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RESEARCH ARTICLE - EARTH SCIENCES

The Integration of GNSS/Leveling Data with Global Geopotential Models to Define the Height Reference System of Palestine

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Abstract In GNSS, 3D geometric position is calculated with high accuracy. This position is the ellipsoidal latitude, longitude and ellipsoidal height (λ, ϕ, h) with respect to WGS84 coordinates system. These coordinates are integrated with local horizontal/projected coordinates (X, Y) by mathematical coordinate's transformations and map projections. The geometric ellipsoidal height (h) obtained by means of GNSS has to be integrated with the physical/orthometric heights (H) obtained by means of precise leveling. The physical surface defining the difference between both height systems is the geoid represented by the geoid undulation (N) at a given position. Different methods are used to build geoid models. Global models using terrestrial and satellite data are available to calculate the geoid heights as function of the earth potential (W). The most recent high degree and order models are EGM2008, Eigen05c, Eigen06c4, etc. (GFZ-Potsdam, 'List of available global models. http://icgem.gfz-potsdam.de/ICGEM/modelstab.html, 2017). Some regional geoid models are available like the European Gravimetric Geoid (EGG97). These models mostly do not fit the local datum due to datum definition problems. Here, a group of precise height reference benchmarks measured with GNSS is used to fit the global models with the local vertical datum to define the local height reference system of Palestine. The accuracy of the different global geopotential models is evaluated before and after the application of the geoid fitting.

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1 The Precise Leveling Network of Palestine

In Palestine, the precise leveling network of benchmarks was built from the end of 1920s to the middle of 1930s. The network was built by means of precise leveling with a start gauge point at Gaza to define the zero level/mean sea level (MSL). The loops were used to connect main cities with main interest of water sources leveling. The elevation differences were measured with level instruments without the use of gravity data. The position of these points was not documented by coordinates [2].

At the current time, the largest amount of these points is missing. Some points are with unknown positions. And the largest number of these points was destroyed due to urban expansions and the unavailability of update and review projects for these points/networks, see Fig. 1. The surveyors are used to define a local datum for engineering project by setting an arbitrary point as benchmark with a fixed elevation to be used in the differential leveling. In the last decade, GNSS became the main tool for surveyors in Palestine with RTK approach. To work with real time in GNSS, a predefined geoid model (height reference surface) became a necessity. Here, the leveling principle with GNSS started to get more complicated. The use of different instruments and different GNSS-service networks meant to get different elevations for the same points. The surveyors set the GNSS receiver to use the global models, local ellipsoidal heights as elevations and or the Israel geoid. All the previous methods used in GNSS leveling do not fit the vertical network of Palestine.

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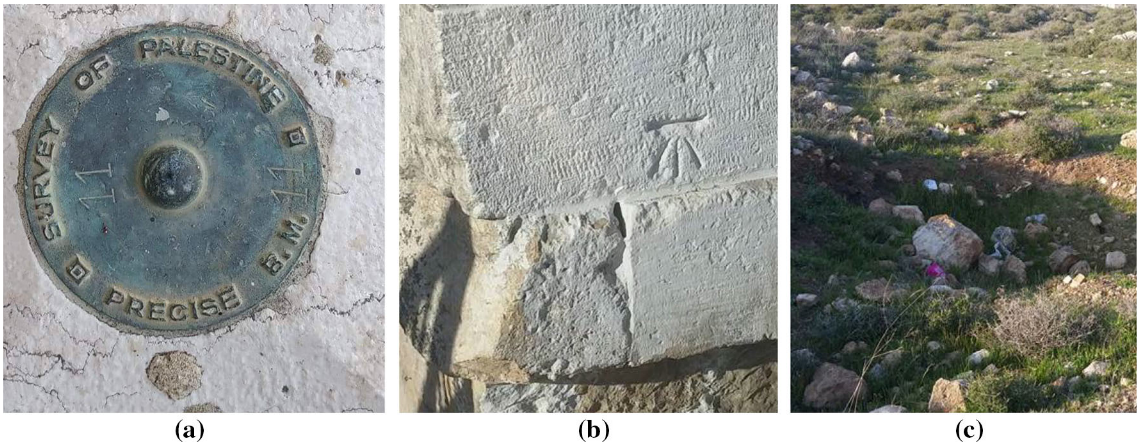

