differently dimensioned, so are on $(\varphi,\lambda,h)_2$ in addition to corrections $(\Delta\varphi,\Delta\lambda,\Delta h)_{(a,b)_1(a,b)_2}$ to add an ellipsoid dimensions transformations. We obtain thus in final general form:

$$\left(\begin{bmatrix}\varphi\\ \lambda\\ h\end{bmatrix}_2 + \begin{bmatrix}\Delta\varphi_{(a,b)_1(a,b)_2}\\ \Delta\lambda_{(a,b)_1(a,b)_2}\\ \Delta h_{(a,b)_1(a,b)_2}\end{bmatrix} - \begin{bmatrix}\varphi\\ \lambda\\ h\end{bmatrix}_1 + \begin{bmatrix}v_\varphi\\ v_\lambda\\ v_h\end{bmatrix}[Molodensky]_{(\varphi,\lambda,h)_1,i}.\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\\ \Delta s\\ t_x\\ t_y\\ t_z\end{bmatrix} \right. \quad (4.202)$$

Where;

$$\Delta\varphi_{(a,b)_1(a,b)_2} = \varphi_{(a_1,b_1|(X,Y,Z)_1)} - \varphi_{(a_2,b_2|(X,Y,Z)_2)} \quad (4.203)$$

$$\Delta\lambda_{(a,b)_1(a,b)_2} = 0 \quad (4.204)$$

$$\Delta h_{(a,b)_1(a,b)_2} = h_{(a_1,b_1|(X,Y,Z)_1)} - h_{(a_2,b_2|(X,Y,Z)_2)} \quad (4.205)$$

$[Molodensky]_{(\varphi,\lambda,h)_1} =$

$$\begin{bmatrix} -\sin\lambda.\frac{a.W+h}{M+h} & \cos\lambda.\frac{a.W+h}{M+h} & 0 & \frac{-\sin\varphi.\cos\varphi.N.e^2}{M+h} & \frac{-\sin\varphi.\cos\lambda}{M+h} & \frac{-\sin\varphi.\sin\lambda}{M+h} & \frac{\cos\varphi}{M+h} \\ \frac{\sin\varphi.\cos\lambda.(N.(1-e^2)+h}{(N+h).\cos\varphi} & \frac{\sin\varphi.\sin\lambda.(N.(1-e^2)+h}{(N+h).\cos\varphi} & -1 & 0 & \frac{-\sin\lambda}{(N+h).\cos\varphi} & \frac{\cos\lambda}{(N+h).\cos\varphi} & 0 \\ -N.e^2.\sin\varphi.\cos\varphi.\sin\lambda & N.e^2.\sin\varphi.\cos\varphi.\cos\lambda & 0 & h+a.W & \cos\varphi.\cos\lambda & \cos\varphi.\sin\lambda & \sin\varphi \end{bmatrix}$$

(4.206)

Where;

$$W = \frac{a}{N} = \sqrt{1-e^2.sin^2\varphi} \quad (4.207)$$

$$e^2 = \frac{a^2-b^2}{a^2} \quad (4.208)$$

Here;

h: ellipsoid height (meters)
φ: latitude in the source system
λ: longitude in the source system
a: semi-major axis of the spheroid (meters)
b: semi-minor axis of the spheroid (meters)
f: flattening of the spheroid
e: eccentricity of the spheroid
M & N: the meridian and prime vertical radius of curvature, respectively, at a given latitude.

The advantage of Molodensky over similarity transformation is that we can easily separate horizontal position (λ,φ) and the vertical position (h), while there is no need to refer to the Geocentric coordinates (X.Y.Z), in addition on difference in the ellipsoid used are included in the transformation. [1]

CHAPTER 5

THE RESULT

5.1. Introduction

5.2.The Coordinate of Trig in WGS 84 & Palestine 1923 Grid

5.3. The Parameter and Residual of Trig in Gaza

5.4. The Parameter and Residual of Trig in West Bank

THE RESULT

5.1. Introduction

The transformation of points from one coordinate system to another is a common problem encountered in surveying and mapping. For instance, a surveyor who works initially in an arbitrary coordinate system on a project may find it necessary to transform the coordinates to the state plane coordinate system to use it in land survey, in addition, every point can be observed will contain an error, so there are several mathematical models which have been developed to make these conversions, all involve some forms of coordinate transformation and calculate the residual in every point to be able to ignore the point taken a high residual.

5.2. The Coordinate of Trig in WGS 84 & Palestine 1923 Grid

These coordinates are taken from a report of a Geodetic Network, the British surveyors provided it before when they once came to Palestine, these points has a neglected height from calculation -the height approximated in calculations-, because the elevation in Palestine isn't accurate enough, for that the British surveyors didn't design it this way, then after transforming from Cartesian to local coordinate, it can be seen that the elevation is approximately equal to zero.

5.2.1. The Cartesian Coordinate of Trig

Shown Table 5.1: this table summarizes the observation sessions and the points computed in the creation of the Geodetic Network for Palestine during the observation campaign of 28th February – 13th March 1999.

Table (5.1): Cartesian Coordinate of Trig

Palestine 1923 Grid_West Bank				WGS 84 _West Bank			
	X	Y	Z		X	Y	Z
042K	4409563.849	3099459.214	3,398,923.835	**042K**	4408860.936	3099860.733	3,399,347.232
132T	4404537.648	3106369.989	3,399,130.473	**132T**	4404298.503	3106294.25	3,399,388.439
341P	4411935.633	3132896.766	3,365,252.736	**341P**	4411698.129	3132818.589	3,365,510.792
044M	4410791.390	3146011.446	3,354,574.042	**044M**	4410609.055	3146088.302	3,354,615.506
1329	4389909.714	3131275.991	3,395,225.982	**1329**	4389671.618	3131200.011	3395482.218
352P	4404619.370	3131116.645	3376398.406	**352P**	4404381.602	3131039.443	3376655.724
045M	4443329.753	3134281.072	3322679.256	**045M**	4443093.427	3134200.246	3322939.09
047M	4449266.071	3134478.384	3314594.821	**047M**	4449030.127	3134396.985	3314854.854
084M	4429221.077	3141031.301	3335034.35	**084M**	4428984.705	3140951.151	3335293.174
419F	4463134.590	3122739.546	3307072.827	**419F**	4462898.54	3122657.628	3307333.949
148T	4391329.019	3116552.235	3406838.217	**148T**	4391089.82	3116477.269	3407095.117
359P	4426880.795	3108202.581	3368500.776	**359P**	4426642.316	3108124.893	3368760.241
523B	4453863.115	3117968.834	3323915.162	**523B**	4453626.48	3117887.942	3324175.857
441F	4467637.784	3118891.316	3304639.635	**441F**	4467401.729	3118809.139	3304901.156
043M	4418488.359	3134658.097	3355069.622	**043M**	4418251.31	3134579.32	3355327.803
087M	4423999.456	3134727.310	3347784.482	**087M**	4423762.588	3134648.08	3348042.996

Palestine 1923 Grid _ Gaza				WGS 84 _Gaza			
	X	Y	Z		X	Y	Z
1361C	4487082.940	3078955.328	3,315,650.010	**1361C**	4486845.458	3078873.423	3,315,913.929
1362C	4484964.028	3082537.084	3,315,191.972	**1362C**	4484714.143	3082459.712	3,315,468.293
94GS	4482235.712	3085310.723	3,316,293.974	**94GS**	4481998.396	3085228.927	3,316,557.400
1365C	4492857.358	3075480.555	3,311,083.451	**1365C**	4492620.075	3075398.232	3,311,347.676
1368C	4494272.168	3076360.521	3,308,363.412	**1368C**	4494035.063	3076277.986	3,308,627.633
1385C	4509344.542	3074638.263	3289529.965	**1385C**	4509108.014	3074554.837	3289794.701
1405K	4499439.712	3076261.781	3301471.284	**1405K**	4499202.787	3076178.908	3301735.733
1406K	4502675.150	3077667.661	3295784.001	**1406K**	4502438.361	3077584.542	3296048.592

5.2.2. The Polar Coordinate of Trig

These coordinates are the coordinates of geodetic network after transforming it to the polar coordinate by excel software functions, to allow it to be used in Moldensky transformation, then the parameters can be calculated between WGS84 and Palestine 1923 Grid Coordinate by using these methods, if you see table 5.2 (h) is approximately equal to zero, because the elevation was approximate in last time in the local coordinate, then the height was deleted to eliminate the error.

Table (5.2): Polar Coordinate of Trig

Palestine 1923 Grid_West Bank				WGS 84 _West Bank			
	PHI	Lamda	h		PHI	Lamda	h
042K	32.4127246	35.10318171	-0.006585076	**042K**	32.41476155	35.11097098	2.68901E-05
132T	32.41493196	35.19401026	-0.006617343	**132T**	32.41520172	35.19481753	4.60297E-05
341P	32.05374677	35.3783993	-0.006644736	**341P**	32.05402996	35.37918041	9.02582E-05
044M	31.94018812	35.4984892	-0.006604884	**044M**	31.93817353	35.50027068	5.74319E-05
1329	32.37323216	35.49986006	-0.006567784	**1329**	32.37348487	35.50067196	4.17167E-05
352P	32.17241961	35.40792132	-0.006589282	**352P**	32.17269081	35.4087148	4.90611E-05
045M	31.60183307	35.19876639	-0.006667101	**045M**	31.60215111	35.19950586	6.48266E-05
047M	31.51626223	35.16444114	-0.0066461	**047M**	31.51658545	35.16517117	3.36962E-05
084M	31.73275722	35.342783	-0.006654291	**084M**	31.73305989	35.34353596	4.09037E-06
419F	31.43671391	34.97948514	-0.006567313	**419F**	31.43705151	34.98020257	3.68394E-05
148T	32.49730616	35.36354817	-0.006681292	**148T**	32.49756171	35.36437074	5.28228E-05
359P	32.08831439	35.07346954	-0.006589596	**359P**	32.08861137	35.07424764	2.22158E-05
523B	31.61492151	34.99430509	-0.006640585	**523B**	31.6152482	34.99503693	9.31993E-05
441F	31.4109961	34.91918105	-0.006681292	**441F**	31.41133883	34.91989341	5.37354E-05
043M	31.94545512	35.35346339	-0.006633094	**043M**	31.94574343	35.35423453	1.63466E-05
087M	31.86805889	35.32036283	-0.006589596	**087M**	31.86835348	35.32112686	-7.46362E-06
Palestine 1923 Grid _ Gaza				**WGS 84 _Gaza**			
	PHI	**Lamda**	**h**		**PHI**	**Lamda**	**h**
1361C	31.52742664	34.45721088	-0.006672309	**1361C**	31.52779058	34.45791455	-9.45572E-06
1362C	31.52258022	34.50092586	-0.006675402	**1362C**	31.52307554	34.50174473	1.98828E-05
94GS	31.53424074	34.5412661	-0.006686294	**94GS**	31.53459921	34.54197351	4.052E-05
1365C	31.47911969	34.39267959	-0.00663213	**1365C**	31.47948857	34.39337521	6.75349E-05
1368C	31.45035766	34.39191124	-0.006609363	**1368C**	31.45072753	34.39260372	-4.75422E-05
1385C	31.25144825	34.28761568	-0.006640218	**1385C**	31.25183069	34.28829093	4.69908E-05
1405K	31.37751836	34.3603715	-0.006661831	**1405K**	31.37789324	34.36105804	2.69227E-05
1406K	31.31745431	34.35338014	-0.006635926	1406K	31.31783285	34.35406299	-3.17637E-05

5.2.3. The Local Coordinate of Trig

These coordinates are the coordinates of geodetic network after transforming it to the local (Palestine 1923Grid) coordinate by excel functions, to allow it to be used in 2D(Affine, conformal, projective) transformation, then the parameters can calculated between WGS84 and Palestine 1923Grid Coordinate by using these methods.

Table (5.3): Local Coordinate of Trig

Palestine 1923 Grid_West Bank			WGS 84 _West Bank		
	E	N		E	N
042K	160007.3002	202122.0049	042K	160740.2602	202347.1623
132T	168551.7001	202361.7044	132T	168627.6441	202391.606
341P	185959.0000	162323.0043	341P	186032.7211	162354.5188
044M	197333.8398	149755.0449	044M	197502.8872	149532.1095
1329	197335.0996	197773.9043	1329	197411.4337	197802.1334
352P	188723.2004	175486.7044	352P	188797.9862	175516.9131
045M	168987.9999	112203.0047	045M	169058.182	112238.2596
047M	165726.2998	102716.2640	047M	165795.6607	102752.0705
084M	182638.2300	126726.7941	084M	182709.5472	126760.4389
419F	148138.7000	93918.9044	419F	148206.9852	93956.19055
148T	184487.0003	211506.2049	148T	184564.2678	211534.6538
359P	157165.8004	166152.3044	359P	157239.3003	166185.1409
523B	149586.9002	113674.7038	523B	149656.4162	113710.7882
441F	142398.0004	91081.2047	441F	142465.8442	91119.02377
043M	183619.7101	150311.9750	043M	183692.582	150344.039
087M	180498.5299	141726.5946	087M	180570.7985	141759.3307
Palestine 1923 Grid _ Gaza			WGS 84 _Gaza		
	E	N		E	N
1361C	98555.78025	104200.1046	1361C	98622.88988	104239.9975
1362C	102704.1804	103634.9644	1362C	102782.3138	103689.38
94GS	106543.6704	104903.6942	94GS	106611.0961	104943.0293
1365C	92386.83021	98887.91399	1365C	92453.23643	98928.32138
1368C	92289.98974	95699.37407	1368C	92356.11856	95739.89306
1385C	82191.20955	73723.50483	1385C	82255.884	73765.36996
1405K	89229.45035	87645.93415	1405K	89295.08027	87686.99445
1406K	88512.46022	80991.3939	1406K	88577.78562	81032.85879

5.3.The Parameter and Residual of Trig in Gaza

This section shows the parameters and residual of Trig in Gaza between WGS84 and Palestine 1923Grid system, by using Transformation methods in 2D (Affine, Conformal, and Projective), 3D (Conformal, Linearzed) and Moldensky.

5.3.1. 2D-Affine

The two-dimensional affine coordinate transformation is also known as the six parameter transformation, these methods deal with linear functions, by it the parameter can be determined between the two coordinate system, if you have at least 3 points, the table (5.4) shows the parameter and the new coordinate of Trig in Gaza after the transform from WGS84 to Palestine 1923Grid.