Fig. 1 A sample of benchmarks of Palestine. **a** Reference point with known elevation. **b** Marked on a wall but the elevation could not be officially found. **c** Completely destroyed

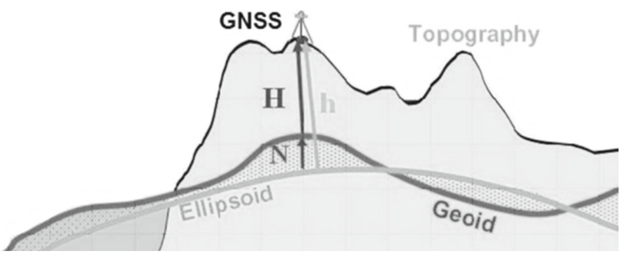

Fig. 2 Relation between ellipsoidal height (h), orthometric height (H) and the geoid undulation (N)

2 GNSS/leveling integration

The GNSS-measured elevations are the geometrical elevations above the reference ellipsoid (WGS84) called the ellipsoidal heights (h). These heights have to be integrated with the engineering measured physical heights above the equipotential surface (geoid) called orthometric heights (H) [3]. The integration is applied by defining the geoid as height reference system (HRS). This is achieved by modeling the difference between the measured GNSS ellipsoidal height (h) and the orthometric height (H) to get the geoid undulation/separation (N) given in Eq. (1), see Fig. 2 [4].

$$H = h - N \tag{1}$$

3 The Global Geoid Models

In global geoid models, the geoid heights are modeled as a function of earth potential (W), which is a result of the summation of the earth masses produced potential/gravitation potential (V) and the centrifugal potential (Ω) produced by the earth's rotation. These models are represented by means of spherical harmonics $\left(\bar{C}_{nm}, \bar{S}_{nm}\right)$ using combinations of satellite and terrestrial data (terrestrial gravity, height fitting points, marine gravity, satellite altimetry, etc.) as given in equation (2) [5].

$$\mathrm{V}(r, \bar{\phi}, \lambda) = \frac{GM}{r} + \frac{GM}{a} \sum_{n=2}^{n\_\max} \left(\frac{a}{r}\right)^{n+1} \sum_{m=0}^{n} \left(\bar{C}_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda\right) \bar{P}_{nm}(\sin\bar{\phi}) \tag{2}$$

In Eq. (2), the gravitational potential V acts as function of the spherical coordinates $(r, \bar{\phi}, \lambda)$. GM is the gravitational constant of the earth defined by the model. a is the semimajor axis of the earth as defined by geoid model. n_{max} is the maximum degree and order of the model. $\left(\bar{C}_{nm}, \bar{S}_{nm}\right)$ are the normalized spherical harmonic coefficients [6]. $\bar{P}_{nm}(\sin\bar{\phi})$ is the normalized Legendre function of integer degree (m) and order (n) given recursive with the abbreviations $t = \sin\bar{\phi}$ and $u = \cos\bar{\phi}$ in Eq. (3a–e) [7]:

$$\bar{P}_{n,m} = a_{nm} t \bar{P}_{n-1,m} - b_{nm} \bar{P}_{n-2,m} \tag{3a}$$

$$a_{nm} = \sqrt{\frac{(2n-1)(2n+1)}{(n-m)(n+m)}} \tag{3b}$$

$$b_{nm} = \sqrt{\frac{(2n+1)(n+m-1)(n-m-1)}{(n-m)(n+m)(2n-3)}} \tag{3c}$$

$$\bar{P}_{0,0} = 1 \quad , \quad \bar{P}_{1,0} = \sqrt{3}t \quad , \bar{P}_{1,1} = \sqrt{3}u \tag{3d}$$

If $n = m$, then $\bar{P}_{n,m}$ reads:

$$\bar{P}_{m,m} = u\sqrt{\frac{2m+1}{2m}} \bar{P}_{m-1,m-1} \tag{3e}$$

The earth potential (W) at a given point is defined as the numerical summation of the gravitational potential (V) calculated by the model coefficients in Eq. (2) and centrifugal