Table (5.4):New Local Coordinate and residual of Trig in Gaza by using 2d-affine

Palestine 1923 Grid(new)						
	E	Residual (E)	N	Residual (N)	Parameter	Value
1361C	98554.0265	-1.753744995	104197.6083	-2.496276498	a	0.999608281
1362C	102711.7945	7.614140719	103645.2345	7.614140719	b	4.81508E-05
94GS	106539.1374	-4.532981721	104897.566	-6.128102096	c	-35.25013049
1365C	92386.53406	-0.2961477	98887.49644	-0.417552847	d	-0.000399778
1368C	92289.3007	-0.689035979	95698.56535	-0.808713888	e	1.000169863
1385C	82191.96451	0.754961637	73724.34746	0.84263639	f	-20.66837787
1405K	89229.07373	-0.376626139	87645.52259	-0.411557053		
1406K	88511.73966	-0.720565878	80990.54341	-0.850496678		
Palestine 1923 Grid(new) , after deleting point (1362c) because it contained high residual						
	E	Residual (E)	N	Residual (N)	Parameter	Value
1361C	98555.90103	0.120782148	104200.1367	0.032112669	a	0.999946306
1362C	102715.1284		103649.7313		b	-4.87619E-05
94GS	106543.644	-0.026373849	104903.6446	-0.049524564	c	-56.61042517
1365C	92386.83786	0.00765125	98887.90621	-0.007784472	d	5.61548E-05
1368C	92289.88067	-0.109066057	95699.34763	-0.026442131	e	1.000039145
1385C	82191.25996	0.050410391	73723.39715	-0.107672485	f	-49.47945073
1405K	89229.39942	-0.050935442	87645.96189	0.027739229		
1406K	88512.46775	0.007531639	80991.52547	0.131571714		

- the maximum value of residual = 0.1316 m, minimum value of residual = -0.1091 m, Standard error = 0.0683 m

5.3.2. 2D-Conformal

The two-dimensional Conformal coordinate transformation is also known as the four parameter transformation, this method deals with linear functions, using it the parameter can be determined between two coordinate system if you have at least 2 points, the table (5.5) shows the parameter and the new coordinate of Trig in Gaza after the transform from WGS84 to Palestine 1923Grid.

Table (5.5):New Local Coordinate and residual of Trig in Gaza by using 2d-conformal

Palestine 1923 Grid(new)						
	E	Residual (E)	N	Residual (N)	Parameter	Value
1361C	98,553.77	-2.01238842	104,196.32	-3.789561722	a	0.999841858
1362C	102,712.58	8.395859178	103,646.10	11.13977255	b	7.68239E-05
94GS	106,540.66	-3.013619694	104,899.85	-3.844788746	Tx	-45.51750927
1365C	92,385.50	-1.332058929	98,885.00	-2.909055011	Ty	-34.7743666
1368C	92,288.64	-1.349158649	95,697.07	-2.300684182		
1385C	82,191.69	0.481910769	73,725.25	1.744612634		
1405K	89,228.71	-0.74532981	87,645.21	-0.721038695		
1406K	88,512.04	-0.42521443	80,992.07	0.680743168		

Palestine 1923 Grid(new), after deleting point (1362c) because it contained high residual						
	E	Residual (E)	N	Residual (N)	Parameter	Value
1361C	98,555.81	0.030240832	104,200.03	-0.075351401	a	1.000014225
1362C	102,715.34		103,649.77		b	8.91171E-05
94GS	106,544.07	0.397268988	104,903.78	0.088801438	Tx	-59.19275532
1365C	92,386.54	-0.28757599	98,887.73	-0.186246242	Ty	-50.24006539
1368C	92,289.71	-0.282219677	95,699.25	-0.128648641		
1385C	82,191.29	0.078040986	73,723.51	0.00480485		
1405K	89,229.34	-0.107016601	87,645.96	0.025314188		
1406K	88,512.63	0.171261471	80,991.67	0.271325824		

- the maximum value of residual = 0.3973 m, minimum value of residual = -0.2876 m, Standard error = 0.1903 m

5.3.3. 2D- Projective

The two-dimensional projective coordinate transformation is also known as the eight-parameter transformation, this method is used to transform the coordinate from WGS84 to Palestine 1923Grid of the Trig in Gaza, so by the parameter result from this method, any point in Gaza can be transformed from WGS84 to Palestine 1923Grid.

Table (5.6):New Local Coordinate and residual of Trig in Gaza by using 2d-Projective

Palestine 1923 Grid(new)						
	E	Residual (E)	N	Residual (N)	Parameter	Value
1361C	98552.83058	-2.949516724	104196.3714	-3.732947928	a1	0.993623611
1362C	102711.2124	7.032183027	103645.0812	10.116974	b1	0.003088483
94GS	106540.0435	-3.626735025	104898.2178	-5.476104376	c1	109.7903328
1365C	92387.14132	0.311229625	98887.42653	-0.487256235	a2	-0.005041731
1368C	92289.66303	-0.326585079	95699.229	-0.144875996	b2	1.001832328
1385C	82191.98555	0.776078052	73724.39495	0.890211684	c2	128.2610224
1405K	89229.55727	0.10701932	87646.78593	0.851933588	a3	-4.68431E-08
1406K	88511.13645	-1.323673197	80989.37585	-2.017934742	b3	3.19502E-08
Palestine 1923 Grid(new), after deleting point (1362c) because it contained high residual						
	E	Residual (E)	N	Residual (N)	Parameter	Value
1361C	98555.83319	0.052943794	104200.0911	-0.013490673	a1	0.999553743
1362C	102715.0808		103649.7234		b1	0.000105812
94GS	106543.6835	0.013047629	104903.6811	-0.0130058	c1	-44.96842807
1365C	92386.8745	0.044287603	98887.90535	-0.008646198	a2	-0.00022461
1368C	92289.89085	-0.098892707	95699.36694	-0.007130185	b2	1.000082145
1385C	82191.29449	0.084933258	73723.44429	-0.060533892	c2	-37.83435659
1405K	89229.41104	-0.039310267	87646.00084	0.066687403	a3	-2.78403E-09
1406K	88512.40321	-0.057009309	80991.43002	0.036119344	b3	1.57371E-09

- the maximum value of residual = 0.0849 m, minimum value of residual = -0.0989 m, Standard error = 0.0511m

5.3.4. 3D- Linearized-Helmert

by this method you can convert from coordinate system to any other coordinate system by using a matrix function, then the table below shows the result of conversion between WGS84 and Palestine 1923Grid coordinate system in the trig of Gaza, in this result the elevation of all points was deleted, because it causes high residual between two coordinates, which the height in Palestine 1923Grid was approximated.

Table (5.7):New Cartesian Coordinate and residual of Trig in Gaza by using 3d-Linearized-Helmert

Palestine 1923 Grid(new)								
	X	Residual (X)	Y	Residual (Y)	Z	Residual (Z)	Parameter	Value
1361C	4487085.591	2.650588041	3078954.872	-0.456163019	3315646.865	-3.144514376	S	0.999827235
1362C	4484954.586	-9.441815955	3082540.528	3.444301911	3315201.482	9.510009542	theta 1	-3.18066E-05
94GS	4482239.305	3.593421256	3085309.196	-1.52665952	3316290.568	-3.406121268	theta 2	-2.92291E-05
1365C	4492859.121	1.762984977	3075480.499	-0.055620115	3311081.121	-2.330196405	theta 3	-1.26331E-05
1368C	4494273.774	1.605421576	3076360.206	-0.315338305	3308361.535	-1.876641845	TX	957.2892103
1385C	4509343.593	-0.949670559	3074638.144	-0.119645367	3289531.362	1.397145476	TY	662.1480446
1405K	4499440.405	0.693260525	3076261.429	-0.35167813	3301470.672	-0.612033026	TZ	339.0228147
1406K	4502675.236	0.085810139	3077667.042	-0.619197455	3295784.464	0.462351903		
Palestine 1923 Grid(new), after deleting point (1362c) because it contained high residual								
	X	Residual (X)	Y	Residual (Y)	Z	Residual (Z)	Parameter	Value
1361C	4487082.873	-0.066828558	3078955.405	0.077527365	3315650.029	0.019222252	S	0.999998135
1362C	4484951.472		3082541.662		3315204.614		theta 1	-4.03597E-05
94GS	4482235.712	0.000757029	3085310.772	0.048812609	3316293.927	-0.047270287	theta 2	-3.55683E-05
1365C	4492857.389	0.031351096	3075480.524	-0.030453271	3311083.438	-0.013212411	theta 3	-2.065E-05
1368C	4494272.259	0.091298173	3076360.416	-0.105194469	3308363.386	-0.025961657	TX	191.4211338
1385C	4509344.549	0.006516646	3074638.341	0.078066359	3289529.883	-0.081103092	TY	128.9002969
1405K	4499439.731	0.019373849	3076261.723	-0.058041425	3301471.311	0.027185826	TZ	-222.3895943
1406K	4502675.068	-0.082468234	3077667.65	-0.010717169	3295784.122	0.121139369		

- the maximum value of residual = 0.1211 m, minimum value of residual = -0.1052 m, Standard error = 0.0739 m

5.3.5. 3D- Conformal

By this method you can convert points from three dimensional coordinate system to any other system, then the table below shows the result of conversion between WGS84 and Palestine 1923Grid coordinate system in the trig of Gaza, in this result the elevation of all point was deleted, because causes high residual between two coordinates for that the height in Palestine 1923Grid was approximated originally, these methods in generally is the same as 3D- Linearized-Helmert from the residual and the parameter.

Table (5.8):New Cartesian Coordinate and residual of Trig in Gaza by using 3d-Conformal

Palestine 1923 Grid (new)								
	X	Residual (X)	Y	Residual (Y)	Z	Residual (Z)	Parameter	Value
1361C	4487085.591	2.650581851	3078954.872	-0.456167287	3315646.865	-3.14451878	theta 1	3.18098E-05
1362C	4484954.586	-9.441807173	3082540.528	3.444307902	3315201.482	9.510015848	theta 2	2.92285E-05
94GS	4482239.305	3.593440246	3085309.196	-1.526646502	3316290.568	-3.406107661	theta 3	1.26287E-05
1365C	4492859.121	1.762968982	3075480.499	-0.055631125	3311081.121	-2.330207829	S	0.999827236
1368C	4494273.774	1.60541171	3076360.206	-0.315345128	3308361.535	-1.876648889	TX	957.2754436
1385C	4509343.593	-0.949669741	3074638.144	-0.119644626	3289531.362	1.397145991	TY	662.1729548
1405K	4499440.405	0.693256733	3076261.429	-0.351680765	3301470.672	-0.612035744	TZ	339.0071934
1406K	4502675.236	0.085817392	3077667.042	-0.61919247	3295784.464	0.462357064		
Palestine 1923 Grid (new), after deleting the point (1362c) because it contained high residual								
	X	Residual (X)	Y	Residual (Y)	Z	Residual (Z)	Parameter	Value
1361C	4487082.873	-0.066828752	3078955.405	0.077527166	3315650.029	0.019222201	theta 1	4.03592E-05
1362C	4484951.472		3082541.662		3315204.614		theta 2	3.55689E-05
94GS	4482235.712	0.000757637	3085310.772	0.048813004	3316293.927	-0.047270245	theta 3	2.06493E-05
1365C	4492857.389	0.031350609	3075480.524	-0.030453662	3311083.438	-0.013212483	S	0.999998137
1368C	4494272.259	0.091297897	3076360.416	-0.105194688	3308363.386	-0.025961695	TX	191.4136107
1385C	4509344.549	0.006516741	3074638.341	0.078066509	3289529.883	-0.081103044	TY	128.8932489
1405K	4499439.731	0.019373792	3076261.723	-0.058041444	3301471.311	0.027185832	TZ	-222.3921199
1406K	4502675.068	-0.082467923	3077667.65	-0.010716884	3295784.122	0.121139433		

- the maximum value of residual = 0.1211 m, minimum value of residual = -0.1052 m, Standard error = 0.0739 m

5.3.6. Moldenesky

Using this method it can be converted directly between two geographic coordinate systems without converting to an X, Y, Z system, the table below shows transformation from WGS 84 to Palestine 1923Grid of Trig in Gaza, in this point the height was deleted because it increases the residual.

Table (5.9):New Polar Coordinate and residual of Trig in Gaza by using Moldenesky

Palestine 1923 Grid (new)								
	Phi		Residual (m)	Lamda		Residual (m)	Parameter	Value
	Radian	degree		Radian	degree		s	-68.79793762
1361C	0.550256373	31.5273678	-3.532765558	0.601390712	34.45714966	-3.237597298	rx	0.467248641
1362C	0.550174474	31.52267537	5.713172119	0.602155897	34.50099153	6.615979108	ry	0.213932815
94GS	0.550375614	31.53419985	-2.455207885	0.602858568	34.54125161	-1.459170056	rz	-1.428015633
1365C	0.54941398	31.47910226	-1.04694281	0.600265674	34.39268973	1.022225073	Tx	-1231999.387
1368C	0.54891249	31.45036899	0.680349859	0.600252305	34.39192371	1.256077869	Ty	-8336900.508
1385C	0.545440481	31.25143751	-0.644451207	0.598432232	34.28764124	2.574532768	Tz	-295639.0716
1405K	0.54764167	31.37755635	2.281001827	0.599702088	34.3603986	2.730543561		
1406K	0.546592402	31.31743774	-0.995169451	0.599578454	34.3533149	-6.572638719		
Palestine 1923 Grid (new), after deleting the point (1362c) because it contained high residual								
	Phi		Residual (m)	Lamda		Residual (m)	Parameter	Value
	Radian	degree		Radian	degree		s	-68.74543944
1361C	0.550256877	31.52739668	-0.281367224	0.601390887	34.45715967	-0.277405361	rx	0.482813639
1362C	0.550175108	31.52271166		0.602156405	34.5010206		ry	0.235818532
94GS	0.550376413	31.53424563	0.190590139	0.602859334	34.54129547	0.21647479	rz	-1.469545617
1365C	0.549414193	31.47911445	-0.204686382	0.600265641	34.3926878	0.22264983	Tx	-1033323.448
1368C	0.548912661	31.45037878	0.216560789	0.600252371	34.39192748	0.223006149	Ty	-8576058.415
1385C	0.545440437	31.25143502	-0.135668839	0.59843215	34.28763651	0.285974907	Tz	-346046.9181
1405K	0.547641701	31.37755813	0.240783004	0.599702183	34.36040406	0.276598161		
1406K	0.546592389	31.31743699	-0.177727896	0.599578615	34.35332414	-0.248286463		

- the maximum value of residual = 0.2859 m, minimum value of residual = -0.2814 m, Standard error = 0.2323 m

- In the table(5.10) which the height of point was used in transformation, the residual will decrease in small deference, then you can conclude that the height causes an error, because the local coordinate (Palestine 1923Grid) can't have height, but in Moldesky can't be causes the result.

Table (5.10):New Polar Coordinate and residual of Trig in Gaza by using Moldenesky

Palestine 1923 Grid (new)								
	PHI		Residual (m)	Lamda		Residual (m)	Parameter	Value
	Radian	degree		Radian	degree		s	-69.30849428
1361C	0.550258083	31.52746579	-2.472935891	0.601392537	34.4572542	-2.266318109	rx	-0.338447942
1362C	0.550173506	31.52261992	3.999220483	0.602156457	34.50102357	4.631185376	ry	0.593058906
94GS	0.550372462	31.53401925	-1.71864552	0.602859234	34.54128977	-1.021419039	rz	-0.68796886
1365C	0.549415358	31.47918119	-0.732859967	0.600264093	34.3925991	0.715557551	Tx	-2249990.999
1368C	0.548913574	31.45043111	0.476244901	0.600250143	34.39179984	0.879254508	Ty	-2342494.116
1385C	0.545438901	31.251347	-0.451115845	0.598435851	34.28784857	1.802172937	Tz	-4486264.84
1405K	0.547642515	31.37760479	1.596701279	0.599700088	34.36028401	1.911380493		
1406K	0.546593084	31.31747683	-0.696618616	0.599577529	34.35326191	-4.600847103		

Palestine 1923 Grid (new)									
	PHI		Residual (m)	Lamda		Residual (m)	Parameter	Value
	Radian	degree		Radian	degree		s	-69.17709367
1361C	0.550258257	31.52747575	-0.196957057	0.601392851	34.45727221	-0.264183753	rx	-0.349685945
1362C	0.550173802	31.52263687		0.602157226	34.50106767		ry	0.588903831
94GS	0.550372847	31.53404131	0.133413097	0.602860298	34.54135069	0.151532353	rz	-0.669372838
1365C	0.549415378	31.47918233	-0.143280467	0.600264245	34.39260782	0.155854881	Tx	-2323528.943
1368C	0.548913636	31.45043468	0.151592552	0.600250301	34.39180891	0.156104304	Ty	-2223506.536
1385C	0.54543879	31.25134064	-0.094968187	0.598435738	34.28784212	0.200182435	Tz	-4501146.86
1405K	0.547642572	31.37760804	0.18548103	0.599700084	34.36028379	0.193618713		
1406K	0.54659319	31.31748291	-0.124409527	0.599577663	34.35326959	-0.173800524		

- the maximum value of residual = 0.2001 m, minimum value of residual = -0.2642 m, Standard error = 0.1708 m

5.4.The Parameter and Residual of Trig in West Bank coordinate

This section shows the parameters and residuals of Trig in West Bank between WGS84 and Palestine 1923Grid system by using Transformation methods in 2D (Affine, Conformal and Projective), 3D (Conformal, Linearzed) and Moldensky.

5.4.1. 2D- Affine

The two-dimensional affine coordinate transformation is also known as the six parameter transformation, these method deal with linear function, by these method you can determine the parameter between two coordinate system if you have at least 3 points, the table (5.11) shown the parameter and the new coordinate of Trig in West Bank after transform from WGS84 to Palestine 1923Grid.

Table (5.11):New Local Coordinate and residual of Trig in West Bank by using 2d-Affine

Palestine 1923 Grid(new)						
	E	Residual (E)	N	Residual (N)	Parameter	Value
042K	160442.9053	435.6050628	202221.5916	99.58669653	a	1.004098444
132T	168362.5071	-189.1930384	202292.1848	-189.1930384	b	-0.002432929
341P	185936.3252	-22.67482815	162356.4203	33.41599582	c	-463.8436609
044M	197484.6971	150.8573151	149586.0442	-169.0007057	d	0.003321496
1329	197275.4312	-59.66841948	197803.3053	29.40095781	e	0.998913216
352P	188680.9004	-42.29998294	175513.6947	26.99029662	f	-439.5611177
045M	169014.1461	26.14627144	112238.2457	35.24102848		
047M	165761.3328	35.03299758	102751.5296	35.26563171		
084M	182686.1293	47.89933119	126789.9856	63.1914851		
419F	148121.9709	-16.72916492	93906.78818	-12.11621413		
148T	184342.2018	-144.7985954	211478.2296	-27.97531481		
359P	157015.5765	-128.4592781	166087.2421	-65.06228644		
523B	149529.2807	-57.61949413	113644.7311	-29.97272227		
441F	142364.2028	-33.79761839	91053.63561	-27.5690399		
043M	183615.8158	-3.894369764	150351.2204	39.24541887		
087M	180502.1238	3.59381155	141765.4729	38.8783095		

Palestine 1923 Grid(new), after deleting point (42K & 44M) because it contained high residual						
	E	**Residual (E)**	**N**	**Residual (N)**	**Parameter**	**Value**
042K	160664.1825		202316.8213		**a**	0.999980793
132T	168551.4118	-0.288268928	202361.7177	-0.288268928	**b**	-6.91431E-05
341P	185958.9228	-0.077182973	162323.2826	0.278310641	**c**	-58.99944228
044M	197429.7552		149500.7781		**d**	5.70839E-05
1329	197334.9659	-0.133782658	197773.6199	-0.284449628	**e**	1.000058486
352P	188723.2247	0.024343222	175486.6045	-0.099860548	**f**	-51.35114119
045M	168988.1749	0.175055895	112203.1233	0.118592512		
047M	165726.3722	0.072389432	102716.1931	-0.070836896		
084M	182638.2739	0.043874265	126726.9313	0.13718606		
419F	148138.6427	-0.057317775	93918.79473	-0.109661228		
148T	184487.0973	0.096922135	211506.2101	0.00515454		
359P	157165.7902	0.608349009	166152.4851	0.180700834		
523B	149586.6799	-0.220264948	113674.6305	-0.073366445		
441F	142397.8082	-0.192209519	91081.13429	-0.070360915		
043M	183619.6591	-0.051047164	150311.9667	-0.008327635		
087M	180498.5291	-0.00086002	141726.5781	-0.016456937		

- the maximum value of residual = 0.6083 m, minimum value of residual = -0.2883 m, Standard error = 0.1771 m

5.4.2. 2D- Conformal

The two-dimensional Conformal coordinate transformation is also known as the four parameter transformation, these methods deal with linear function, by these methods you can determine the parameter between two coordinate system if you have at least 2 points, the table (5.12) shows the parameter and the new coordinate of Trig in West Bank after transform from WGS84 to Palestine 1923Grid.

Table (5.12):New Local Coordinate and residual of Trig in West Bank by using 2d-Conformal

Palestine 1923 Grid (new)						
	E	**Residual (E)**	**N**	**Residual (N)**	**Parameter**	**Value**
042K	160,541.27	533.9715965	202,293.38	171.3779371	**a**	0.999878919
132T	168,427.63	-124.067961	202,349.99	-11.71412959	**b**	0.001542864
341P	185,892.37	-66.62651979	162,344.60	21.60024566	**Tx**	132.6683052
044M	197,380.93	47.09419229	149,541.44	-213.6002972	**Ty**	-277.2793807
1329	197,215.02	-120.082193	197,805.48	31.57853299		
352P	188,637.00	-86.2043934	175,509.67	22.96712091		
045M	168,997.21	9.212345104	112,208.22	5.219315004		
047M	165,749.72	23.42203674	102,718.15	1.885901714		
084M	182,624.52	-13.71118961	126,749.71	22.91312988		
419F	148,176.75	38.0468267	93,896.20	-22.70637271		
148T	184,348.22	-138.7805946	211,516.52	10.31417438		
359P	157,096.53	-69.27149796	166,130.34	-21.9658978		
523B	149,595.52	8.62346319	113,650.64	-24.06381169		
441F	142,440.68	42.67799875	91,050.52	-30.68768253		
043M	183,571.05	-48.66189336	150,331.97	19.99335888		
087M	180,462.89	-35.64221654	141,743.48	16.88847504		

Palestine 1923 Grid(new), after deleting point (42K & 44M) because it contained a high residual						
	E	Residual (E)	N	Residual (N)	Parameter	Value
042K	160,662.61		202,315.68		a	1.000043709
132T	168,550.33	-1.366326519	202,360.80	-0.905021729	b	8.56264E-05
341P	185,959.60	0.599765719	162,323.45	0.448309997	Tx	-67.35081771
044M	197,431.37		149,501.46		Ty	-54.09193225
1329	197,335.77	0.674775961	197,773.59	-0.313468874		
352P	188,723.86	0.658311552	175,486.66	-0.04542916		
045M	168,988.61	0.610114923	112,203.55	0.544644398		
047M	165,726.76	0.458547191	102,716.67	0.402268174		
084M	182,639.33	1.098451059	126,727.53	0.738293525		
419F	148,138.07	-0.632786914	93,918.90	-0.008607719		
148T	184,486.87	-0.129171848	211,505.61	-0.593475079		
359P	157,164.59	-1.207959191	166,151.78	-0.527739748		
523B	149,585.87	-1.030163286	113,674.48	-0.222848518		
441F	142,396.92	-1.082123367	91,081.11	-0.09125024		
043M	183,620.39	0.676641787	150,312.25	0.272352459		
087M	180,499.20	0.671922928	141,726.90	0.301972518		

- the maximum value of residual = 1.0985 m, minimum value of residual = -1.3663 m, Standard error = 0.6788 m

5.4.3. 2D- Projective

The two-dimensional projective coordinate transformation is also known as the eight-parameter transformation, this method it used to transform the coordinate from WGS84 to Palestine 1923Grid of the Trig in West Bank, so by the parameter result from this method you can transform any point in West Bank from WGS84 to Palestine 1923Grid .

Table (5.13):New Local Coordinate and residual of Trig in West Bank by using 2d-Projective

Palestine 1923 Grid(new)						
	E	Residual (E)	N	Residual (N)	Parameter	Value
042K	160433.7323	426.4895552	202230.9645	109.0387039	a1	1.006013191
132T	168365.4548	-186.1768508	202304.4043	-57.21336559	b1	-0.006677338
341P	185936.1062	-22.80353125	162334.3639	11.43708088	c1	-320.4877962
044M	197468.4548	134.7203837	149582.8599	-172.1049623	a2	0.007115945
1329	197300.5445	-34.44437106	197817.5037	43.71023658	b2	0.993670241
352P	188688.1961	-34.90885866	175497.9436	11.32680152	c2	-481.576139
045M	169006.9743	19.039156	112250.6816	47.72069798	a3	1.70862E-08
047M	165756.7301	30.49084706	102781.7272	65.50259603	b3	-2.58495E-08
084M	182669.5972	31.44998403	126796.2897	69.55395748		
419F	148134.9746	-3.684260761	93924.79923	5.921158974		
148T	184367.8426	-119.0654525	211517.0328	10.93690176		
359P	157007.3011	-158.4471166	166037.7438	-114.5056559		
523B	149533.7237	-53.13372792	113617.8341	-56.83914574		
441F	142382.3762	-15.58874631	91067.28769	-13.89461718		
043M	183610.0621	-9.562006899	150330.3714	18.46562356		
087M	180494.0738	-4.375003163	141747.4758	20.94398805		

Palestine 1923 Grid(new), after deleting point (42K & 44M) because it contained high residual						
	E	Residual (E)	N	Residual (N)	Parameter	Value
042K	160664.3455		202316.6327		a1	0.999918805
132T	168551.5236	-0.176538145	202361.6151	-0.089270296	b1	-6.76443E-05
341P	185958.9233	-0.076747666	162323.2676	0.263312267	c1	-53.55375363
044M	197429.7611		149500.7549		a2	3.40833E-05
1329	197335.0588	-0.040800765	197773.8102	-0.094078914	b2	1.000029288
352P	188723.2517	0.051338801	175486.6402	-0.064183197	c2	-47.44735334
045M	168988.1252	0.125312251	112203.0591	0.054421095	a3	-1.67511E-10
047M	165726.3244	0.024627612	102716.1529	-0.111044116	b3	-1.5966E-12
084M	182638.2105	-0.019433906	126726.8357	0.041642603		
419F	148138.7422	0.042140933	93918.8921	-0.012294388		
148T	184487.1853	0.184956774	211506.3029	0.097978567		
359P	157165.9194	0.119013499	166152.3404	0.036070513		
523B	149586.7964	-0.103840621	113674.6423	-0.061516786		
441F	142397.9783	-0.022095499	91081.28767	0.083016364		
043M	183619.6376	-0.072557504	150311.917	-0.058043837		
087M	180498.4946	-0.035375763	141726.5086	-0.086009876		

- the maximum value of residual = 0.2633 m, minimum value of residual = -0.1765 m, Standard error = 0.0974 m

5.4.4. 3D- Linearized-Helmert

by this method you can convert from coordinate system to any another coordinate system by using a matrix function, then table below shows the result of conversion between WGS84 and Palestine 1923Grid coordinate system in the trig of West Bank, in this result the elevation of all point was deleted, because causes high residual between the two coordinates, which the height in Palestine 1923Grid was approximated.

Table (5.14):New Cartesian Coordinate and residual of Trig in West Bank by using 3D- Linearized-Helmert

Palestine 1923 Grid (new)								
	X	Residual (X)	Y	Residual (Y)	Z	Residual (Z)	Parameter	Value
042K	4409180.466	-383.3832161	3099844.147	384.9335004	3399069.847	146.0118812	(S)	0.999853015
132T	4404613.534	75.88628189	3106272.988	-97.00137028	3399121.274	-9.199474211	(theta 1)	-0.001054731
341P	4411965.08	29.44762608	3132835.136	-61.63025374	3365270.997	18.26052602	(theta 2)	-0.000754329
044M	4410857.222	65.83169093	3146113.509	102.0629375	3354392.128	-181.9146121	(theta 3)	-0.000808478
1329	4389965.72	56.00597965	3131167.384	-108.6071111	3395252.923	26.94115568	(TX)	909.517036
352P	4404659.472	40.1023448	3131038.584	-78.06090054	3376417.933	19.527001	(TY)	459.9754436
045M	4443322.538	-7.215438994	3134286.864	5.792213227	3322683.331	4.075289894	(TZ)	278.4733335
047M	4449252.109	-13.96185845	3134496.898	18.51441914	3314596.013	1.191660728		
084M	4429219.751	-1.326547424	3141012.344	-18.95749563	3335053.359	19.00891834		
419F	4463122.301	-12.28929766	3122778.409	38.86264722	3307053.374	-19.45332823		
148T	4391404.374	75.35424369	3116435.705	-116.5296528	3406847.519	9.302051992		
359P	4426929.483	48.68781536	3108153.723	-48.85777364	3368482.656	-18.12014396		
523B	4453868.162	5.04754688	3117984.168	15.33400169	3323894.769	-20.39307688		
441F	4467626.104	-11.67949633	3118936.691	45.37517582	3304613.483	-26.15235399		
043M	4418508.194	19.83550886	3134611.645	-46.45220065	3355086.419	16.79726982		
087M	4424013.112	13.65681682	3134692.532	-34.77813658	3347798.599	14.11723468		

Palestine 1923 Grid (new), after deleting point (42K & 44M) because it contained a high residual								
	X	Residual (X)	Y	Residual (Y)	Z	Residual (Z)	Parameter	Value
042K	4409100.46		3099936.471		3399088.605		(S)	1.000017202
132T	4404537.785	0.137769984	3106369.981	-0.00847351	3399130.308	-0.165562355	(theta 1)	-4.82534E-05
341P	4411935.497	-0.135984261	3132896.599	-0.167097331	3365253.059	0.322698169	(theta 2)	-4.03234E-05
044M	4410845.626		3146167.039		3354358.27		(theta 3)	-2.54953E-05
1329	4389909.856	0.142350466	3131275.986	-0.00507175	3395225.811	-0.171532	(TX)	105.63878
352P	4404619.338	-0.03167409	3131116.699	0.053366699	3376398.392	-0.013803732	(TY)	74.0398819
045M	4443329.582	-0.17101024	3134281.135	0.063182725	3322679.426	0.17025584	(TZ)	-288.900789
047M	4449266.053	-0.017391911	3134478.419	0.034726509	3314594.821	-0.000509262		
084M	4429220.944	-0.133062373	3141031.2	-0.100736579	3335034.617	0.266554664		
419F	4463134.701	0.111004327	3122739.576	0.029985263	3307072.661	-0.166190862		
148T	4391328.927	-0.092806366	3116552.466	0.231151315	3406838.142	-0.075688071		
359P	4426880.701	-0.093696996	3108202.703	0.121730795	3368500.77	-0.006091947		
523B	4453863.283	0.168146658	3117968.759	-0.07465559	3323915.002	-0.160143702		
441F	4467637.968	0.184088667	3118891.253	-0.062912282	3304639.458	-0.177033387		
043M	4418488.335	-0.024166605	3134658.019	-0.077558102	3355069.716	0.093940279		
087M	4423999.412	-0.04356726	3134727.273	-0.037638162	3347784.565	0.083106365		

- the maximum value of residual = 0.3227 m, minimum value of residual = -0.1770 m, Standard error = 0.1291 m

5.4.5. 3D- Conformal

by this method point can converted from three dimensional coordinate system to any other coordinate system, then the table below shows the result of conversion between WGS84 and Palestine 1923Grid coordinate system in the trig of West Bank, in this result the elevation of all point was deleted, because it causes a high residual between the two coordinate, which the height in Palestine 1923Grid was approximated.

Table (5.15):New Cartesian Coordinate and residual of Trig in West Bank by using 3D- Conformal

Palestine 1923 Grid								
	X	Residual (X)	Y	Residual (Y)	Z	Residual (Z)	Parameter	Value
042K	4409180.467	-383.3824905	3099844.148	384.9340061	3399069.847	146.0124324	theta 1	0.0010544
132T	4404613.535	75.88684031	3106272.988	-97.00098204	3399121.274	-9.199050227	theta 2	0.000754755
341P	4411965.08	29.44754737	3132835.136	-61.63030338	3365270.997	18.26046638	theta 3	0.000808111
044M	4410857.221	65.83121122	3146113.508	102.0626041	3354392.127	-181.9149769	S	0.999854182
1329	4389965.72	56.00584467	3131167.384	-108.6072127	3395252.923	26.94105186	TX	904.482512
352P	4404659.472	40.10230077	3131038.584	-78.06092866	3376417.933	19.52696738	TY	456.0789905
045M	4443322.537	-7.215712824	3134286.864	5.792023062	3322683.331	4.075082153	TZ	274.6158161
047M	4449252.108	-13.96220733	3134496.898	18.5141728	3314596.012	1.191395745		
084M	4429219.75	-1.326928914	3141012.343	-18.95775895	3335053.359	19.00862867		
419F	4463122.3	-12.28943492	3122778.409	38.8625444	3307053.374	-19.45343245		
148T	4391404.374	75.35446754	3116435.705	-116.529506	3406847.52	9.30222079		
359P	4426929.483	48.68840388	3108153.724	-48.8573532	3368482.656	-18.11969579		
523B	4453868.162	5.047693406	3117984.168	15.33410645	3323894.769	-20.39296478		
441F	4467626.104	-11.67957431	3118936.691	45.37511225	3304613.483	-26.15241323		
043M	4418508.194	19.83537456	3134611.645	-46.45228819	3355086.419	16.79716812		
087M	4424013.112	13.65666511	3134692.532	-34.77823612	3347798.599	14.11711985		

Palestine 1923 Grid (new), after deleting point (42K & 44M) because it contained high residual								
	X	Residual (X)	Y	Residual (Y)	Z	Residual (Z)	Parameter	Value
042K	4409100.46		3099936.471		3399088.605		theta 1	4.82526E-05
132T	4404537.785	0.137768645	3106369.981	-0.008474117	3399130.308	-0.16556151	theta 2	4.03243E-05
341P	4411935.497	-0.135983126	3132896.599	-0.167096968	3365253.059	0.322698138	theta 3	2.54944E-05
044M	4410845.626		3146167.039		3354358.27		S	1.000017204
1329	4389909.856	0.142351625	3131275.986	-0.005071725	3395225.811	-0.171531877	TX	105.6281014
352P	4404619.338	-0.03167302	3131116.699	0.05336695	3376398.392	-0.013803658	TY	74.03014019
045M	4443329.582	-0.171009837	3134281.135	0.063182978	3322679.426	0.170255426	TZ	-288.9040207
047M	4449266.053	-0.017391733	3134478.419	0.034726664	3314594.821	-0.000509773		
084M	4429220.944	-0.133060955	3141031.2	-0.100736051	3335034.617	0.266554182		
419F	4463134.701	0.111002971	3122739.576	0.029984936	3307072.661	-0.166191161		
148T	4391328.927	-0.092806665	3116552.466	0.231150838	3406838.142	-0.075687501		
359P	4426880.701	-0.093698386	3108202.703	0.121730497	3368500.77	-0.006091277		
523B	4453863.283	0.168145339	3117968.759	-0.074655809	3323915.002	-0.160143672		
441F	4467637.968	0.184086789	3118891.253	-0.062912786	3304639.458	-0.177033622		
043M	4418488.335	-0.024165436	3134658.019	-0.077557671	3355069.716	0.093940136		
087M	4423999.412	-0.043566211	3134727.273	-0.037637734	3347784.565	0.083106169		

- the maximum value of residual = 0.3227 m, minimum value of residual = -0.1770 m, Standard error = 0.1291 m

5.4.6. Moldenesky

In this method you can convert directly between two geographic coordinate systems without converting to an X, Y, Z system, in the table below shows transformation from WGS 84 to Palestine 1923Grid of Trig in West Bank, in this point the height was deleted because its increases the residual.

Table (5.16):New Polar Coordinate and residual of Trig in West Bank by using Moldenesky

Palestine 1923 Grid (new)								
	phi		Residual (m)	Lamda		Residual (m)		
	Radian	degree		Radian	degree		Parameter	Vlue
042K	0.565717291	32.41321316	3.908454366	0.612726852	35.10666264	27.84718842	s	-71.30367262
132T	0.565733341	32.41413277	-6.39346519	0.614206407	35.19143489	-20.6027827	rx	-0.037136824
341P	0.559456953	32.05452222	6.203550845	0.617472026	35.37854106	1.13409137	ry	0.585591793
044M	0.55742831	31.93828954	-15.18846028	0.619591227	35.49996232	11.78478636	rz	-1.109273506
1329	0.565023573	32.37346604	1.870970902	0.619613338	35.50122922	10.95314423	Tx	-926246.4066
352P	0.561526423	32.17309412	5.395989301	0.617992027	35.40833491	3.308751491	Ty	-5196039.808
045M	0.551558077	31.60194995	0.935071869	0.614320124	35.19795035	-6.528293053	Tz	-3531547.338
047M	0.550059218	31.51607168	-1.524444147	0.613719864	35.16355802	-7.064944481		
084M	0.553845752	31.73302411	2.135047101	0.616835047	35.34204484	-5.905261298		
419F	0.548668802	31.43640669	-2.45774045	0.61051483	34.97992307	3.503387407		
148T	0.567174785	32.49672141	-6.722364844	0.617195535	35.36269928	-13.3715387		
359P	0.560050341	32.08852084	1.651595661	0.612122217	35.07201958	-11.59954183		
523B	0.551788986	31.61518006	2.06831613	0.610770121	34.99455018	1.960683525		
441F	0.54821815	31.41058627	-3.278550074	0.609472797	34.92021902	8.303674857		
043M	0.55756637	31.94619978	5.957240338	0.61703229	35.35334603	-0.938895413		
087M	0.556214417	31.86873861	5.437703082	0.61645055	35.32001477	-2.784450181		

Palestine 1923 Grid (new), after deleting point (42K & 44M) because it contained high residual								
	phi		Residual (m)	Lamda		Residual (m)		
	Radian	degree		Radian	degree		**Parameter**	**Vlue**
042K	0.565719987	32.41336764		0.612764516	35.10882058		**s**	-72.17002934
132T	0.565733979	32.41416933	-0.305050769	0.614236413	35.19315409	-0.256848312	**rx**	0.10436441
341P	0.559455907	32.05446231	0.286214376	0.617477088	35.37883107	0.172706599	**ry**	0.437933124
044M	0.557423832	31.938033		0.619586055	35.49966599	0	**rz**	-1.154540088
1329	0.565019961	32.3732591	0.170112303	0.619615645	35.50136136	0.257363027	**Tx**	-1264728.95
352P	0.561525232	32.1730259	0.264561525	0.617997965	35.40867516	0.226149385	**Ty**	-5860107.57
045M	0.551552769	31.60164583	-0.081704197	0.614318254	35.19784321	-0.221560484	**Tz**	-2443681.771
047M	0.550052066	31.51566192	-0.261954329	0.613715187	35.16329003	-0.276265339		
084M	0.553840101	31.7327003	-0.170764513	0.616832223	35.34188305	-0.21598693		
419F	0.548666172	31.43625603	-0.199800033	0.610512046	34.97976358	0.111374462		
148T	0.567170664	32.49648533	-0.287032666	0.617211967	35.3636408	-0.291974505		
359P	0.560057342	32.08892196	0.265117158	0.612149594	35.07358815	0.071170602		
523B	0.551791938	31.61534924	0.186642727	0.61077758	34.99497753	0.268975327		
441F	0.548217343	31.41054002	-0.1990128	0.609470331	34.9200777	0.283151712		
043M	0.557564746	31.94610673	0.284336911	0.617035832	35.35354895	0.102677089		
087M	0.556212354	31.86862041	0.245024571	0.616453513	35.32018456	-0.071307388		

- the maximum value of residual = 0.2862m, minimum value of residual = -0.3051 m, Standard error = 0.2269 m

- In the table (5.17) which the height of points was used in transformation, the residual changed in small degree, then you can conclude that the height can't cause an error in Moldensky method, because in Moldensky it is separate between lat, long, h.

Table (5.17):New Polar Coordinate and residual of Trig in West Bank by using Moldenesky

Palestine 1923 Grid (new)								
	phi			**Lamda**				
	Radian	**degree**	**Residual (m)**	**Radian**	**degree**	**Residual (m)**	**Parameter**	**Vlue**
042K	0.56572359	32.41357407	2.735918056	0.612723566	35.10647435	19.4930319	**s**	-70.69161806
132T	0.565732188	32.41406671	-4.475425633	0.614214627	35.19190585	-14.42194789	**rx**	-0.111082852
341P	0.559454927	32.05440617	4.342485592	0.617477566	35.37885845	0.793863959	**ry**	0.434451901
044M	0.557445656	31.93928341	-10.6319222	0.61955254	35.49774572	8.249350449	**rz**	-0.871024347
1329	0.565021508	32.37334776	1.309679631	0.619598748	35.50039327	7.667200958	**Tx**	-2188131.332
352P	0.561519752	32.1727119	3.777192511	0.618003473	35.40899074	2.316126043	**Ty**	-3901105.766
045M	0.551554599	31.60175068	0.654550309	0.614339735	35.19907402	-4.569805137	**Tz**	-3087772.259
047M	0.550054522	31.5158026	-1.067110903	0.613741684	35.16480822	-4.945461136		
084M	0.553850913	31.73331978	1.494532971	0.616835981	35.34209838	-4.133682908		
419F	0.548665756	31.43623219	-1.720418315	0.610510853	34.97969523	2.452371185		
148T	0.567165493	32.49618902	-4.705655391	0.61720773	35.36339801	-9.36007709		
359P	0.560056627	32.08888103	1.156116963	0.612111601	35.07141131	-8.119679284		
523B	0.551792683	31.6153919	1.447821291	0.610752838	34.99355993	1.372478468		
441F	0.548214096	31.41035399	-2.294985052	0.609463065	34.91966138	5.8125724		
043M	0.557563052	31.94600967	4.170068237	0.617045939	35.35412808	-0.657226789		
087M	0.556215568	31.86880456	3.806392157	0.616455304	35.3202872	-1.949115127		

Palestine 1923 Grid (new), after deleting point (42K & 44M) because it contained high residual								
	phi			Lamda				
	Radian	degree	Residual (m)	Radian	degree	Residual (m)	**Parameter**	**Vlue**
042K	0.565735479	32.41425527		0.612755491	35.1083035		**s**	-72.05836824
132T	0.565743839	32.41473428	-0.2135355	0.614242627	35.1935101	-0.179793819	**rx**	-0.114647706
341P	0.559450344	32.05414358	0.200350064	0.617476222	35.37878145	0.120894619	**ry**	0.534164873
044M	0.557421512	31.93790005	0	0.619519319	35.49584233		**rz**	-0.953473261
1329	0.565015811	32.37302134	0.119078612	0.619592866	35.50005624	0.180154119	**Tx**	-1573073.476
352P	0.561518329	32.17263039	0.185193067	0.618006657	35.40917317	0.158304569	**Ty**	-4237886.419
045M	0.551552435	31.60162672	-0.057192938	0.614334602	35.19877988	-0.155092339	**Tz**	-3553697.745
047M	0.550053257	31.51573012	-0.18336803	0.613734945	35.16442211	-0.193385737		
084M	0.553838915	31.73263237	-0.119535159	0.61681808	35.34107273	-0.151190851		
419F	0.548668936	31.43641441	-0.139860023	0.610512846	34.97980944	0.077962123		
148T	0.567174125	32.49668363	-0.200922866	0.617224172	35.36434009	-0.204382153		
359P	0.560062938	32.08924263	0.185582011	0.612136613	35.0728444	0.070458896		
523B	0.551794358	31.61548786	0.130649909	0.610761123	34.99403465	0.188282729		
441F	0.548219552	31.41066659	-0.13930896	0.609469271	34.92001696	0.198206198		
043M	0.557558842	31.94576846	0.199035837	0.617044121	35.35402394	0.101650318		
087M	0.556209037	31.86843033	0.1715172	0.61644949	35.31995408	-0.049915172		

- the maximum value of residual = 0.2004m, minimum value of residual = -0.2135 m, Standard error = 0.1597m

CHAPTER 6

CONCLUSION AND RECOMMENDATION

1.1. Conclusion

1.2. Recommendation

CONCLUSION AND RECOMMENDATION

6.1. Conclusion

1- The process of converting coordinate systems was done in a way to fit and suit the requirement of the Palestinian surveying and operations using the different techniques of surveying.

2- the previous chapters it was shown and concluded in the results of converting between WGS84 and Palestine 1923grid, that the 3D conformal method was the best one between the mentioned systems from the side of having less residual, but it can't be used between the WGS84 and Palestine 1923grid because the Palestine 1923grid doesn’t have any heights and at the same time the WGS84 has heights taken in account. Moreover the equations of this method requires taking the heights in account in both sides of transforming systems, so it may have a perfect transformation process.

3- It was also concluded that in Moldensky method of transformation there is a separation between the coordination's (latitude, longitude and the height) therefore not having the height in one of the two transforming systems won't effect on the result of conversion, the table(6.1)shows the result of all method of transformation.

Table (6.1): Comparison between method

Comparison between method in West Bank				Comparison between method in Gaza			
	Max of residual	Min of residual	Standered error		Max of residual (m)	Min of residual (m)	Standered error (m)
2D-Affine	0.6083	-0.2883	0.1771	**2D-Affine**	0.1316	-0.1091	0.0683
2D-Conformal	1.0985	-1.3663	0.6788	**2D-Conformal**	0.3973	-0.2876	0.1903
2D-Projective	0.2633	-0.1765	0.0974	**2D-Projective**	0.0849	-0.0989	0.0511
3D-Linearized	0.3227	-0.177	0.1291	**3D-Linearized**	0.1211	-0.1052	0.0739
3D-Conformal	0.3227	-0.177	0.1291	**3D-Conformal**	0.1211	-0.1052	0.0739
Moldensky	0.2859	-0.2814	0.2323	**Moldensky**	0.2862	-0.3051	0.2269
Moldensky - contain hight	0.2001	-0.2642	0.1708	**Moldensky - contain hight**	0.2004	-0.2135	0.1597

6.2. Recommendation

Using the result of this study the following are recommended:-

1- It recommended to set an official group of transformation parameter by level authority of Palestine, so that any surveyor, instrument or services companied will be forced to use the same parameter to graduate the result in any case

2- The point used in this transformation were measured in 1999, in is recommended to get new group of observation for these point, in additional, more densification point are recommended.

3- It recommended to set a modern and precise geoids model for Palestine, so that height in any coordinates system can be obtain easily precisely.